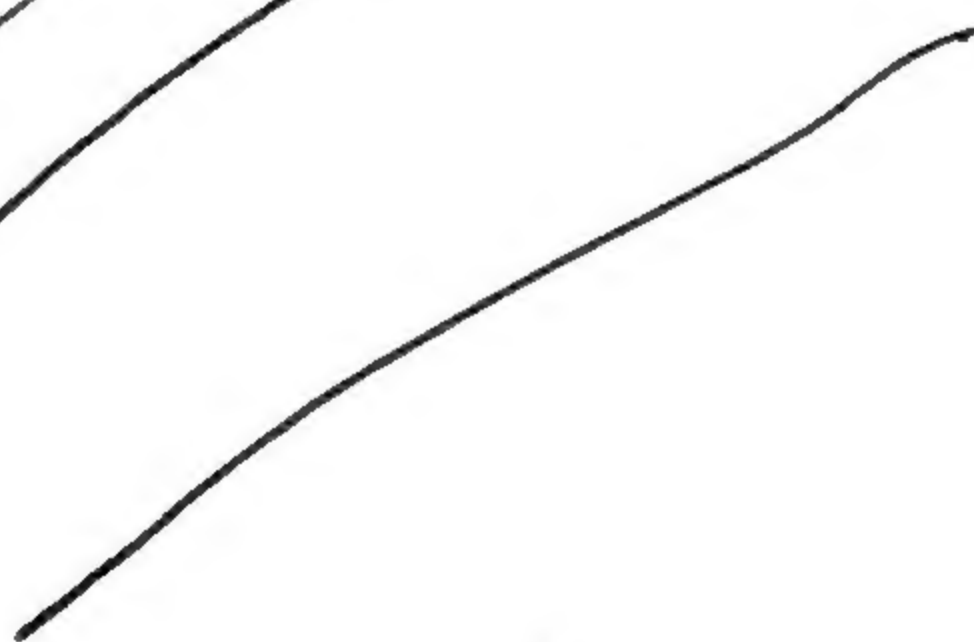


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**Color
in Business,
Science,
and Industry**

Color in Business, Science, and Industry

DEANE B. JUDD

*Photometry and Colorimetry Section
National Bureau of Standards*

1958
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To My Wife

PREFACE

I can make no claim of intimate knowledge of all, or even most, of the color problems arising in industry. It has been my privilege, however, in my twenty years at the National Bureau of Standards, to come into contact with hundreds of colorimetric sore spots in our industrial life. I have seen victories that paid off in dollars and cents won by applying the sciences of mathematics, physics, and psychology to these problems. In particular, a whole new science has been developed, largely since the turn of the century, applying to many of these color problems. This is the science of visual psychophysics. The key to color problems of the future is to be found in visual psychophysics mixed with a liberal sprinkling of common sense. This book is an attempt to present visual psychophysics in terms that are practically useful.

I am indebted to the Committee on Colorimetry, Optical Society of America, for developing under the able direction of Dr. L. A. Jones, Eastman Kodak Company, the system of color terms used in this book. Without it I could not say what I have to say.

Grateful acknowledgment goes to my many friends in the Inter-Society Color Council whose frank answers and equally frank questions have both been valuable in revealing the polyglot role of color measurement in industry. My particular thanks are due to Miss Dorothy Nickerson, Secretary, Inter-Society Color Council, to Dr. K. S. Gibson, my immediate superior at the National Bureau of Standards, to Prof. I. A. Balinkin, Chairman, Inter-Society Color Council, to Mr. Walter C. Granville, Container Corporation of America, to Dr. E. I. Stearns, American Cyanamid Company, and to Mr. Carl E. Foss, Color Consultant, for taking time from their busy schedules to read and suggest improvements in the early drafts of *Color in Business, Science, and Industry*.

DEANE B. JUDD

Washington, D. C.
March, 1952

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Introduction

The judgment of color permeates our industrial life. The farmer judges the worth of farm land by the color of the soil. He judges the worth of fertilizer by the color of growing crops. His vegetables and fruits are often graded and priced according to color. His prime beef and pork are identified by their colors. The miner selects the commercially valuable minerals by color and discards those having no commercial value in the same way. Mica is graded by color. Ruby mica is much in demand; nonruby mica brings a much lower price.

The manufacturer is both buyer and seller. He judges the worth and uniformity of his raw material by its color; and his product is judged in turn by its color. Raw wool, cotton and silk are graded and priced according to color. In this way the textile manufacturer can produce goods of uniform color at a price low enough to sell. Building stone, sand, gravel, lumber, and plaster are color graded. The architect has to know these colors so that the structures erected according to his design will satisfy the customer. Oils and resins used for paint are measured by their colors with extreme care, and pigments and dyes likewise. The refining of crude oil to produce the hundreds of lubricating oils, liquid fuels, and by-products, is measured by the color changes produced. Intermediates in this complicated process are bought and sold according to prices determined by color. Vegetable oils are judged by color. Poor-quality cottonseed oil, peanut oil, coconut oil, and olive oil can often be detected by color; and light-colored cooking oils bring a premium price apart from quality because bakers know they can sell the resulting light-colored cakes and pies without trouble.

America excels at mass production. To make a complicated machine like a refrigerator or automobile takes a whole series of steps, each one performed by a specialist. In each of these steps the color question may, and often does, arise for the article or part both before and after processing. The way we have solved the problems of color measurement and control in mass production is the envy of the rest of

the world. Take a refrigerator for example. The walls may be of sheet steel coated with white vitreous enamel; the door to the freezing compartment, of white plastic. It is no accident that these two whites are alike. To get this symbol of the machine age into our houses at a price we can afford, a hundred thousand plastic doors have to come out the same white, and not be one of the several hundred other whites and near-whites that the eye can distinguish. The essence of mass production is interchangeable parts; any one of the 100,000 doors must be suited to any one of the 100,000 refrigerators in size, shape, and color.

The upholstery of your automobile is not a one-piece job. Pieces cut from different rolls of cloth have to be sewed together. It is not by luck that they seem to have been cut out of the same piece of goods. Only by color control on a large scale can customer-pleasing merchandise be produced at the prices that will move this merchandise in large quantities. Color measurement is an integral part of the miracle of American business.

The color of the finished product takes on this great importance only in a land whose business is organized on the principle that the customer must be pleased. In a competitive system such as ours, in which we usually have a wide choice, no manufacturer can afford to overlook the fact that the act of purchasing is an emotional experience. We may not even notice that the upholstery on the back seat fails to match that on the front, but we get a vague impression of misfit that often completely nullifies our enthusiasm to buy.

Groceries packaged in faded cartons are continually pushed aside in favor of those in brightly colored cartons. Sometimes we think the product may be stale; more often we reach as a matter of habit for the carton having the colors we are used to. On this account color tolerances on trade-marked containers have to be remarkably small.

The importance attached by the American consumer to the emotional effect of color is well illustrated by the trouble taken to color oleomargarine yellow. The food value is not enhanced by the addition of the yellow dye, but the consumer acceptance is. And, if the yellow color keeps the family from feeling slightly nauseated at table, who can say it is not worth the trouble! The colors of consumer goods profoundly influence consumer acceptance. Some, like textiles, cosmetics, and paint, we buy primarily and purposely for the emotional effect produced by the colors. Others we buy primarily for other purposes, and often respond to their colors without knowing it.

In the complicated network of processes that make up our economic life, we use our eyes time and again to choose among competing

products, be they raw materials from below and above ground or processed materials. To carry out this inspection, color measurement, and selection, there have been developed various tools and techniques. It is the purpose of this book first to tell how the eye works, second to list the tools available to assist in color measurement, and third to show how to select the best tool for a given color-measuring job. Even though American colorimetry is the envy of the rest of the world, and even though we excel at applying it to mass production, there is room for great improvement. Furthermore, it is the essence of our industrial system that no one firm shall have a monopoly in any given activity. New companies spring up whenever there seems to be a chance to improve the efficiency of producing a product. Other firms already established continually take on new enterprises. In this continual readjustment springing from our freedom to embark on new enterprises lies our hope for a continuously increasing standard of living. Color inspection, measurement, and control play important parts in reducing industrial waste; and it is to be hoped that this book may supply to the American business man and technologist the information required for this purpose.

PART I

Basic Facts

Color perception is what we see apart from variations in time and space. It does not refer to size, shape, texture, gloss, transparency, or flicker, though it is influenced by all of them. Color is what is immediately responsible for the color perception; it is one aspect of the light entering our eyes.

Color perception is one of the fundamental ways by which we find out about the things around us. To understand color we must know something about the human eye, how it is built, and how it works. We must know about the various kinds of eyes, the good ones, the defective ones, the well, and the sick—how they can give us amazingly detailed information, and how they can fool us. We must also know something about the physics of radiant energy—how the rays from a light source are altered by striking an object, and how the rays leaving the object are bent and altered on entering our eyes. We must understand the important part played by the chemist in developing, supplying, and controlling the materials (pigments and dyes) used to change the color of objects. For there is a chemical as well as a physical basis for color. But color, itself, is not purely physical or purely psychological. It is the evaluation of radiant energy (physics) in terms that correlate with visual perception (psychology). This evaluation rests squarely on the properties of the human eye.

THE EYE

Our eyes permit us to perceive size, shape, texture, gloss, transparency, flicker, and color. It was once remarked by a designer of photoelectric colorimeters that the human eye is much too good for designers to compete with at reasonable cost. It is provided with thermostatic equipment that keeps it at constant temperature to about plus or minus one degree centigrade. It has built into it a device (lids) that cleans off the refracting surface (cornea) many times a minute. The photocells have the correct spectral sensitivity by definition. And this equipment is standard, supplied to most of us at no

extra charge. The chief trouble with it is that it is connected, not to a well-behaved amplifier, but to a brain.

It is true that the eye in good working order does an amazingly versatile job and does it well. It seeks out objects of interest. It focuses an image of the object onto the light-sensitive layer (retina), protects this image from stray light, transforms this optical image into a pattern of nerve activity, and feeds the nerve-activity image to the brain by means of a cable (optic nerve). Figure 1 is a sketch showing the arrangement of parts that does this.

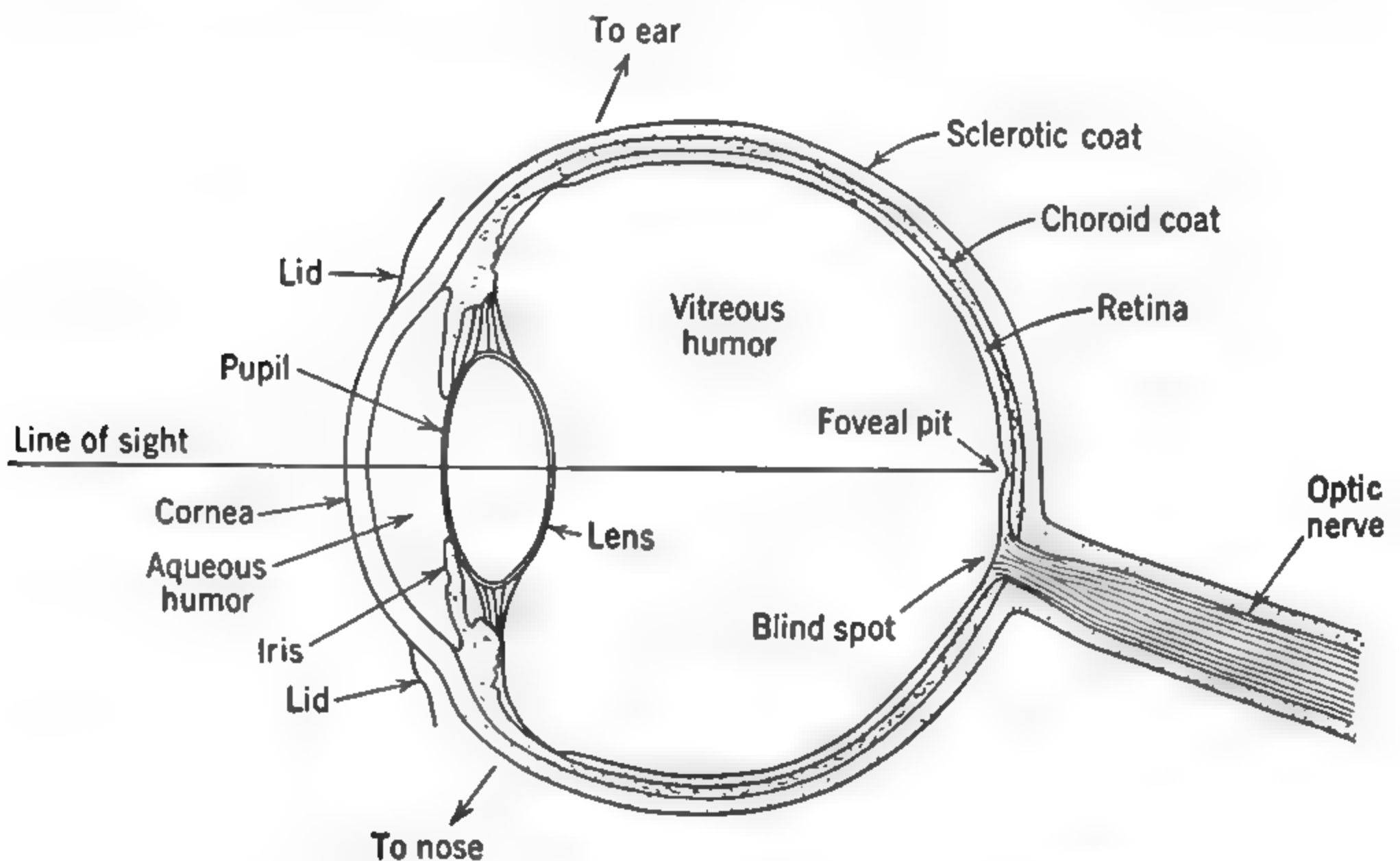


FIG. 1. Sketch of a horizontal cross section of the human eye.

Focusing Elements. The rays from an object strike the cornea and are thereby nearly focused on the retina. The focusing is completed by the lens. If there is plenty of light the iris diaphragm contracts so that the pupil is small and only the center of the lens is used. Exclusive use of this part of the lens gives the sharpest images.

Cornea. The cornea is a transparent extension of the sclerotic coat, itself an opaque white (the white of the eye). Its shape is closely like a part of a sphere, and its outer surface is kept in good optical condition by a film of salt water supplied by the tear ducts. This surface is frequently swept by winking of the eyelids, which carry away dust-polluted salt-water film and deposit a clean film. If an object becomes embedded in the cornea its presence is detected by nerve endings there and on the under side of the lids. The nerve endings are so sensitive that prompt measures usually are taken to remove the object and so restore the cornea to good optical condition.

If too much aqueous humor is fed into the space between the cornea and lens, the cornea bulges out and has so much curvature relative to the axial length of the eye that the image comes to a focus in front of the retina (nearsightedness). If too little pressure is built up by the aqueous humor the cornea flattens out so that the image comes to a focus behind the retina (farsightedness) or not at all. But in most eyes the cornea has a curvature nearly enough correct that the lens can make a precise final adjustment of the focus.

Pupil. The pupil is the hole in the iris diaphragm through which the light passes. Since very little of the light entering the eye ever gets out again, the pupil ordinarily looks black. The iris surrounding the pupil may be black, brown, green or blue depending on whether it has much, some, little, or no brown pigment in it. The blue color comes from selective scattering of light by the tissues of the iris. It is like the blue of the sky.

In dark adaptation, radial muscles pull the iris away from the center thus enlarging the pupil. The pupil of the dark-adapted eye may be as much as 8 mm in diameter. If either eye receives a sudden increase of light, the pupils of both eyes automatically become smaller. This comes from the contraction of the sphincter (circular) muscles around the inner edge of the iris. Because of this contraction only the best part of the eye's optical system, the center part, is used when there is enough light to spare. The result is that the retinal image is sharpened (spherical aberration reduced), and colored borders between light and dark areas (chromatic aberration) are almost wholly eliminated. By using an excess of light, such as by working in full sunlight, we make the best of the optical equipment of our eyes. But there is a price to pay. The lid muscles are automatically contracted by an excess of light. We may have to choose between maximally sharp vision with a headache and somewhat inferior vision without.

Lens. The lens is held in place by radial muscles tending to flatten it, and a sphincter muscle around the base of these radial muscles tends to take tension off the lens, which is only semi-rigid, and allows it to thicken again to its normal shape. To accommodate the eye for sharp vision of near objects the sphincter muscle must contract and so allow the lens to assume its normal thick shape. To accommodate the eye for sharp vision of distant objects, the sphincter must relax and allow the radial muscles to flatten the lens. It is normal for the lens to increase in rigidity with age so that it refuses to resume its normal thickness even with no pull from the radial muscles. Thus there comes a time when all of us need bifocals. The lens also yellows with age and sometimes degenerates by losing its transparency, as in cataract.

Prolonged exposure to infra-red energy from furnaces can cause this. As the lens becomes more and more cataractous, all objects in the visual field appear as through a fog, until finally the eye yields only the perception of color without detail. Removal of the lens by surgery restores detail vision, but high-power spectacle lenses must be used to bring the image back into focus on the retina, and, of course, the power of accommodation is lost. As already mentioned, the lens system of the eye suffers from defects known as spherical and chromatic aberration. Because of chromatic aberration it brings blue or violet rays to a focus at a point closer to the lens than the focal points for green, yellow, and red rays.

Vitreous Humor. The vitreous humor is a somewhat viscous fluid filling the interior of the eye. It maintains nearly constant the distance between lens and retina. Its optical quality is far from perfect and it usually has trash floating around in it or partly clinging to the retina. These bits of tissue degrade the retinal image particularly when they are near to the retina. They then cast visible shadows onto the image of any bright, uniformly colored surface, commonly appearing as nearly transparent strings of beads, intertwined snakes, and so forth, floating slowly around. In addition to these rather large pieces of light-scattering tissue, the vitreous humor also contains very fine light-scattering particles such as would make the scattered light appear blue. Its optical properties are therefore something like those of air filled with a combination of dust and tobacco smoke. In spite of these imperfections, the optical system of the eye can give about as sharp an image as can be appreciated by the receptor layer, or retina, itself.

Sensitive Elements. The photosensitive part of the eye is a mosaic of receptor cells, the rods and cones of the retina. The noses of the rods and cones are in close contact with the choroid coat away from the incident light. By means of the rods and cones the optical image is transformed into a pattern of nerve activity. Each central cone and each group of rods have a direct line to the brain by way of the inner surface of the retina and the optic nerve. In addition to these direct connections there are myriads of local cross connections in the retina. The light traversing the vitreous humor first penetrates the retinal layer of nerve tissue and blood vessels, then comes to the layer of rods and cones. The designer of a television camera based on such a mosaic of photocells would probably take care to have the wires come in from behind so as not to obstruct the light falling on the photocells. Not so, the eye. Instead the nerve tissue is between the light and

the rods and cones. This means that the nerve tissue has to be nearly transparent, which it is, and the blood vessels, which are not, have to be rendered invisible. This we will come to later.

Rods—Night Vision. The rods can respond to very small amounts of radiant energy. They are responsible for our ability to see by moonlight, starlight, or even with the stars obscured by clouds. The sensitivity of the rods is maximal at wavelength 510 $m\mu$ (green) in the

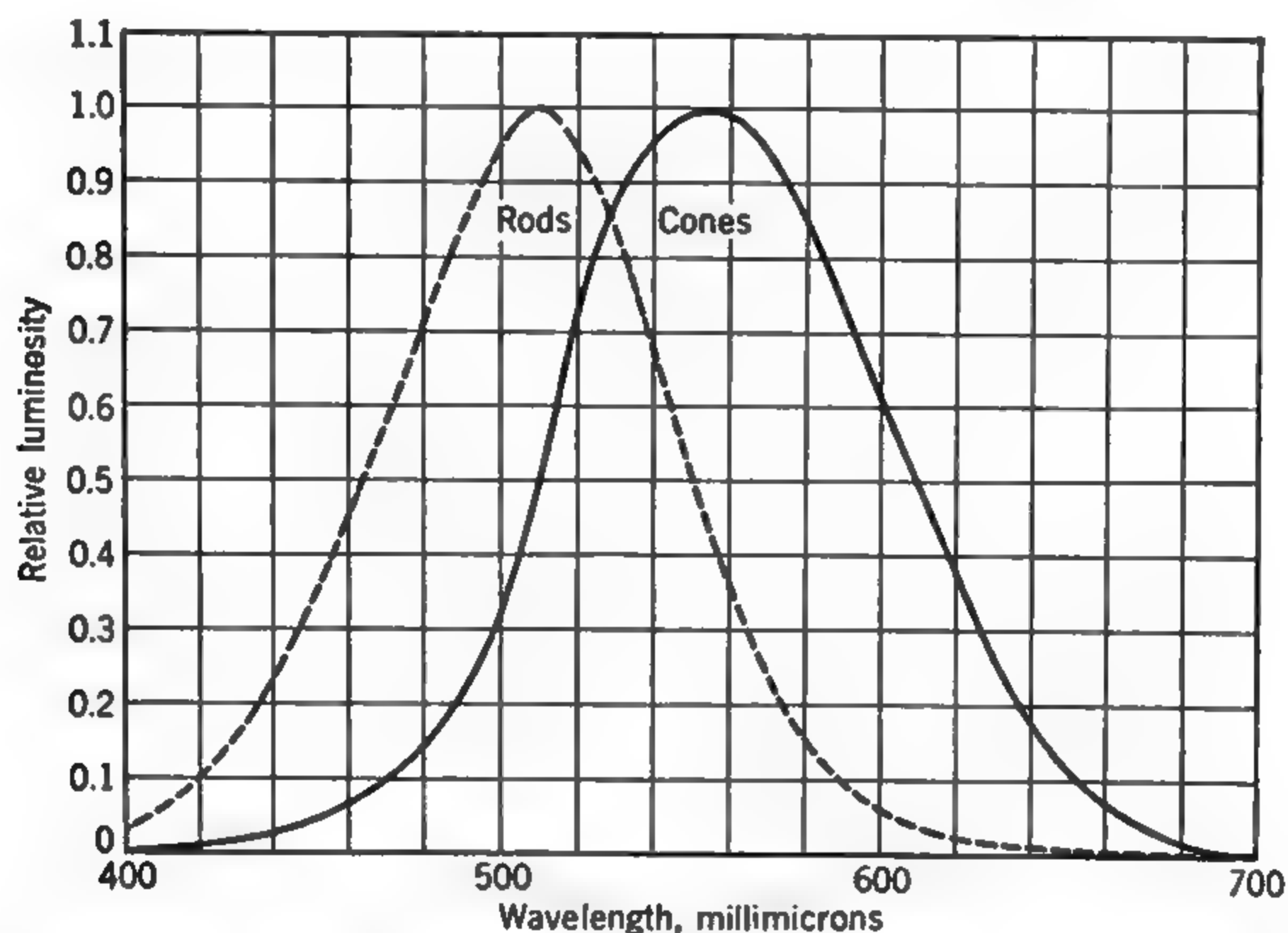


FIG. 2. Relative spectral sensitivities of the rods and cones. Rod vision (dotted curve) serves us in very dim light such as starlight; cone vision (solid curve), in daylight. The solid curve refers only to the light-dark aspect of daylight vision and is called the relative luminosity curve.

spectrum. Figure 2 (dotted curve) shows the dependence of sensitivity on wavelength. The rods, however, do not yield perceptions of red, yellow, green or blue. They yield only the achromatic or neutral color perceptions, white, gray and black. Furthermore, the rods do not have private lines to the brain, but are connected in bunches. This arrangement favors high sensitivity, but prevents seeing fine detail. These facts account for the generally gray and indistinct character of twilight vision. "At night all cats are gray."

In the foveal pit (1° in diameter) there are no rods, nor any in the central 2° field; but, from about this point on, the number of rods per unit retinal area increases relative to that of the cones until at the extreme periphery there are scarcely any cones. The rods reach maximum density about 20° from the fovea. The lack of any rods in the

very center of the eye where day vision is most acute has given rise to the saying, "To see a very weak light it is necessary not to look at it."

Cones—Day Vision. The cones have a more complicated response than the rods. Instead of simply detecting light and dark and giving us a series of grays, they also give us our perceptions of chromatic color. That is, by cone vision we can also see yellow-blue differences, and red-green differences. The wavelength distribution of light-dark sensitivity that cone vision gives us is shown in Fig. 2 (solid curve); it is called relative luminosity. The rods are much more sensitive to the short-wave (blue) part of the spectrum than the cones are and about equally sensitive to the long-wave (red) part, but the cones retain their sensitivity even when the density of radiant flux making up the retinal image is many times too high for the rods to respond to. We get our daylight vision almost wholly from the cones. The shift in wavelength distribution of sensitivity from cone (daylight) vision to rod (night) vision is called the Purkinje effect. This Purkinje shift, so called because of its discovery by Purkinje, in 1823, makes an object that looks red by day appear black by starlight, whereas a daytime blue object appears light gray.

The sensitivity of the rods is due to absorption of radiant energy by a photosensitive pigment in them called rhodopsin. Their insensitivity under daytime conditions comes from the fact that nearly all of the rhodopsin is bleached out while the eye is adapting to daylight. This bleaching goes on rapidly so that daylight adaptation may be completed within a few minutes. The only way that the sensitivity of the rods can be regained is to wait until the nutritive processes of the eye have built up the concentration of rhodopsin in the rods again. This takes about half an hour. If a sailor has to go on lookout duty at night, he must for about half an hour before that look at things that are dark for rod vision. He may do this by shutting his eyes, or by staying in a dark room; but he can also do it by wearing red goggles, that is, goggles that transmit only the long-wave end of the spectrum, say of wavelength greater than 600 m μ . By this transmitted red light he can read his instructions or play cards, but, since everything he looks at is rather dark as far as rod vision is concerned, the accumulation of rhodopsin in his rods goes on almost as fast as in a dark room. He can use his eyes and rest them at the same time. It is a great advantage to have two kinds of receivers (rods and cones) built into the same eye. Not all animals are so favored. Hens have only cones; they must roost at sundown. Owls have only rods; they must blink by day.

The cones, about 7,000,000 in all, are distributed over the whole retina except the place (blind spot) where the nerve fibers join and lead out of the eyeball to form the optic nerve (Fig. 1). They are most plentiful in the foveal pit where there are no rods. They are quite plentiful up to 5° from the center (parafoveal region) where only

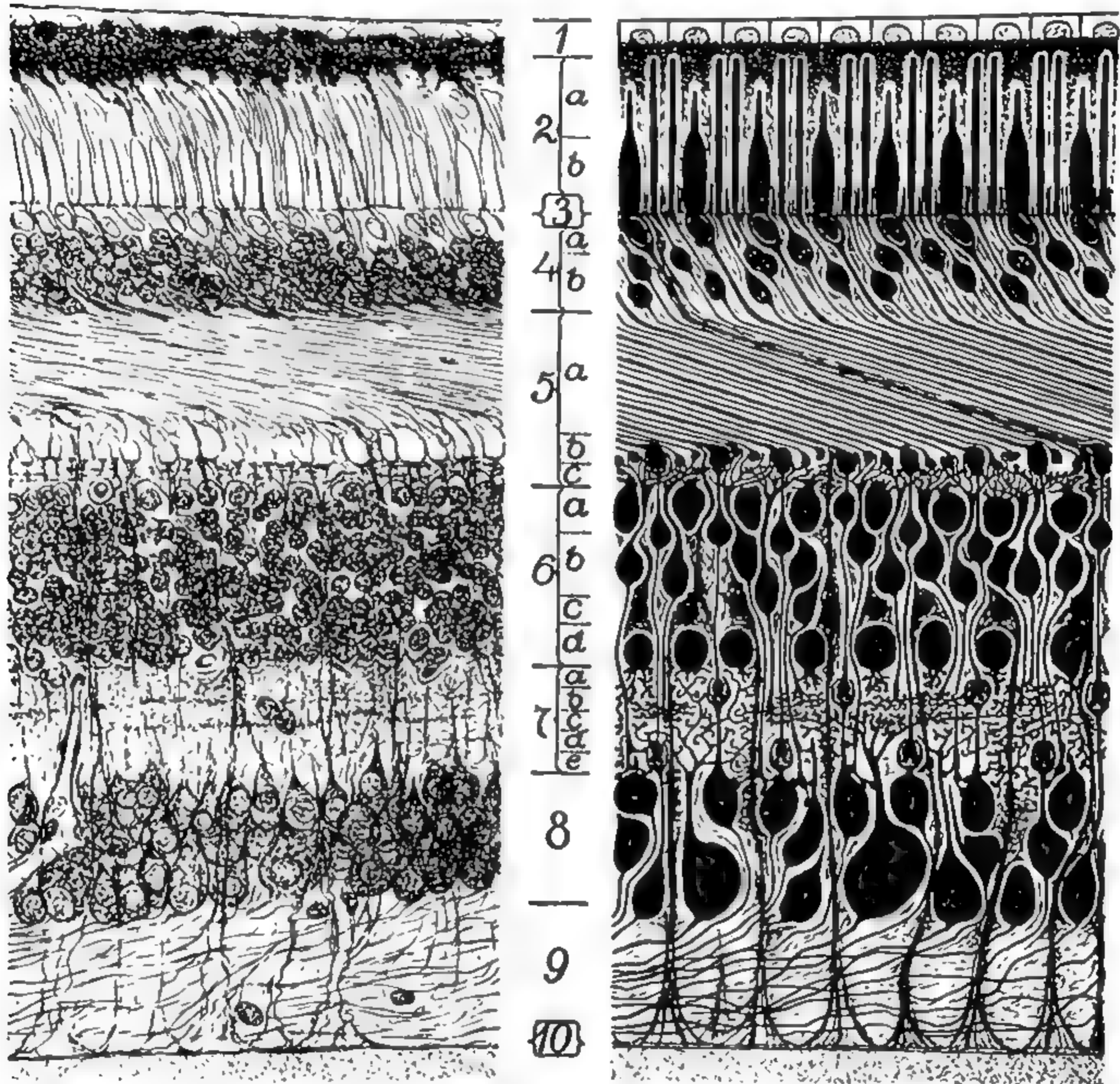


FIG. 3. Cross section of the retina near the foveal pit. (From Polyak, *The Retina*, 1941.)

a few rods are intermingled. And there is still an occasional cone mixed in with the rods in the extreme periphery of the retina used only for seeing out of the corner of the eye. Figure 3 (Polyak, 1941) is a cross section of the retina in a region where the ratio of rods to cones is about 4 to 1. The left half of Fig. 3 shows the cross section of the retina as it appears in the microscope; the right half is a diagram based on the left. The cones terminate with a brush of nerve ends giving plenty of chance for side connections. This structure corresponds to their complicated duties. In the center of the retina the cones are

very closely packed thus permitting the perception of fine detail. In fact, the optical system of the eye is such that closer packing would scarcely improve our visual acuity. The rod-free region (2° in diameter) is about one square millimeter in area and contains about 50,000 cones. Figure 4 shows that these "cones" have lost their char-

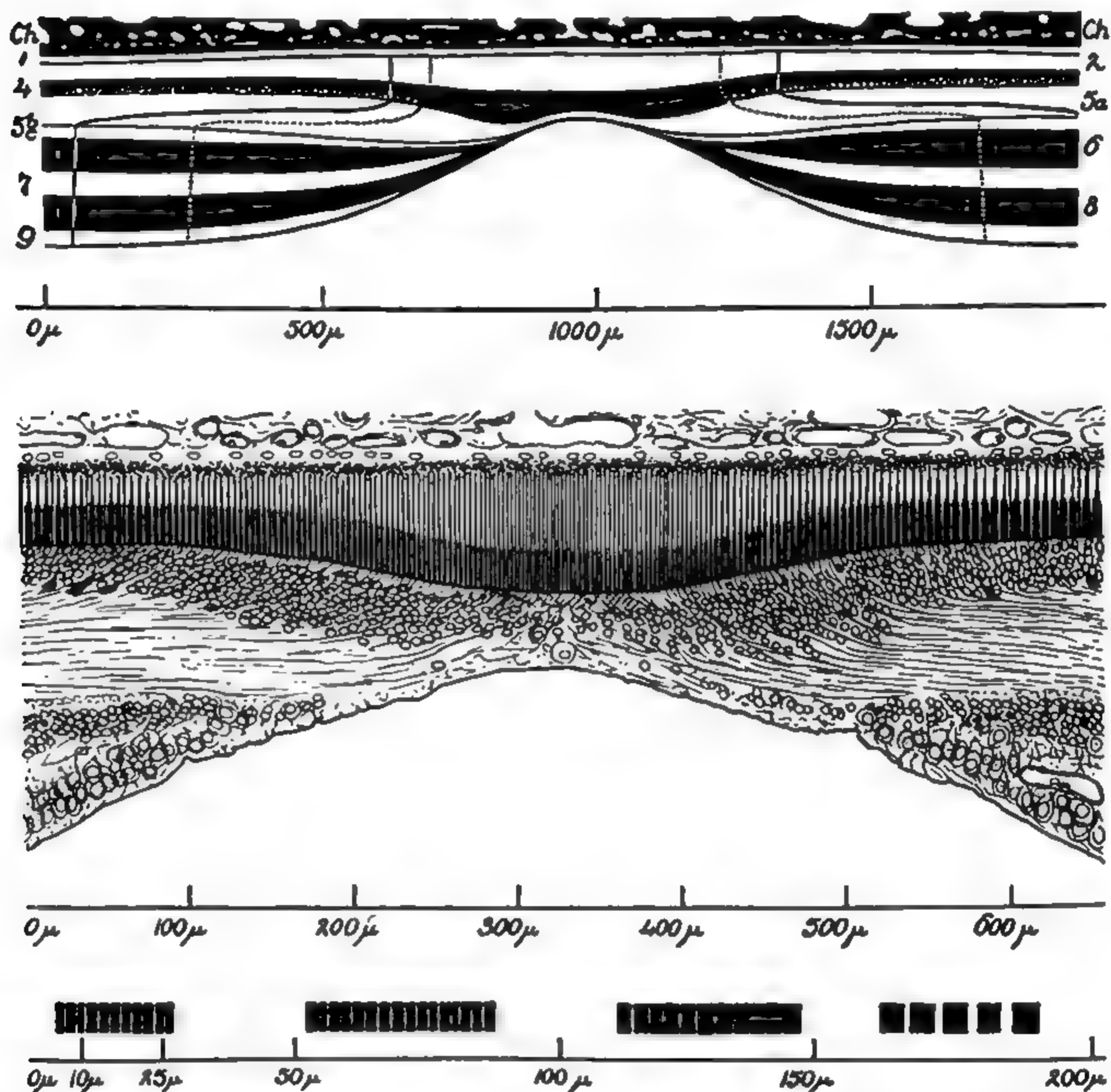


FIG. 4. Cross section of the foveal retina. (From Polyak, *The Retina*, 1941.)

acteristic shape by being squeezed to less than half the diameter of the more peripheral cones. But they have the brush of nerve ends and private lines to the brain. They are thus fitted to give us acute vision and color vision at the same time. Note from Fig. 4 how the nerve ends go off to the side leaving the very center with the thinnest possible layer of nerve tissue. This layer is also free of blood capillaries. The thinning out of this layer produces the foveal pit (Fig. 1). Upon the properties of this single square millimeter of retina with its more than 50,000 cones is built the measurement of color.

Rods and Cones—Twilight Vision. In twilight vision both rods and cones are active. This is the range of illumination corresponding to that from the sky with the sun more than a few degrees below the horizon and extends down to the illuminations produced by light from the half-moon high in a clear sky. It includes vision in dimly lit interiors such as those by candlelight. Because the relative participation of rod vision and cone vision continually varies, color judgments are unreliable in the extreme, yet sometimes manufacturers permit color inspections to be carried out in dim light. There are a few manufactured products that have to be evaluated by means of this mixed vision, however, because they are intended for use in dim light. An example is phosphorescent black-out paint. Materials that phosphoresce different colors can be very useful to people that have to work in the dim light of a military black-out or in the photographic dark room. But the measurement of these colors poses problems that have yet to be solved.

Eye Pigments. The radiant energy penetrating the cornea is not only brought to a sharp focus by the cornea and lens but also modified in spectral composition by pigments (lens, macula), transformed to nerve activity by pigments (rods and cones), and most of it is finally absorbed by pigment in the choroid coat.

Lens. Young children have clear, relatively pigment-free lenses. But as the years go on the lenses develop a (melanin-type) pigment like that in the skin. It is yellowish or brownish and prevents some of the incident energy, particularly the short-wave energy, from reaching the retina. Figure 5 shows for the various parts of the spectrum the fraction of incident energy transmitted by the ocular media of the average 21-year-old person (Ludvigh and McCarthy, 1938). The variations of transmittance with wavelength shown are partly due to the cornea and humors, but chiefly to the lens. Because of the accumulation of lens pigment, old inspectors may get results that young inspectors disagree with. Inspectors become less sensitive to violet and blue light as they grow older.

Macula. The nerve layer of the retina in and near the fovea between the vitreous humor and the cone layer is colored with a yellow or brownish pigment (Wald, 1945), probably xanthophyll. This brownish spot is known as the macula lutea (yellow spot), or simply as the macula, and the pigment is called the macular pigment. It can be seen by lighting up the retina strongly and looking into the pupil of the subject's eye by means of an ophthalmoscope, but it ordinarily is invisible to the subject himself. The lens pigment protects the whole retina from overstimulation by ultraviolet and violet energy, and the

macular pigment protects the all-important central part from overstimulation by blue light such as causes long-lasting afterimages. Figure 6 shows for the various parts of the spectrum the fraction of in-

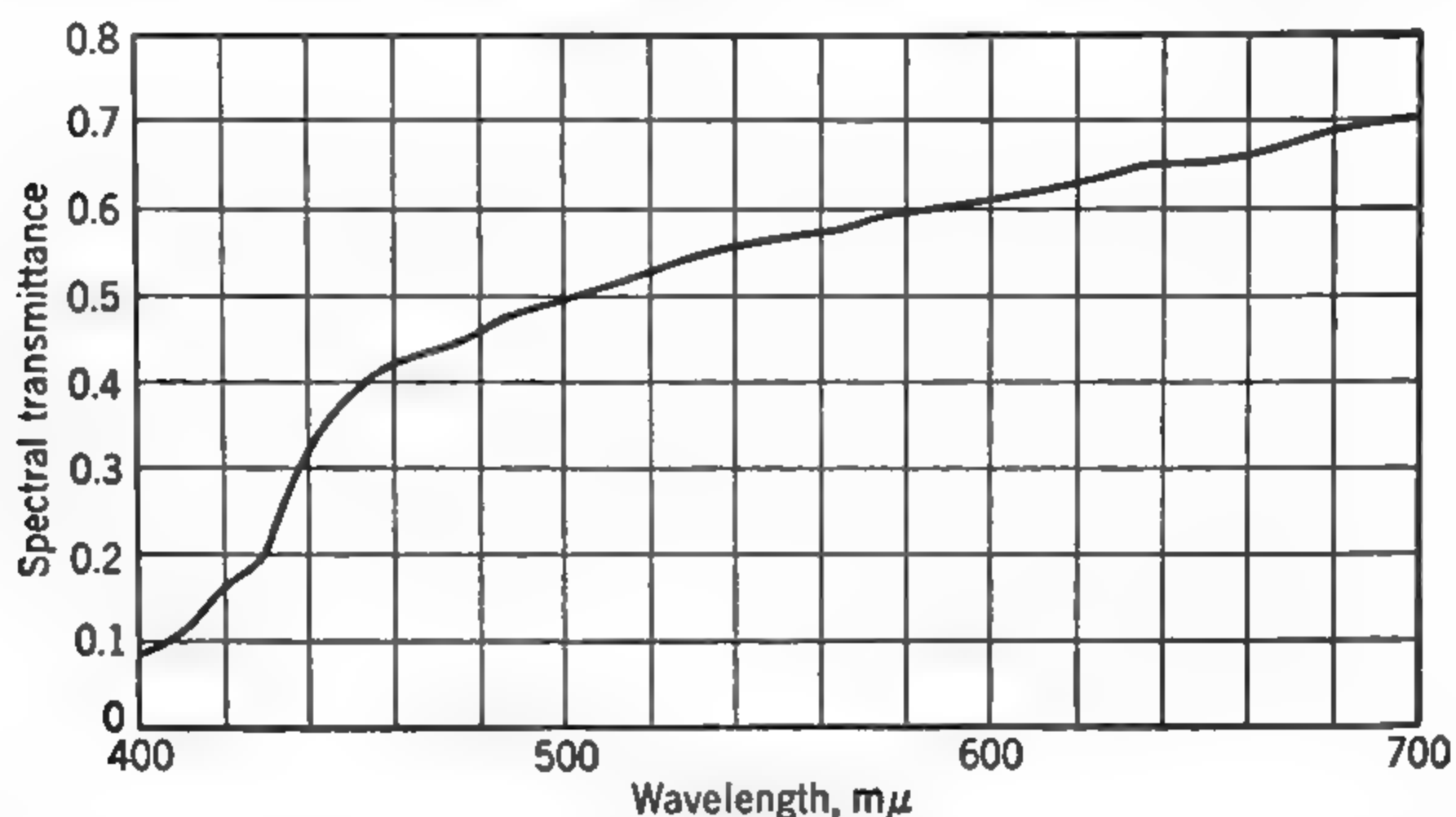


FIG. 5. Spectral transmittance of the ocular media, according to Ludvigh and McCarthy (1938). Macular pigmentation is not included.

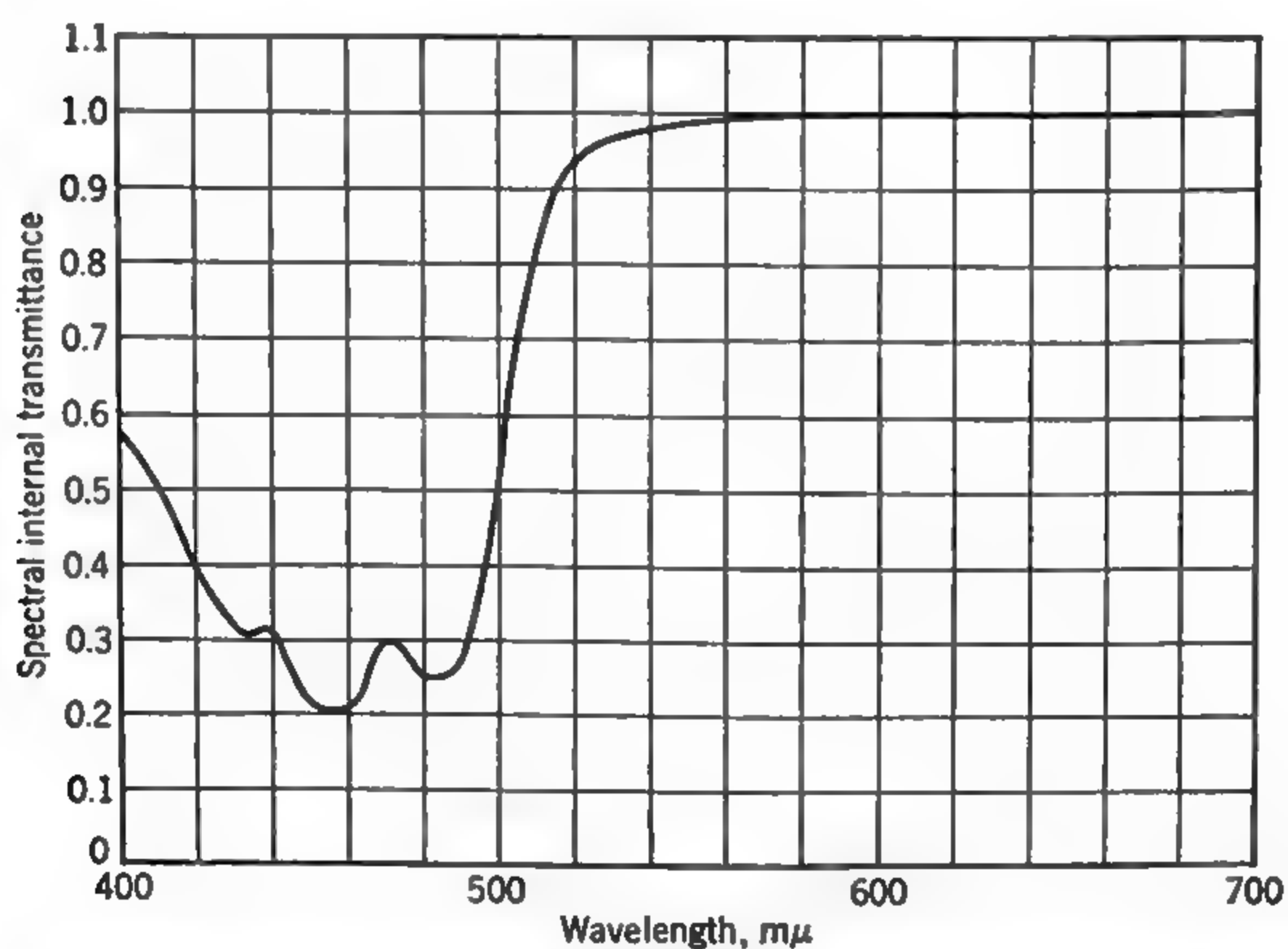


FIG. 6. Spectral internal transmittance of the macular pigment, according to Wald (1945).

cident energy transmitted by the macular pigment according to Wald (1945). Because of the macular pigment the cones in the center of the retina are permanently understimulated by short-wave (blue) energy, compared to the surrounding cones. Because of this permanent under-

stimulation, perhaps by a factor of 4, the central cones ordinarily have a super-sensitivity for short-wave energy by nearly the same factor. As a result, when we look at a uniform field we do not detect any central spot corresponding to the macular pigment. The field appears as it is, quite uniform.

There is only one basis on which the observer may see his own macular pigment. That is to catch the central cones before they have had a chance to get all their super-sensitivity to blue relative to the surrounding cones. Place a uniform surface like a sheet of white paper before you, and close your eyes for 15 to 20 seconds. When you open them, a yellow spot, the projection of your macular pigment, may be seen wherever on the uniform surface you happen to be looking. It will disappear in a few seconds, probably before you are sure that you have seen it. Another way is to look at a bright, uniform surface such as the clear blue sky alternately through a purple filter and a neutral gray filter (Miles, 1945). If you alternate filters about once a second, the picture of the macular pigment can be kept fresh and clear. It will appear as an irregularly shaped red spot of about 3 to 4° diameter against the purple background. Sometimes there is reported a central region apparently unpigmented, and sometimes within this central region there is reported a second spot apparently pigmented like the surrounding ring.

The macular pigment is not noticeable in daily life, but it is such a serious handicap to the use of the eye in certain kinds of color measurement that a photoelectric cell is preferable. In order to know when to use the eye and when to use a photoelectric method we shall have to look more closely into these kinds of color measurement. But this will come later.

Blood. The capillaries of the retina are almost opaque because of the blood pigment (haemoglobin). They cast sharp, dark shadows onto the underlying cones. Like the macular pigment, these shadows are ordinarily invisible because of local adaptation of the cones. It is easy, however, to make the capillary system of the retina visible by casting the shadows on near-by parts of the retina that are not used to them. If in a dark room you look straight ahead of you and turn on a concentrated-filament lamp, such as an auto headlight lamp, held off to the side, the image of the filament on the retina will be bright enough, because of slight reflection of light there, to serve as a secondary source of light within the eyeball. Light from this illuminated spot, of course, strikes the capillaries at an angle far different from the usual perpendicular. The whole visual field will be seen dimly illuminated with the capillaries appearing as a system of black

lines. Moving the lamp back and forth slightly changes this angle and keeps the pattern from disappearing by adaptation. Figure 7 shows (black lines) the system of retinal capillaries of a macaque. These capillaries radiate out from the blind spot and do not extend quite to the macula.

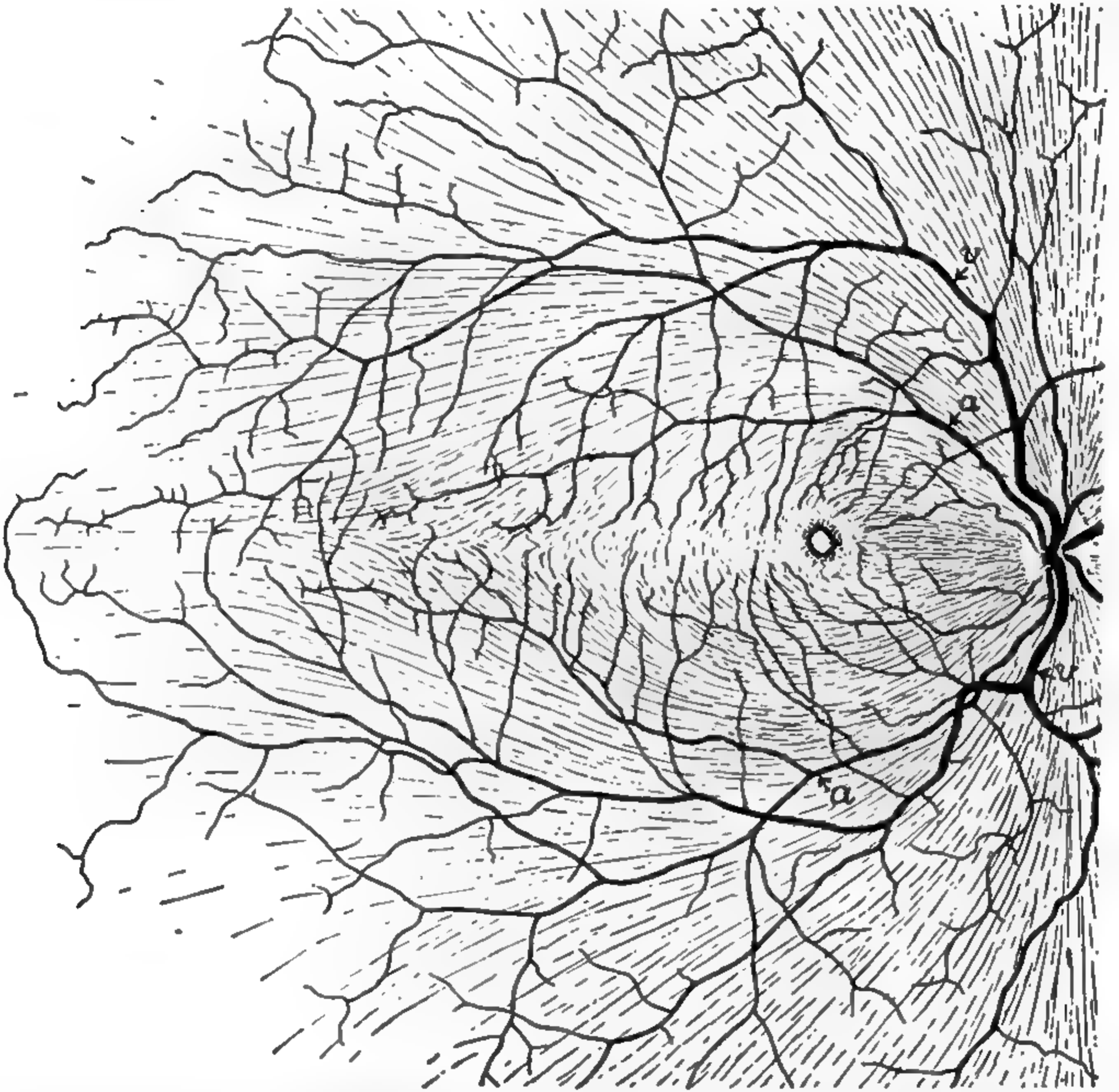


FIG. 7. Courses of the capillaries (heavy lines) and nerve fibers (light lines) across the inner face of the retina (of a macaque) toward the exit point (optic nerve). Note that these are laid down in such a way as to avoid the fovea, thus permitting image formation at this important central spot to be maximally sharp. (From Polyak, *The Retina*, 1941.)

The very dark appearance of these shadows shows that very little light gets through the capillaries; so the spectral character of haemoglobin has little to do with the effect. However, we know that some light gets through the capillaries because we can see the motion of the blood corpuscles in the capillaries of our own eyes. Look at a bright uniform field like the sky. After a few seconds' adaptation you

will see not only the slow-moving strings of beads corresponding to trash in the vitreous humor, but also little points of light and shadow darting about in short curved paths in every direction all over the field of view except the very center. These "flying gnats" (*muscae volitantes*) correspond to spaces between the red corpuscles in the capillaries.

Choroid Coat. The choroid coat has a brownish black (melanin-type) pigment. This pigment serves to absorb the radiant energy after it has passed through the rod-cone layer of the retina. It prevents the image cast by the lens system of the eye from being overly degraded by stray light. Persons (albinos) that are free from melanin-type choroid pigment have to depend upon the relatively poor absorption of haemoglobin in the blood to keep down stray light within the eyeball. They have to contend with retinal images very much diluted by stray light. This gives them low visual acuity and fear of light (photophobia). The pupils of their eyes instead of being black have a reddish glow.

Rod and Cone Pigments. The retinal image consists of a pattern of radiant flux of varying density. The only way this image can be trans-

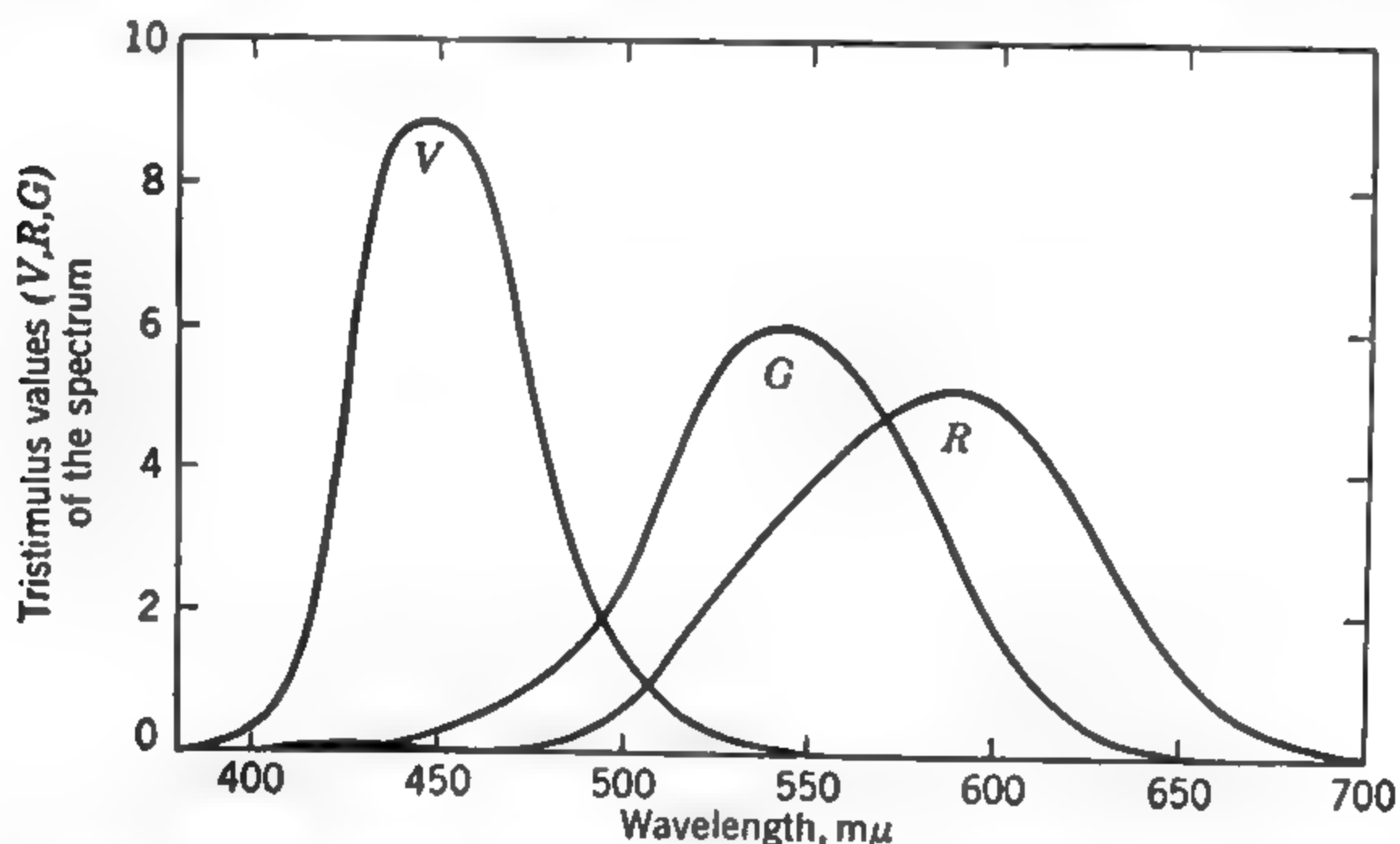


FIG. 8. Approximate spectral sensitivities of the presumed three cone photopigments. To get more precise spectral sensitivities these values must be divided by the spectral transmittances of the ocular media (Fig. 5) and the macular pigment (Fig. 6).

formed into a pattern of nerve activity is by the presence of substances in the rods and cones that absorb a part of this radiant flux. The rod pigment has been successfully extracted and studied. It is called rhodopsin, and its ability to absorb radiant energy is found to vary with wavelength in proportion to the sensitivities of the rods

to radiant energy (Fig. 2), provided that a correction for the influence of the lens pigment be applied (Fig. 5).

Wald's work (1949) indicates indirectly the existence of one cone pigment which he calls iodopsin, though it has not yet been extracted and isolated. We know, however, from the fact that we get three independent kinds of information from the cones (light-dark, red-green, yellow-blue) that there must be at least three cone photopigments, or photopigment-filter combinations. Furthermore, we know closely how their ability to absorb radiant energy varies with wavelength; see Fig. 8. The cones are anatomically alike, though their connections differ. In spite of their similarity it is generally presumed that some of them contain short-wave absorbing (*V*) pigment, some contain a preponderance of long-wave absorbing (*R*) pigment, and some contain a preponderance of middle-wave absorbing (*G*) pigment. How the pigments are supplied to the right cones in the right proportions is a matter of conjecture. Measurement of color rests upon the known properties of the foveal cones acting in unison; but the analysis of the roles of individual cones is incomplete.

Interpretive Elements. The pattern of nerve activity once initiated by the rods and cones has to be combined and sorted so as to make sense. The combination and sorting starts in the side connections of nerves in the retina itself, the resultant impulses pass laterally along the retinal nerve fibers between the vitreous humor and the rod-cone layer, out through the exit point (blind spot) of the optic nerve, and thence through various structures of the brain to terminate in the occipital lobes. Other nerve cells take over at this point so that this visual information may be coordinated with what our ears, nose, tongue, digestive apparatus, and so on have to tell us, and a course of action, or inaction, is laid down. We need not go into the anatomical details and speculative explanations of most of this, but some of it has a bearing on color measurement and inspection. This is the part dealing with cone vision.

Retinal Side Paths. It is generally held that each cone can respond in only one characteristic way, depending upon the photosensitive substances that it contains. That is, it is supposed that there are red-reporting cones that ought to have long-wave sensitive stuff in them; there are green-reporting cones that ought to have middle-wave stuff in them; and violet-reporting cones that ought to have short-wave stuff. If the cones have the wrong stuff in them color blindness results. By this view one of the chief jobs of the retinal side paths is to connect neighboring cones together, perhaps in triads, so that the retina, if uniformly stimulated, can report a uniform color. That is, if you are

looking at the clear sky you would have a chance, with local side connections in the retina, of seeing sky blue; but without them you ought to see a mosaic of red, green, and violet spots with the red ones somewhat weaker than the green and violet.

The exact connections required to produce this result are anybody's guess. Under the microscope all the cones look alike; there is no telling which is red-reporting, which green and which violet. And there are so many side connections that almost any system is possible; the wires are all there. The action of a nerve fiber, however, is not very closely like that of a wire conducting an electric current. It is more like a self-renewable bomb fuse. When one end of a nerve is rendered active, a burst of local activity runs rapidly down the nerve. Then, a fraction of a second later, another impulse can go over it. A strongly stimulated nerve responds by very frequent discharges; a weakly stimulated nerve, by less frequent discharges. All these impulses, detected by the microelectrode, are of the same intensity. It is all or nothing. To give a complete color response, at least three cones must feed impulses along three independent fibers. These impulses must be combined at some stage in the nervous system, either in the retina or in the brain. We know that this combination can take place in the brain because red from one eye combined with green from the other can make the subject see yellow. Probably the combination can take place either in the retina or in the brain.

When the resultant nerve impulses have reached the layer of nerve fiber in contact with the vitreous humor, there is no more chance for side connections in the retina. The impulses are merely conducted over the inner face of the retina to enter the optic nerve. Figure 7 shows how the retinal nerve fibers (fine lines) combine to form the optic nerve. It will be noted that they generally take the shortest paths except where that path would cross the center of the eye (macula lutea). Cones located on the nose side are already near the exit point and present no problem. Cones located on the ear side of the retina, however, have their nerve fibers carefully led around the center. This prevents the image falling on the center from being degraded by having to be projected through a mass of more or less light-deflecting nerve fibers.

The nerve fibers curving around the center of the eye can be seen, when they are active, by the subject himself. If you go into a nearly dark room and look slightly to the right or left of a vertical slit of light, you will see more than the image of the slit. Two pairs of reddish blue arcs become visible, one pair corresponding to the projection of the retinal nerve fibers from above and below the fovea to the right-eye blind spot, the other pair to the left-eye blind spot (Judd,

1929; Newhall, 1937). Once you have seen the blue arcs, you will notice them very often, sticking out of the quarter-moon, from a crack in the door to a lighted room, and so on. Green, yellow, orange and red slits work best. The nerve activity of these fibers makes them visible to cones beneath them. Perhaps they glow from actual radiant energy, or perhaps the local electrical impulses associated with a nerve discharge stimulate the near-by cones. They serve to remind us of the rather strange way in which we get the most in color and detail out of the center of our eyes.

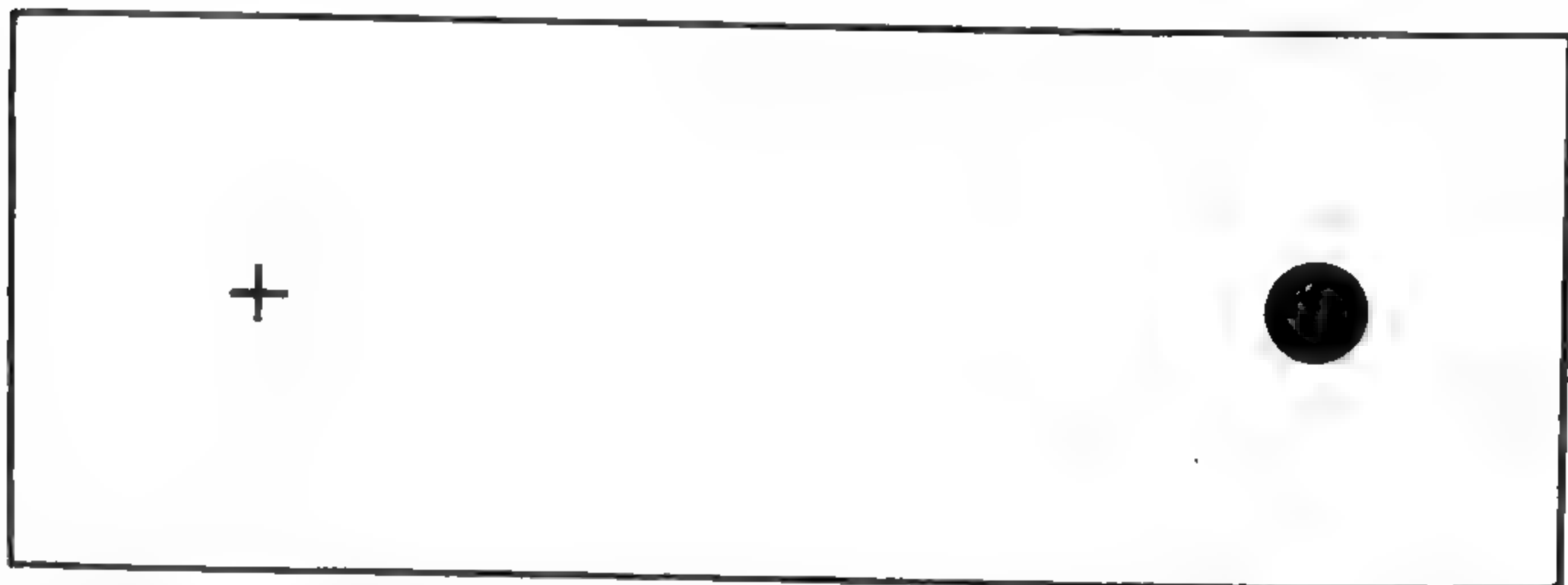


FIG. 9. Demonstration of the blind spot, or exit point of the optic nerve. Hold the page about 8 inches from the right eye and, with the left eye closed, look at the cross. The black spot will then be seen to disappear.

At the exit point of the optic nerve (Fig. 1) no rods or cones are to be found by microscopic examination. Indeed, it is hard to see how any could be there except possibly a few rudimentary, scattered, and poorly fed receptors. We may expect, therefore, that radiant energy incident on the optic disk (as the exit point is called) would be invisible. This is ordinarily true, and, although careful experimental studies (Helson, 1929) have shown some sensitivity there, the optic disk is often called the blind spot. Figure 9 serves to demonstrate this. Look at the cross with the right eye, and at the same time hold the left eye closed. If the page is about 8 inches distant from the eye, the spot may be caused to disappear. Sometimes the page has to be turned a few degrees in its own plane to line up the black spot with the optic disk. In this way you will discover that your right-eye blind spot is about 16° to the right of center and has a diameter of about 6° . It is big enough to hide a row of 10 contiguous full moons. Of course, this great gap in our visual field is ordinarily filled by vision from the other eye whose blind spot, being 16° to the left of center, does not overlap. The fact that with one eye closed we see this gap filled with the same kind of color and texture that surrounds it illustrates the great im-

portance of what goes on after the nerve impulses have left the retina. Somehow this retinal gap is filled in by post-retinal processes.

Brain. The brain has been likened to a huge combat-information center. It tells the muscles when to make us duck; it closes the hundreds of switches required to make us throw a rock at someone lurking in the shadows. It cancels all these operations in short order if the lurker steps out of the shadows and is seen to have an olive-drab uniform. Most of the switching equipment in the brain is held in a stand-by condition; our normal operations take only a small part of it.

Efforts to untangle the millions of connections of this immense switchboard have been successful to a considerable degree. We know, for example, that the optic nerve from one eye joins up with that from the other (optic chiasma) in such a way that the fibers from the right half of one retina run side by side with those from the right half of the other retina and after passing through a relay-station in the mid-brain, terminate in nearly the same place in the occipital lobe at the back of the head. There is a projection of the retinas in this lobe, but the part corresponding to the center of the eye (fovea) is immensely magnified compared to the rest of the retina. At the relay station there is a chance for side connections, and the occipital lobe itself is richly connected to all other parts of the brain.

The organizing principle of this complicated mass of nerve connections seems to be very simple though precise information is lacking. It would appear that the more a certain set of connections is used the more likely it is that an impulse will follow that set instead of one of the side connections. This is called learning. We are born with certain patterns of nerve connections already established. It is no accident, for example, that the projection of the retina normally occurs in the occipital lobe. The child learns to respond to light, movement, form, objects, and color in about that order. During this learning process patterns of nerve connections are being established in the occipital lobe. At the same time, random movements give way to coordinated movements. Any time the occipital lobe is destroyed, these patterns are lost. A soldier having a gunshot wound in the back of the head usually does not survive. But if he does he is often found to be worse off as far as vision is concerned than a child. Ultimately, the veteran can distinguish light from dark, then movement without form, then both form and movement (Poppelreuter, 1925). These shapes and movements in the visual field, however, have no meaning. The veteran cannot interpret them as objects, cannot reach for them, cannot tell which of them refer to parts of his own body. After discouraging months of relearning, he may be discharged from the hospital, able

to identify objects again, able to look after himself. It is presumed that he has made use of some of the side connections normally kept in a stand-by condition. A return to a reasonable approximation of normal vision including color has been reported. The ability to identify visual objects is the result of a learning process. It is one of the most important interpretative gifts of the brain, and it is profoundly important in color measurement.

No one has yet successfully explained the complete mechanism by means of which visual objects are perceived, but we know some of the end results. If you look at one corner of the room, then at another in quick succession, the optical image of the room has swept across the retina. Each cone has received a pulsating stimulation. But did you perceive any flicker? No. Did you perceive that the room moved? You certainly would have from the same pattern of retinal stimulation if your head and eyes had remained stationary. No, you perceived that the room was stationary as it was in fact, and that you had moved your own eyes and head. This shows how well you have learned to use your eyes to find out about things around you. You have built up a mechanism that automatically corrects the perceived image for head and eye movements accomplished in the usual way. This ability is basic to the perception of a visual object.

But now try an extension of this experiment. Close one eye and press gently against the eyeball of the other through the lid. You will note that the whole room moves with this pressure. Strike the side of your head above the ear with the heel of your hand. Again note that each blow causes the room to jerk. This proves that, although the mechanism works to keep the visual field steady in spite of head and eye movements initiated by itself, movements of the same parts of the body made through the arm cause the visual field to move. It is thus a matter of control. Only the eye and head movements made as a part of the visual act are automatically compensated.

Experiment has shown that the eye normally moves to fixate one object in the visual field after another by a series of jerks. As you read this line your eye will fixate somewhere near the first word of it, will make two or three intermediate stops, and end with a fixation somewhere near its end. What happens to the confused pattern of retinal images during the jerks? These do not form a part of the visual experience at all. The optical image is there coursing rapidly over the retina. The nerve pattern is formed and started on its way toward the occipital lobe. But somewhere along the line, a switch is opened up. The same mechanism that controls the eye movements automatically shuts off the confusing information caused by it.

Our name for this marvelous property is attention. We say that we do not see the confusing images because we have learned not to attend to them. But this is a very meager negative way of describing a most important positive gift. Our disregard of confusing information arising from the seeing act, itself, is basic to the perception of objects. And, once learned, it can be unlearned only with considerable practice.

The visual mechanism, acting as an integrated whole, does more than keep visual objects stationary regardless of voluntary head and

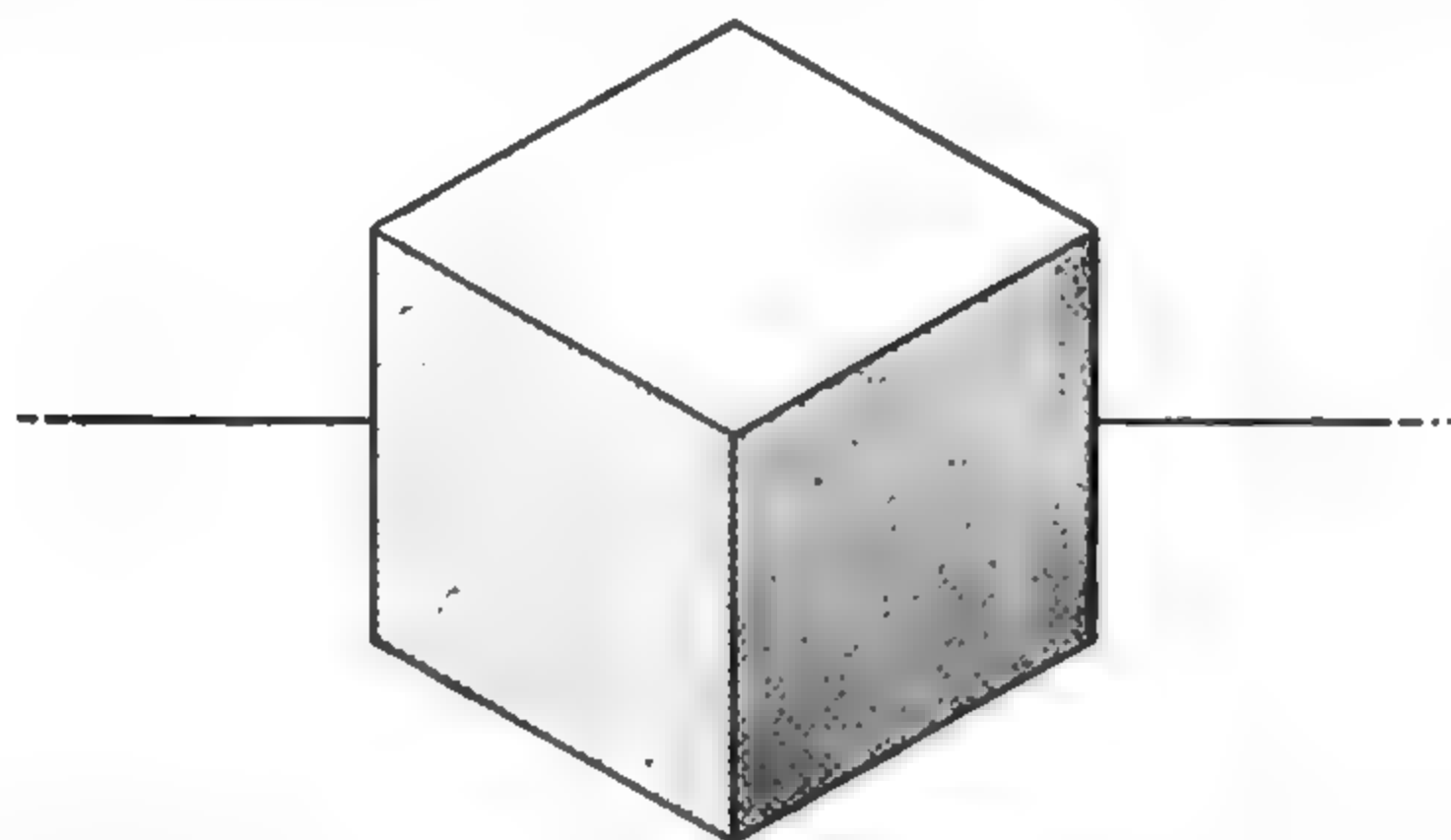


FIG. 10. Connection between color perception and depth perception. If the three diamond-shaped areas are seen as a picture of a cube, the cube is usually perceived as white with the three faces unequally illuminated. If the areas are perceived as three diamond shapes in the same plane, they will be perceived to have different colors (white, light gray, middle gray).

eye movements. It also keeps their colors nearly constant regardless of the amount of light striking them. This adjustment is of two kinds, local and general. By general adaptation we can identify a white object both in full sunlight and in moonlight. Figure 10 illustrates the local adjustment. It is a picture of a cube. The areas of the figure representing the cube are white for the top, light gray for the left side, and middle gray for the right. But you will not perceive Fig. 10 as a picture of a cube with a white top and gray sides. You will perceive it as a cube whose faces all have the same white or light gray color illuminated from the top and from the left of the top. You do this because you have seen hundreds of such patterns every day from infancy and have learned what they mean on the average. As often quoted by Evans (1948) from Prof. A. Ames, Jr., Dartmouth College, "What you see is your best guess as to what is out front." The basis for this guess may reside in the established pathways of response in the brain; but it is hard to understand how a simple turning on or off of certain groups of switches, however complicated, can give

us the marvelous faculty of seeing objects as solid. If Fig. 10 can be seen as a flat picture of three diamond-shaped areas touching each other, they will be seen to have the colors: white, light gray, and middle gray. As soon as the much more usual perception of a cube is obtained, the same retinal pattern that gave us a white and two grays in the same plane, now gives us three faces of a solid object of uniform color but unequally illuminated. The retinal patterns within the eye are identical, but the response patterns within the brain must be essentially different. The retinal pattern tells us that the cube has differently colored faces, but the brain takes this wrong information and comes out with the answer that, on the average, is right. On this adjustment for local differences in illumination largely depends our ability to recognize objects around us and deal with them promptly and successfully. Seeing an object in an illuminated space is quite different from seeing a patch of color filling a hole in a screen. The colors of the former are called related colors because they are always perceived in intimate relation to the amount of the illumination. The latter are called unrelated colors or aperture colors; the customer almost always sees related colors. Practical colorimetry must take account of this fact.

ASPECTS OF COLOR (BASIC TERMS)

It should be plain by now that the perception of color is a complicated process. The various parts of this process touch the domain of various sciences—chemistry, physics, physiology, and psychology. So far we have been dealing mostly with physiology; now we must go on to the others.

To have an object perceived as red with a blue top, for example, ordinarily requires four things—an inclosure containing a light source, an observer, and the object. The source must emit radiant energy well distributed in the spectrum between 380 and 770 $m\mu$ at a rate sufficient to make the eye of the observer respond. The observer must have at least approximately normal color vision. The inclosure must be filled with a reasonably clear optical medium and have boundaries capable of revealing the object character of the object (as by cast shadows). The object must be pigmented with two kinds of pigment one of which (blue) absorbs strongly the long-wave and middle-wave portions of the incident energy and reflects the short-wave remainder, the other of which (red) reflects the long-wave energy and absorbs strongly the remainder.

The events leading up to the perception of a red object with a blue top take place in the sequence shown in Fig. 11. Radiant energy from

the source (1) strikes the object (2) and the walls of the inclosure. Some of this energy is reflected toward the eye of the observer, entering the pupil (3) and forming on the retina a radiant-energy pattern (4), the elements of which have different densities and spectral compositions of radiant flux. Some of this energy is absorbed by the photosensitive pigments of the rods and cones of the retina.

Thus far the processes are the concern of physics.

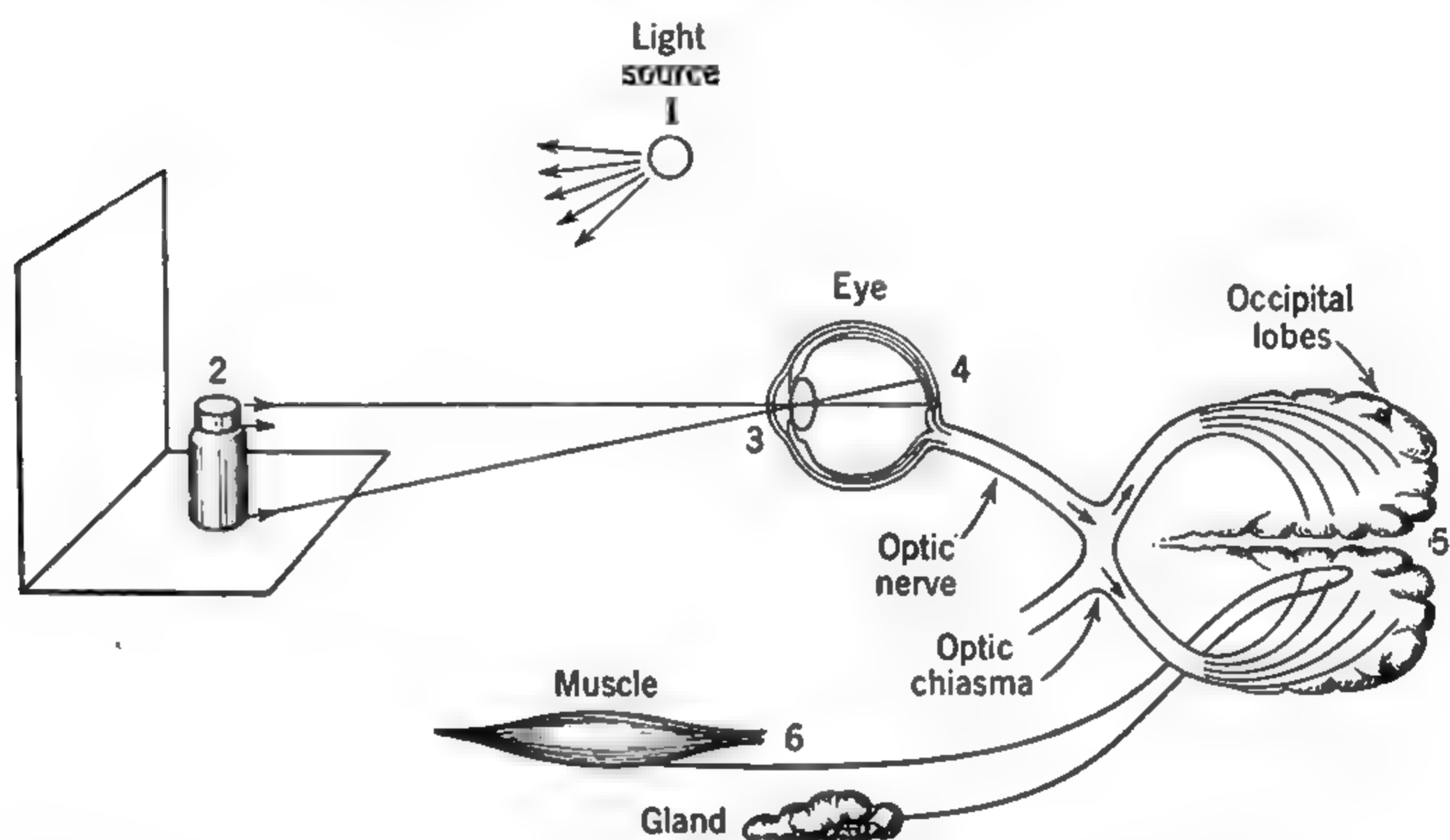


FIG. 11. Conditions required for the perception of a red object with a blue top. There must be a light source, an illuminated space, the object and its top, and an observer with at least approximately normal color vision. The formation of the retinal image, the cortical pattern, and the muscular and glandular pattern are also diagrammed.

The energy absorbed within the rods and cones, however, causes a change in them that permits them to initiate a pattern of nerve impulses in the fibers of the optic nerve. These impulses find their way from fiber to fiber through synapses and produce a corresponding pattern of nerve activity in the occipital lobe of the cortex (5), and from this pattern there flow other nerve impulses controlling the action of the observer's muscles and glands (6). They may cause the observer to move his head, make his eyes traverse the object, his face light up in a smile, and even make him say, "I see a red bottle with a blue top on it." This segment of the action caused by the radiant energy leaving the light source is the concern of physiology.

The subjective aspect of the nerve activities in the cortex is the color perception itself and is the concern of psychology. Reduced to its simplest terms, this color perception is by no means simple. It involves

a complete grasp of the essential elements of the whole situation—the light source, the inclosure, and the object. Yet this is the aspect of color in which industry finds its primary concern. All the color control in industry comes down to just one thing—What will the customer see?

Chemical—Pigments and Dyes. We have just outlined the physical, physiological, and psychological aspects of color. These aspects are described by means of terms of the corresponding sciences, and some of them will be presented in due time. But, if a manufacturer wants to make sure that his customers will see red bottles with blue tops, he will first be concerned with the chemical aspect of color, that is, he will have to procure some red and blue colorants.

Most objects owe their color to substances that absorb radiant energy within the visible spectrum. These substances are called colorants; if insoluble, pigments; if soluble, dyes. The property of a colorant that makes it absorb more of one part of the visible spectrum than another is its chemical constitution. Pigments and dyes in early days were extracted from animal (cochineal, Tyrian purple) and vegetable (indigo, madder) sources, but now because of the advance of organic chemistry these colorants and many besides are produced by synthesis. The chemical theories of color seek to account for the selective absorption of radiant energy in terms of chemical constitution. These theories are incomplete but have been of immense value in guiding chemists in their search for more useful colorants.

The chief things a manufacturer needs to know about a colorant are: (1) will it produce the color his customers demand, (2) will it go onto or into his product, (3) will it resist change resulting from washing, sunlight, and chemical agents (acid, alkali, salts), (4) how many pounds will be required to do the job, and (5) what is the price per pound. Most of this information he will have to get from the organic chemist and the colorant technologist. Precisely what makes a dye combine with a textile fiber, what makes a paint film harden, and why some colorants can be used in ceramics and some cannot are not completely known; but research and development work are constantly producing new colorants, new methods of dyeing, and new formulations for paint and ceramic tile. Permanence of dyed textiles, dyed plastics, and paint films is being studied extensively by outdoor exposure to natural weathering conditions and by artificial weathering produced by exposure to radiant energy from the carbon arc combined with fresh-water and salt-water spray. The chemistry of color is worthy of important mention because a great deal of color measurement in

commerce has to do with assessing the commercial value of what the chemist has achieved.

It should be remembered, however, that a dye chemist pays only secondary attention to description of the colors he produces. He tends to think of and talk about these colors in terms of the chemical formulas of the organic compounds used in the colorants.

Physical—Radiant Energy and the Spectrum. Objects about us are made visible by the radiant energy that comes from them to our eyes. The analysis, measurement, and explanation of radiant energy is a part of physics. If we are going to measure color in a fundamental way, we must measure radiant energy; and, if we are to understand what we see, we must know something about the complicated nature of radiant energy.

The idea of energy is one of the most useful physical discoveries. We know energy in various forms: thermal energy, kinetic energy, mechanical work (foot-pounds), chemical energy (calories), electrical energy (watt-hours), radiant energy (ergs), and, most recently, atomic energy (electron volts). These various forms of energy can be converted from one to another, but the energy itself cannot be destroyed. All physical events have their energy aspect and may be partly described in terms of a transfer of energy from one form to another. If the light source is an incandescent lamp, it gets its energy at a fixed rate from the electric current passing through its filament. In this filament the energy is changed in form from electrical to thermal energy, causing the temperature of the filament to rise until the rate at which the filament gives off radiant energy is equal to the rate at which it gains thermal energy from the electric current. Some of this radiant energy is incident on the object and its background. Of this a part is absorbed and thereby converted back into thermal energy going to heat up the object; part is re-emitted in the direction of the observer's eye. Of the radiant energy incident on the cornea so as to enter the pupil of the observer's eye, part is reflected by the cornea and the lens surfaces, part is absorbed by the eye media and thus transformed into thermal energy or heat, and a large part of it is absorbed by the choroid coat which also is thereby heated up. A small fraction, however, is absorbed from the radiant energy pattern passing through the retina by photosensitive pigments in the receptors, rods and cones. This small fraction absorbed is transformed, not into thermal energy, but into chemical energy. That is, a number of molecules of the photosensitive pigment are so energized by the small additional energy picked up from the retinal image that they break into pieces or decompose.

The results of this decomposition on the activity of the retinal nerves theoretically can also be traced in terms of energy; but most of the energy resides in the nerve itself, the radiant energy absorbed in the receptor merely acting to trigger the nerve into a burst of activity under its own power.

The nature of radiant energy is such that it is hard to form even an approximate concept of it. In some experiments, radiant energy acts as if it were composed of a stream of projectiles shot out at high speed (186,000 miles per second) by the source. For example, it travels in a straight line from the source and exerts a small but measurable pressure on an object in its path. In other experiments, radiant energy acts as if it consisted of transversely vibrating waves starting from the source and traveling through space at this same high speed. A complete idea of radiant energy must, therefore, include both the idea of a corpuscular nature and the idea of a periodic or wave nature; and this is beyond the ability of most of us to conceive.

Fortunately, it is not necessary in the measurement of color to take into account the corpuscular nature of radiant energy. We may confine our attention to its periodic or wave nature. The most important variable in the periodic aspect of radiant energy is the frequency with which the electric or the magnetic vector reaches its maximum value, that is, the frequency of alternation. This variable may be expressed directly in terms of cycles (or kilocycles, or megacycles) per second, or it may be expressed indirectly by giving the distance the beam travels from one crest to another of the alternation. This distance is called the wavelength. To compute the wavelength from the frequency it is necessary to know the distance traveled by a beam of radiant energy in one second. The speed of radiant energy has been found to be exactly the same in a vacuum, regardless of the frequency of alternation. This distance traveled in unit time, or speed, c , of radiant energy in empty space is approximately 30 billion centimeters per second (more precisely 2.998×10^{10} cm/sec). Converted into miles per second, this speed is:

$$c = \frac{2.998 \times 0.3937 \times 10^{10}}{12 \times 5,280} = 186,500 \text{ miles/sec}$$

where 0.3937 is the number of inches in a centimeter, 12 is the number of inches in a foot, and 5,280 is the number of feet in a mile. Frequency, ν (nu), in cycles per unit time obviously is connected to wavelength, λ (lambda), in empty space by the simple reciprocal relation:

$$\nu = c/\lambda \quad (1)$$

Speed of radiant energy through other media (air, water, glass, plastics, paint vehicles) is less than c and is given by the ratio c/n , where n is known as the index of refraction of the medium. For air the index of refraction is about 1.0003 and in technical applications is usually taken simply as 1.000. For water the index is about 1.33; and for glass, plastics, and paint vehicles it usually ranges between 1.45 and 1.6.

Radiant energy is known to us under many names which refer simply to differences in wavelength or frequency. Table 1 lists a few

Table 1. Names for the Various Parts of the Spectrum of Radiant Energy *

Name	Wavelength			Frequency		
	Meters	Milli-meters	Milli-microns	Cycles	Kilo-cycles	Mega-cycles
Radio						
broadcast	1×10^3	1×10^6	1×10^{12}	3×10^5	3×10^2	3×10^{-1}
ultra-high						
frequency	5×10^{-1}	5×10^2	5×10^8	6×10^8	6×10^5	6×10^2
Infrared	1×10^{-6}	1×10^{-2}	1×10^4	3×10^{13}	3×10^{10}	3×10^7
Light						
red	7×10^{-7}	7×10^{-4}	7×10^2	43×10^{13}	43×10^{10}	43×10^7
green	5×10^{-7}	5×10^{-4}	5×10^2	60×10^{13}	60×10^{10}	60×10^7
violet	4×10^{-7}	4×10^{-4}	4×10^2	75×10^{13}	75×10^{10}	75×10^7
Ultraviolet	1×10^{-7}	1×10^{-4}	1×10^2	3×10^{16}	3×10^{12}	3×10^9
X-rays	1×10^{-10}	1×10^{-7}	1×10^{-1}	3×10^{18}	3×10^{15}	3×10^{12}
Gamma rays	1×10^{-11}	1×10^{-8}	1×10^{-2}	3×10^{19}	3×10^{16}	3×10^{13}
Cosmic rays	5×10^{-14}	5×10^{-11}	5×10^{-5}	6×10^{21}	6×10^{18}	6×10^{14}

* Numbers are fairly representative of the middle of each range but do not indicate the extent of the ranges.

of these names together with the frequencies and wavelengths that are fairly typical. Three units of wavelength in vacuum (meter, millimeter, and millimicron) and three units of frequency (cycles per second, kilocycles per second, and megacycles per second) are used in this table. It will be noted that radio waves, X-rays, and cosmic rays have the same nature as light, differing only in frequency or wavelength. It may also be pointed out that two World War II developments in “seeing” through haze, fog, and clouds make use of hitherto somewhat neglected portions of the spectrum of radiant energy. These are the high-frequency portion of the radio spectrum, used in radar, and the low-frequency portion of the infrared spectrum, used for signaling through fog. Both combine the penetrating nature of radio waves with the directional nature of light.

Every single element of area in the universe that can form a part of an observer’s visual field has passing through it radiant energy at a

certain rate. This energy comes from matter because of thermal and other agitation of its molecules (thermal radiant energy), or because of instability of the atoms composing the molecules (atomic radiant energy, cosmic rays), or from man-made radiators (radio, X-rays). This energy can be completely specified by stating how much radiant energy passes through the element per unit of time for each part of the radiant-energy spectrum. Time rate of flow of radiant energy is called radiant flux or radiant power, and, when it is considered for each part of the spectrum separately, it is called spectral radiant flux. By giving the complete distribution of spectral radiant flux passing through a given area in the direction of the observer the physicist has completely defined the property that causes the observer to experience a color belonging to that area. As we have seen, however, the human eye is sensitive to only a minute fraction (little over an octave) of the immense spectrum of radiant energy. It is seldom necessary as far as color is concerned to specify spectral distribution of radiant flux for wavelengths less than $380\text{ m}\mu$ and more than $770\text{ m}\mu$, because the eye is so little sensitive beyond those wavelength limits.

The analysis of a beam of radiant energy into its spectral components may be accomplished either by means of a prism or by means of a diffraction grating. The quantitative comparison of radiant flux in the different parts of the spectrum so produced may be accomplished by various sensitive receivers (bolometer, thermopile, thermocouple, photoelectric cell). The combination of a dispersing element (prism or grating) with a radiometric detector calibrated to indicate radiant flux in absolute terms is called a spectroradiometer. If the device serves merely to measure radiant flux of one beam relative to that of a standard beam, the photographic plate or the human eye may also be used as a receiver, and the device is then called a spectrophotometer. This is the physicist's great color-measuring tool, and more will be said about it in due time. With it the physicist can specify completely not only the physical basis of the color of a small, distant source of light, or a large, uniformly luminous area but also the basis by which either one imparts to objects their colors. He can also specify the physical basis for the colors of transparent and opaque objects, natural or synthetic, by showing how these objects modify the spectral composition of the energy incident upon them.

If two neighboring elements of the visual field are shown to deliver in the direction of the observer's eye radiant energy at identical rates with identical spectral distributions, we may be quite certain that these two elements will be perceived to have identical colors. The physicist thus supplies us with a complete, unambiguous specification of the

stimulus to color; for, if two uniform fields supply radiant energy to the eye at exactly the same rate and of exactly the same spectral distribution, there is no way for the eye, or any other receiver, to tell them apart.

A manufacturer of lamps may employ a physicist to find out why certain lamps cannot be sold to butcher shops. The butcher may complain that the lamps make meat look greenish, or perhaps hardly any redder than it appears by natural daylight. We may be sure, however, that the physicist is not thinking "redder, greener, daylight color of meat," and so on. He will be thinking "spectral reflectance of meat, spectral distribution of incident radiant flux in watts per square meter per millimicron" both for the lamp that cannot be sold and for the competitor's lamp that can, getting in this way to the crux of the problem. His primary attention is given to the stimulus.

Psychological—the Customer's Angle. The ultimate consumer, and that means each of us, judges what to buy on the basis of all the senses at his command. We touch, taste, smell, lift, and look at the merchandise. We glance at the trade-mark, and sometimes even read the fine print on the label. In nearly every transaction, color plays an important part. The customer perceives color as belonging to the merchandise or to the package; that is, color for the ultimate consumer nearly always means object color. For each package or each type of merchandise we carry in our heads a memory of the color or range of colors that is acceptable. Bread whose crust is too dark is passed by as probably burnt. Tomatoes and apples cannot have too green a color or they may be rejected as unripe. Beef to sell well must have a red color that is neither too dark, too grayish, too greenish, nor too purplish. Packages of merchandise must show the background and trade-mark colors to which the buying public is educated; if the colors of the package are too pale (light and grayish) they suggest to the customer that the package has been on the shelf so long that the colorants have had time to fade, and the contents to spoil. Package colors too dark and grayish indicate dirt from excessive handling and suggest that other customers have found something wrong with this particular package or brand, causing them to put it back on the shelf. Color that is the wrong hue (such as orange instead of red) is the worst of all. The customer may not even recognize the package at first as the brand she has been buying, and may not even reach for it. If she does recognize it, she may get an impression of carelessness on the part of the maker. If he cannot take enough care to put the right color on the package, he probably misses out on the contents, too, and she

will not take the next package even if it is correctly colored but will switch to another brand.

This dependence of the customer on a mental standard of color is subject to various kinds of uncertainty. The spectral character of the light by which the merchandise is viewed sometimes influences color perception to an important degree. Some fluorescent lamps give meat a greenish color suggestive of putrefaction; others give it a redder color than it has by daylight. A suit or necktie picked out under incandescent-lamp light may be returned the next day because its daylight color is unsuitable. Color of the surroundings influences the color judged by simultaneous contrast. Colors previously seen influence the color judged by successive contrast. Adaptation to blue adds yellow to the color judged, and vice versa; and adaptation to green adds red. Furs that have an undesirable brownish or rusty color are sometimes mistakenly selected in daylight if too many neon signs are visible from the window admitting the daylight. By exposure of his eyes to light from the neon signs, the customer without knowing it will respond too little to orange-red light and so fail to detect the undesirable rusty color. By and large, however, our eyes and our ability to use them to see color belonging to objects stand us in good stead, regardless of wide change in light and surroundings, and enable us to make wise selections.

The names used by the buyer for his various mental standards of colors are unimportant; tomato red, pea green, eggplant, beaver, Lucky Strike green, butter yellow, and so on. The essential part of the judgment is the character and amount of the difference between the actual color perceived and the mental standard to which it must conform. The language of the perceived color differences refers to each customer's own subjective world of color perception. This language has been fairly well indicated by the examples given. The kind of difference is indicated by three pairs of opposites: lighter or darker, grayer or more saturated, and a pair of the hue terms, red, yellow, green, blue. Thus, a green color may be perceived to vary in hue either toward yellow or toward blue by being either a yellowish green or a bluish green; and an orange color may vary in hue either toward red or yellow. Variations of lighter or darker are called lightness differences; variations of grayer or more saturated are called saturation differences, and variations toward any contiguous member of the series, red, yellow, green, and blue, are called hue differences. An object color perceived as a part of any observer's visual field may vary in lightness, saturation, and hue, but in no other way not describable in terms of com-

binations of these. On this account we say object color perception is tridimensional.

The use of lightness, saturation, and hue terms to describe object color experience is very common. Most people have organized their color experiences along these lines, even though they have not given formal names to the three variables. We should say a word, however, about other ways of organizing color experience and describing color differences.

Combinations of lightness and saturation differences have fairly well-recognized names. If we perceive one object to have a color that is both lighter and grayer than that of another, we say it is paler. And the opposite of pale is deep. Thus:

Paler = Lighter and grayer = Whiter
Deeper = Darker and more saturated

and, similarly:

More brilliant = Lighter and more saturated = Cleaner
Duskier = Darker and grayer = Dirtier = Blacker

Differences between object colors perceived to be of the same lightness may be described in terms of hue and saturation alone. These are called chromatic differences, and the variable embracing hue and saturation is called chromaticness. An alternate way of describing chromatic differences is in terms of the opposing pairs: yellow-blue and red-green. To say of two yellows that one is perceived as bluer than the other is simply another way of saying that one is less saturated, or grayer, than the other. Thus blue stone, blue cheese, and blue cotton usually refer to objects having yellow colors rather grayer, and sometimes darker, than the usual yellowish colors of stone, cheese, or cotton. They may be grays or even yellowish grays, not blue at all by themselves.

Since it is possible to pass by fine gradations from any one surface color to any other, the perceptions of these colors may be represented by points in a space diagram whose dimensions in cylindrical coordinates are lightness, saturation, and hue. Figure 12a shows these dimensions. Lightness is represented by distance above the base plane. The grays are plotted along the central axis with black at the bottom and white at the top. Saturation is represented by perpendicular distance from the black-white axis, and hue by angle about the axis. Object colors perceived to be equally light will be represented by points on the same horizontal plane in the object color solid. Those perceived to be of the same hue will be represented by points in some one of the vertical

planes intersecting the black-white axis. Those perceived to be of the same saturation will be represented in some one of the circular cylinders coaxial with the line of grays representing color perceptions of zero saturation, that is, colors perceived to have no hue.

If the alternate description of chromatic differences in terms of yellow-blue and red-green is used, the color solid may be laid out in Cartesian coordinates; that is, by using three axes that are mutually

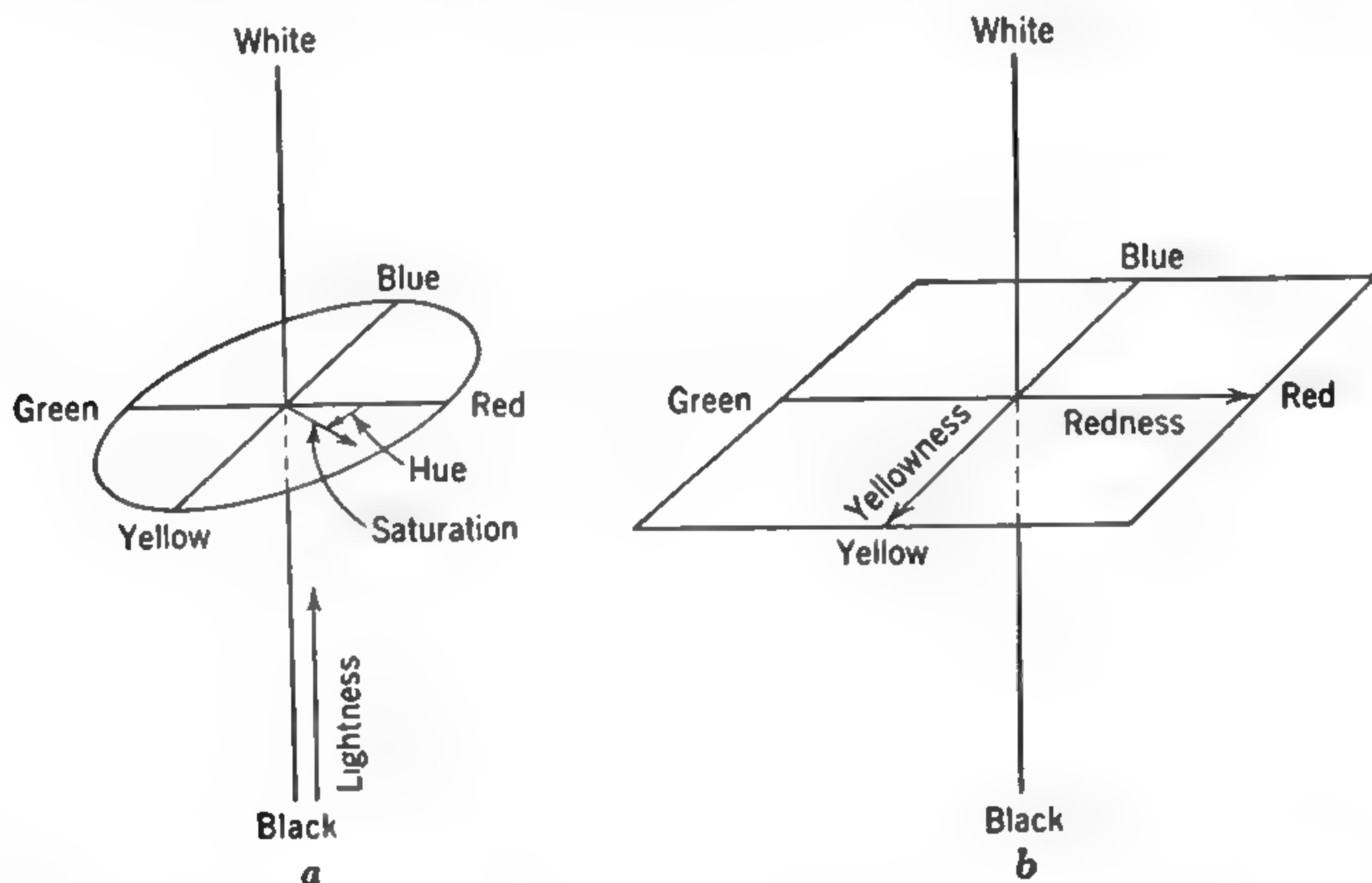


FIG. 12. Organizations of color-perception space: (a) cylindrical coordinates arising from the ideas of hue, lightness, and saturation, and (b) Cartesian coordinates arising from the ideas of black-white, red-green, and yellow-blue.

perpendicular. Figure 12b shows the dimensions of the object color solid laid out in this way. The lightness scale extending from black at the bottom to white at the top is unchanged. Perpendicular to the lightness scale at its mid-point (middle gray) in the plane of the paper is laid out the redness scale whose negative is greenness. Perpendicular both to the black-white axis and to the red-green axis is the yellow-blue axis. This axis, of course, does not lie in the plane of the paper but is merely indicated in oblique projection. Surface color perceptions corresponding to zero in redness are represented by points in the vertical plane containing the yellow-blue axis. Those corresponding to zero yellowness are represented in the vertical plane containing the red-green axis. Those corresponding to both zero redness and zero yellowness are called achromatic, neutral, or gray. They are represented by points along the black-white axis.

The spacing of the points in the surface color solid should come from the customer's estimates of the amounts by which the merchandise colors are perceived to differ from the mental color standard. These estimates, it must be admitted, are scarcely semiquantitative at best. The color of the merchandise may be obviously satisfactory, doubtful, or obviously off-color; but uncertainty in the mental standard, or memory of the color, prevents the customer from trying to make a more precise estimate. However, if she brings along a swatch from junior's worn-out pants as a check on the suitability of the replacement to be worn with the coat, which is still good, the situation is more favorable. Various possible replacements may be found as she shops around. These might be classified as being off color by very much, considerably, noticeably, or just perceptibly. Every customer carries around as a part of his or her mental equipment a color tolerance, more or less strict, applicable to any color. This tolerance varies with the use to which the article is to be put, but it is always related to the customer's ability to detect the color difference. The spacing of the surface color solid, therefore, has to be in terms of the just perceptible color difference. The conception of the surface color solid is that each point in it represents the perception of a surface color, and each color just perceptibly different is represented by points separated from the first point by unit distance. That is, these points fall on the surface of a sphere with the first point at its center.

The scale of the color solid must therefore be psychologically spaced. That is, unit distance must correspond to the just perceptible color difference for all three scales. The unit for the lightness scale is the just perceptible difference in lightness starting with zero for black. The unit for the saturation scale is the just perceptible difference in saturation starting with zero for the gray having the same lightness as the color perception to be designated. The unit for the hue scale is just perceptible difference in hue at some arbitrary lightness (such as that corresponding to middle gray) and some arbitrary saturation (such as 50 steps) starting with zero for some arbitrarily selected hue (such as red).

The conception of a solid representing the perceptions of surface colors has proved surprisingly useful to industry. The most valuable of the applications comes from the simple qualitative idea that there are only three ways for the perception of one surface color to vary from that of another. The quantitative application of the idea is hampered by our lack of precise knowledge regarding the effects of simultaneous and successive contrast. Points in the color solid to represent a color perception have to be found by estimate; they cannot

be found precisely by any measurement yet devised. Some attempts to devise such measurements suggest that color perceptions cannot really be represented by a space diagram. Go 10 saturation steps away from gray, then, at this saturation, step off the hue circuit. If you get significantly more or less than $2\pi \times 10 \doteq 63$ steps, you are in geometric trouble with your surface color solid. Mathematicians have had a field day inventing special geometries to deal with things like this—non-Euclidian geometries, of course. They talk about the curvature of space. It is at about this point that geometry ceases to explain color. The geometers thereafter are using color to exemplify geometry. With ordinary space the color worker can get a useful conception of how one color is related to another. As long as we recognize that the precise location of a point in color space to represent the perception of a given color is impossible, we see that we do not need any fancy geometry for it.

The idea of the color solid applies not only to surface colors but also to the colors of transparent volumes. But there is a minor difference. The color perception represented at the top of the solid must here be, not white, but the color perceived to belong to a perfectly transmitting medium (vacuum, air, distilled water, and tap water, to take progressively less perfect examples). The artist buying varnish to protect his finished painting looks for a light-colored varnish. The customer of the wine dealer has a good idea of the color he should see not only through the thickest part of the bottle but also through the progression of colors produced by the reduced thickness of wine in the neck of the bottle. Kerosene, lubricating oil, and vegetable oils are also judged by color.

A similar arrangement of the colors perceived to belong to self-luminous areas is also possible. The purchaser of an advertising sign will choose one having a distinctive color. It must be both bright and chromatically different from near-by signs. The solid representing the perceptions of the colors of self-luminous areas, however, differs in one important way from those referring to object colors. Self-luminous areas do not give rise to the perception either of black or of white. The lower terminus of the solid is therefore invisible, or no color at all; the upper terminus, very bright, or dazzling, the brightest impression that our eyes can give us. Because of this essential difference between self-luminous and nonself-luminous objects, the vertical dimension takes a different name. The name is brightness. Brightness varies from invisible to dazzling, the maximum brightness the observer can experience, and is a property of objects perceived to be self-luminous. Lightness varies from black to white (or per-

fectly clear) and is a property of objects perceived to be nonself-luminous.

The language of the customer can thus be reduced to such terms as lighter, brighter, grayer, redder, greener, yellower, and bluer. It is a far cry from these familiar ideas that we all know about from our own subjective experiences to the physicist's measurement of spectral reflectance, spectral transmittance, and spectral composition of radiant flux. But to make practical use of physical measurements of color we must find how to compute from these physical measurements quantities that will correlate with what the customer sees. We must bridge the gap between physics and psychology.

Psychophysical—How to Predict What the Average Customer Will See. If a manufacturer wishes to know how his product looks to the average customer, he will in the great majority of cases resort, as a matter of course, to a very direct method. He will pick out someone near at hand and ask him. This may be the vice president of the company, a man in overalls in the plant, the chief designer, a typist, or anyone who might be a typical customer. The manufacturer may even trust himself, if he thinks he can adopt the combination of gullibility and suspicion typical of the average customer. Or he will ask his chief designer or his advertising agent to make a survey. The big question is "Is the color right?"

What the customer will buy and how much at what price are questions largely responsible for making business and merchandising a considerable gamble. Skepticism regarding surveys is fairly widespread and rises sharply with each widely publicized conspicuous failure of a public poll. Many times an enterprising sign company could have, and perhaps has, made a tidy profit by promoting the sale of office signs saying, "So You Made a Survey—So What?" Nevertheless surveys, formal or informal, can serve the manufacturer and industrial designer well even regarding so difficult a subject as choice of color for a product or package (Rahr, 1947). After due consideration of the many conditions under which the product is seen by the customer, the kind of light, the distance, the things around it, and so on, some fairly wise selection of a color can be made.

After the selection of the color, which might, for example, be a deep red for a portable radio set, the manufacturer may have to duplicate the color in ten or a hundred thousand items, and he may have to plan to do the same thing year after year. The first operation may seem relatively simple. He submits the design and a sample of the color to a molder of plastics and asks for a bid. There is some talk of dimensional tolerances, and some haggling over the price and delivery.

Then the molder says, "We can match exactly the deep red color you want by making up a special formulation, but our standard Red XG-12S is very close to your color and it would simplify the molding process for us to use Red XG-12S. We have used it successfully in scores of jobs already, and we could shave a little off the price." The manufacturer remembers the results of the color survey which indicated that some deep reds were rejected because the make-believe customers thought they would not look well in the house, or on the beach, or with the rest of the luggage. Furthermore, he is not sure he wants the same color that has already been used on scores of jobs. He has visions of making this particular rich, deep red a distinctive symbol for the rich, satisfying tone of the radio. So he says, "I'll think it over. Let me have a sample of your Red XG-12S." He notices that Red XG-12S is a brownish red of moderate saturation or what might well be called a muddy red. So he submits the two reds to another molder and gets more bids. He finds that he could indeed save a few cents on every radio if he used the brownish red. But then he thinks of the color survey; he didn't get that for nothing either. So he decides to pay the few extra cents and stick to the original deep red. The first bidder was low; so the contract is written specifying a commercial match for the original deep red.

So far, nothing about physics or psychophysics has come up. Nine times out of ten, perhaps, the manufacturer gets the color he wants simply by cutting the original color chip in two, giving one half to the bidder as a guide, and keeping the other half himself to check on deliveries. There is no need for measurements. But all too frequently his experience may be that deliveries of the plastic cases for the portable radios gradually drift away from the original deep red toward the brownish red produced by the cheaper colorant mixture. The manufacturer becomes irked; he is paying a good price for the right color, but the molder is getting away with a liberal admixture of the cheaper brownish red. A few cents off on each of 50,000 cases is a lot of money. The manufacturer rejects the rest of the order, claiming they are off-color, and withholds payment. The molder threatens to sue, claiming the plastic covers are a commercial match for the agreed-on standard. The manufacturer says to himself that he will prove that the molder is mixing in some of that no-good red XG-12S. So he sends samples of red XG-12S to a dye chemist, together with two of the plastic cases, one acceptable as deep red, the other unacceptable. The chemist reports that each of these plastics contains a mixture of the same dyes; he cannot distinguish them by chemical means. The manufacturer still thinks he is being cheated. The

molder suspects that the radios have not been selling well, and the manufacturer is looking for an excuse to break the contract. Suspicions, lawyers, long-distance phone calls, conferences, plane trips, headaches. It develops that the two halves, one much thumbed, of the original color chip no longer have the same color. Finally, the delivery is accepted but at a price slightly lower than originally stipulated. The manufacturer has learned two things: first, a color chip sometimes changes color; second, no one knows exactly what a commercial color match is.

It is now time to place an order for more plastic cases. The manufacturer has learned from his previous experience. He becomes curious to know just what gave the first few radio cases delivered the right deep red color. He would like to put a clause into the next contract that would make clear when the color of the plastic is acceptable and when unacceptable. In spite of the molder's suspicions the radios have been selling well, and the manufacturer would like to get the color right again. He is no color specialist; he knows where to buy the various parts of a radio set and how to assemble and test them efficiently. But he has heard of a machine that analyzes colors—a spectrophotometer, a gadget that measures exactly how much of each part of the spectrum is reflected by a plastic specimen. A little inquiry discloses that there are several places where this tool of the physicist can be placed at his disposal for a modest fee per specimen. He can send his specimen to the Electrical Testing Laboratories in New York, or to some other of several commercial testing laboratories, or to the physics department of a near-by university, or even to the National Bureau of Standards, U. S. Department of Commerce, if the specimen is to serve as a standard in a commercial transaction. So he obtains curves of spectral reflectance for the visible spectrum (400 to 700 $m\mu$ is usually adequate) not only for the deep red specimen that he wants to duplicate and the brownish red that he wants to avoid but also for specimens of other colors, such as white, black, and tan, that he might be using for plastic covers for small radio sets. Figure 13 shows the variation of reflectance with wavelength for these five specimens.

The manufacturer is rather interested in the results of this test. He notes that the white specimen reflects copiously (80 to 90 percent) throughout the whole visible spectrum, but that the black specimen reflects only about 4 percent, regardless of the wavelength of the incident light. He is a little puzzled by observing that none of the specimens has a reflectance less than 4 percent anywhere in the spectrum, but it is pointed out to him that each glossy plastic specimen reflects

about 4 percent from the surface itself by mirror reflection; so any specimen that reflects as little as 4 percent at any wavelength is really absorbing all of that kind of radiant energy that penetrates the surface. He is also rather intrigued by the complicated shape of the spectral reflectance curve of the tan specimen and is interested to learn that this curve reveals that there are at least three pigments contrib-

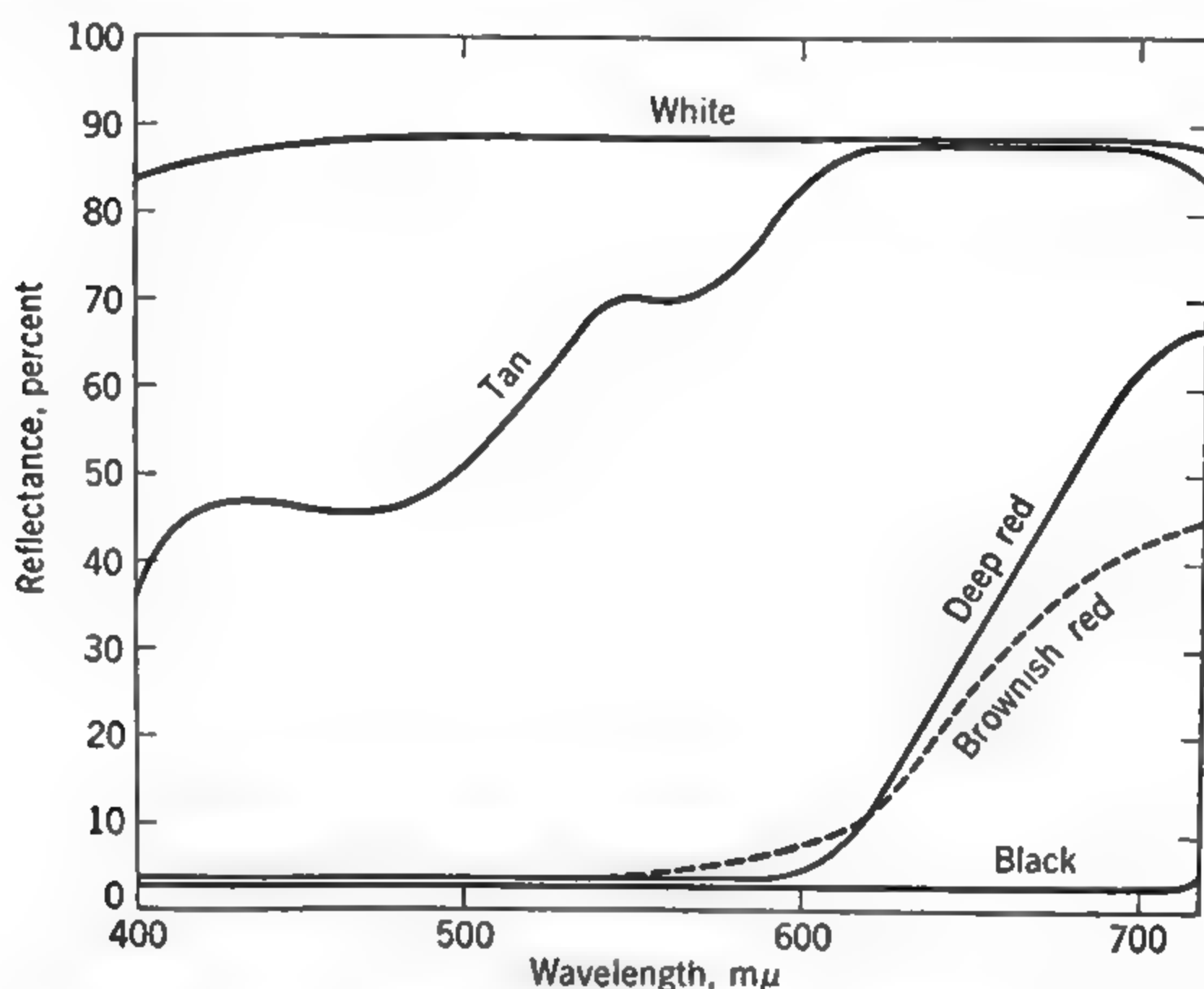


FIG. 13. Spectral reflectance curves of five imaginary plastics—white, tan, brownish red, deep red, and black. The plastics are imaginary in the sense that these curves do not correspond accurately to actual plastic colorants but have been dreamed up to illustrate the story, likewise dreamed up, of the radio manufacturer trying to buy plastic cases.

uting to the tan color, a red pigment producing an absorption maximum at about 550 mμ, an orange pigment absorbing most strongly near 470 mμ, and a white pigment with an absorption maximum somewhere in the ultraviolet (less than 400 mμ). But he is most concerned with the difference between the curve of the deep red specimen that he wants to duplicate and the brownish red specimen (dotted curve) that he wants to avoid.

By study of Fig. 13 the manufacturer has already made some correct generalizations. He has noted that the white specimen, which has the lightest color in the lot, also has the highest reflectance. And the black specimen, which has the darkest color, reflects the least. Furthermore, the tan specimen, whose color is considerably lighter than either of the reds, also reflects more at every part of the spectrum

than the red specimens do. It is evident that reflectance is related in a general way to the lightness of the color perceived. The higher the percentage reflected by the specimen, the lighter is the color perceived to belong to it.

He has made other generalizations. Both the white and the black specimens are essentially nonselective in their reflectance, that is, they do not select any one part of the spectrum much more than another to be returned to the eye of the observer. The result is that these specimens do not resemble any part of the spectrum particularly; they have neutral, or grayish, colors. The tan, the brownish red, and the deep red specimens produce progressively more saturated colors, that is, progressively more distinct from any gray; and it may be noted that the corresponding curves of spectral reflectance are progressively steeper, that is, the specimens are progressively more selective in their reflectance. The deep red specimen resembles closely the color of the spectrum near its long-wave end; and the tan specimen resembles somewhat the appearance of the spectrum near $585\text{ m}\mu$ but is a very pale, washed-out version of this hue. As a general rule, then, the hue of the perceived color corresponds to that of the region of the spectrum selected for copious reflectance by the specimen, and the saturation of the perceived color corresponds to the selectivity of the reflectance, or steepness of the reflectance curve.

The manufacturer can see now why the deep red specimen was perceived as a more saturated color; the curve of spectral reflectance is steeper. But, when he tries to apply his general rule for lightness to determine why the brownish red specimen is perceived to have a lighter color than the deep red specimen, he realizes the qualitative nature of that rule. For some parts of the spectrum the deep red specimen reflects much more copiously than the brownish red specimen. It must be that the spectral region between 550 and $600\text{ m}\mu$, where the brownish red specimen reflects more strongly than the deep red specimen, counts for more than the long-wave end of the spectrum, where the reverse is true. Now for the first time the manufacturer has realized the need for some reliable psychophysical information. He needs a basis for a purchase specification that will ensure that his customers will see the plastic cover as neither too light nor too dark, neither too yellowish nor too bluish, and not too grayish. He realizes that the first requirement, that the cover appear neither too light nor too dark, can be set as some kind of average of the curve of spectral reflectance, but the weights of the various parts of the spectrum in this average must correspond to the way in which his customers see the merchandise. This set of weights is, as a matter of

fact, given by the relative luminosity function for cone vision (see Fig. 2). This function is the sensitivity of an average normal observer to radiant energy of various wavelengths as far as the judgment of lighter or darker is concerned. This function is without doubt the most important psychophysical information required for fundamental color measurement. By means of the luminosity function we can use the techniques of physics to answer unambiguously the question, which of two colors will be perceived lighter or darker when the specimens are compared side by side by the average consumer.

The determination of the relative luminosity function is straightforward in principle. The experimenter requires the observer to adjust two halves of a photometric field to be equally bright. One half is illuminated by one part of the spectrum, the other by a neighboring part. When the adjustment is made, the radiant flux emitted toward the observer's eye from the adjustable half of the photometric field is compared with that from the fixed or standard half. The inverse of the flux ratio is taken as the relative luminosity of the two parts of the spectrum; that is, the part requiring a greater radiant flux to produce an equally bright field is taken as having the lesser luminosity. This procedure is followed throughout the spectrum, step by step, and Fig. 2 shows for daylight (cone) vision the average result for about 200 observers (International Commission on Illumination, 1924).

The relative luminosity function given in Fig. 2 has been made a part of our standard of light. It supplies the essential basis on which luminaires (gas light, tungsten-filament lamps, mercury-arc lamps, fluorescent lamps) are bought and sold. It provides the crucial test to decide which of two competing lamps supplies more light and by how much, and it has many other applications.

To determine the more complicated questions: grayer or more saturated, redder or greener, bluer or yellower, and the like, and in general the question whether one specimen color-matches another, requires the curve of spectral reflectance to be averaged by other weighting functions, sometimes called color mixture curves, one form of which is graphed in Fig. 8. The determination and use of these psychophysical functions, second in importance only to the luminosity function, are fraught with pitfalls and shadows. We must approach them warily. Figure 14 shows the most generally used form of these weighting functions. The central curve (\bar{y}_λ) is the luminosity function, and, by weighting the curve of spectral reflectance by each in turn (first by \bar{x}_λ , then by \bar{y}_λ , then by \bar{z}_λ) and averaging each separately, we obtain the tristimulus values, X , Y , Z , of which Y

is the luminous reflectance. The chromaticity of the color is usually indicated by chromaticity coordinates (x, y, z) computed from the tristimulus values, X, Y, Z , as fractions of their total, thus: $x = X/(X + Y + Z)$, $y = Y/(X + Y + Z)$, and similarly for z . From the chromaticity coordinates may be computed dominant wavelength and purity. Dominant wavelength of a color is the wavelength of the part of the spectrum required to be mixed with some fixed light (like daylight) to produce the color. Purity is the ratio of the amount of

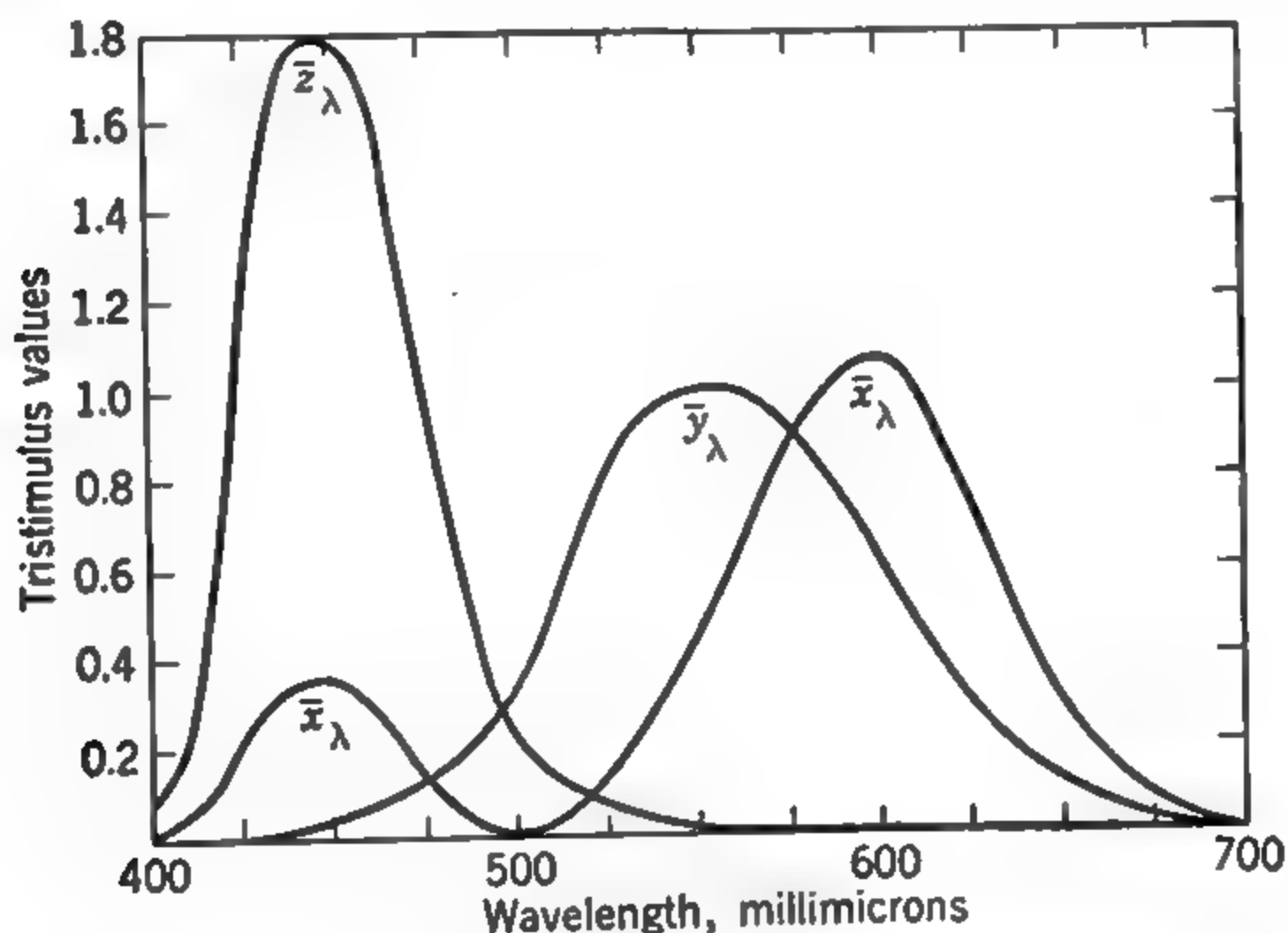


FIG. 14. Weighting functions used to reduce spectrophotometric data to colorimetric terms (tristimulus values, X, Y, Z). These functions define the standard observer recommended in 1931 for colorimetry by the International Commission on Illumination—the 1931 CIE standard observer.

the spectrum component in this mixture to the sum of the spectrum and the daylight components. Dominant wavelength correlates somewhat with judgments of hue; and purity correlates with judgments of grayness, though in an inverse and restricted way.

Visual psychophysics is a relatively new branch of science, and the application of psychophysical data to the solution of practical business problems is a relatively new technic. It is enough at this stage merely to mention these few psychophysical color terms. Complete definitions and fuller discussion will come later. Suffice it to say, that in a few industries, the terms, chromaticity coordinates, tristimulus values, and luminous reflectance are already recognized as essential parts of the solutions to the perplexing problems of color measurement and control. These are terms of visual psychophysics.*

* We are indebted to the Committee on Colorimetry, Optical Society of America, for developing and bringing into wide acceptance the convenient and logical

Summary of Basic Terms. Most of the terms mentioned have had to do with the color of opaque surfaces, and these are summarized in Table 2 so as to emphasize the distinctions between the physical, psychological, and psychophysical aspects of color. However, there are quite analogous sets of terms applicable to the colors of transparent volumes and to the colors of self-luminous objects, and for the sake of completeness these have also been included in Table 2. It is intended that Table 2 will serve chiefly to orient the reader at this stage, and further discussion of these terms is given at more appropriate places later in the book.

Table 2. Basic Color Terms for Various Kinds of Objects

Kind of Object	Physics (Radiant Energy)	Psychophysics (Color)	Psychology (Color Perception)
Opaque surface	Spectral reflectance	Luminous reflectance	Lightness (black to white)
Transparent volume	Spectral transmittance	Luminous transmittance	Lightness (black to perfectly clear)
Self-luminous area	Spectral radiance (areal density of spectral radiant intensity)	Luminance (photometric brightness)	Brightness (invisible to dazzling)
All	Spectral selectivity	Chromaticity dominant wavelength, purity or chromaticity coordinates	Chromaticness hue, saturation or red-green, yellow-blue

COLOR MATCHING

If an observer with normal color vision attempts to adjust one controllable element of his central visual field so that it matches a neighboring element, he will ultimately discover that three independent adjustments have to be at his disposal. If he is using the red, yellow, and blue paints frequently found in primary grade schools, only by chance will he obtain a match from a mixture of two of them. Even a brown color requires blue in addition to red and yellow. Within the color gamut of the three paints, an exact match for any given color is easily possible, but three primary paints are the irreducible minimum. The same considerations apply to the magenta, yellow, and set of color terms used here; see published report of the committee (O.S.A., Committee on Colorimetry, 1944, 1945). Particular acknowledgment should go to Dr. L. A. Jones, Chairman, and Dr. D. L. MacAdam, who jointly prepared most of the report, and to Eastman Kodak Company, which has borne most of the cost of the twenty-year study.

cyan colorants used in color photography, lithography, and color printing. Similarly, if he is trying to color-match one spot of light by shining several spotlights of different color onto the same neighboring spot of a white screen, he finds, in general, that either three lights of fixed spectral composition are required, or, if two lights are added together, not only the amounts of both but also the spectral composition of at least one has to be adjustable. The same rule applies to rotary mixture on a sector disk: four sectors, giving three independent adjustments, are necessary and sufficient. Normal color vision is tridimensional.

We have mentioned the tridimensional nature of normal color vision several times. It was pointed out in connection with Fig. 8 that there must be at least three different photosensitive pigments or filter-pigment combinations in the retina to account for what we see. There must also be three weighting functions (\bar{x}_λ , \bar{y}_λ , \bar{z}_λ) as in Fig. 14 to interpret the results of the spectrophotometer and so obtain a color measurement from them. From Table 2 it is seen that the color of any kind of object requires a triad of values to specify it; this triad consists of the luminous term appropriate to the kind of object combined with a two-part specification of chromaticity. And, finally, the description of the color perception likewise requires three variables, one (lightness or brightness) appropriate to the mode of appearance of the color, the other two describing the chromaticness of the perception.

The various ways by which one part of someone's central visual field may be color-matched to a neighboring part reveal the tridimensional nature of normal color vision, but we must analyze what happens to the radiant flux on its way from the source to the retina in each case.

By Addition of Lights. One of the most direct ways to study how our eyes permit us to perceive color is by addition of lights. First, set up a spotlight (say, incandescent-lamp light) to shine on a white screen. The radiant energy from this spotlight is reflected and refracted by the pigment particles of the screen and leaves the screen well distributed according to direction so that an observer can see the spot equally well from many directions. In any one direction only a small fraction of the radiant flux reflected by the screen enters the eye. Since the particles of white pigment act on radiant energy of one wavelength in the visible spectrum closely in the same way as on that of another (see Fig. 13), this small sample of radiant flux is distributed in the spectrum closely in the same way as that incident on the screen. The observer immediately grasps the meaning of this complicated (that is, physically complicated) situation. He sees a white screen

with a spot of yellow light on it. More exactly, it is a bright spot of poorly saturated reddish yellow color. The color does not belong to the screen but to the light shining on it.

Then let a neighboring spot on the screen be illuminated simultaneously by three lights of widely different colors, say red, green, and blue, as in Fig. 15. As before, the white pigment particles diffuse the radiant energy from each of these three lamps quite impartially. The

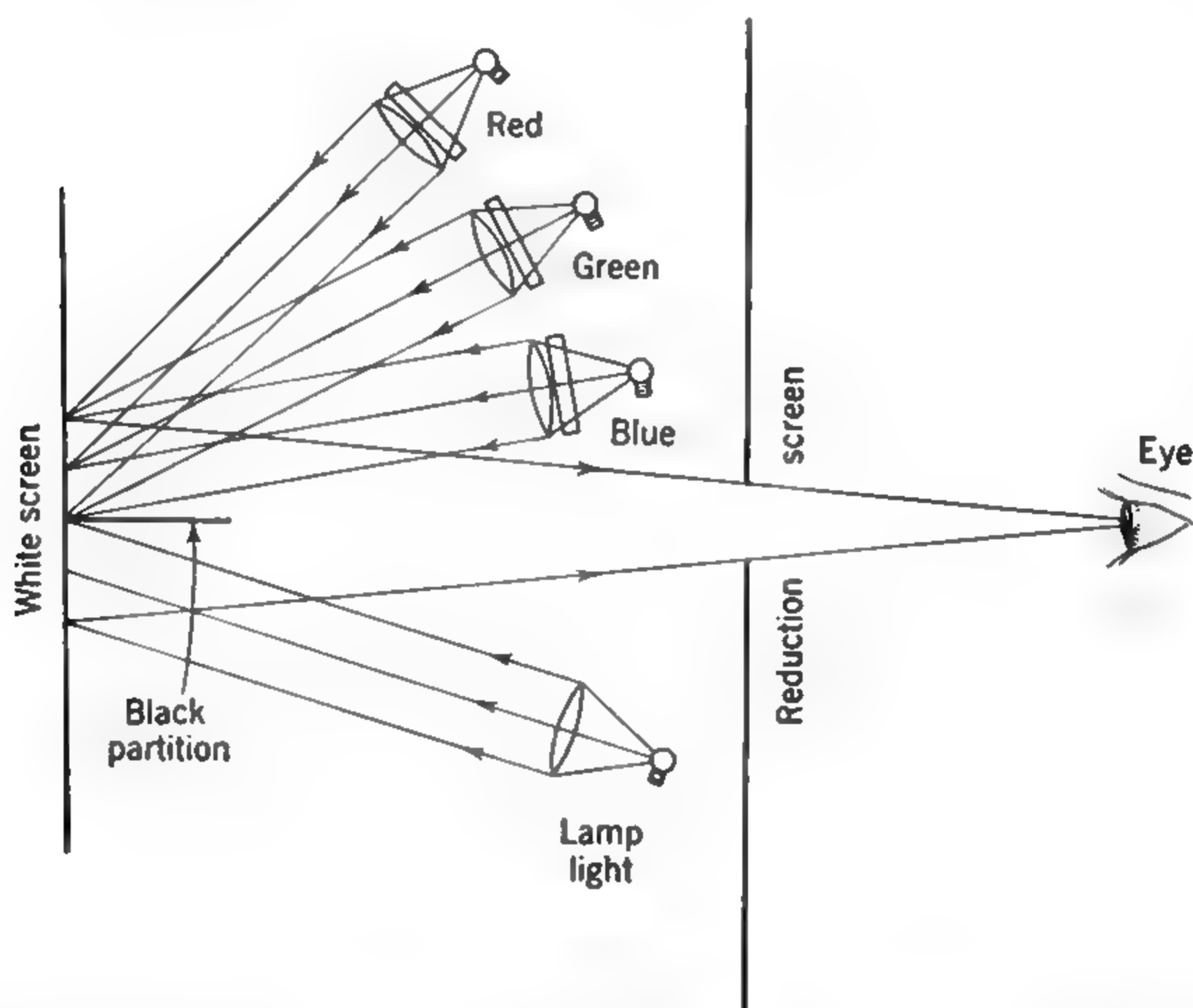


FIG. 15. Diagram of an arrangement of spotlights to demonstrate additive color mixture. This simple arrangement illustrates the principle of tristimulus colorimetry.

radiant flux entering the eye of the observer is the sum of the separate fluxes that would enter the eye from each lamp separately if the other two were turned off; and the spectral radiant flux from the combination can be found from the wavelength distributions of spectral radiant flux from each separately by simple addition. This may be checked by the spectrophotometer. Here the observer sees a second spot as red, green, blue, or some intermediate color, depending on the relative amounts of radiant flux received from the red, green, and blue lights. Again, the color is seen as belonging to the light rather than to the screen, which, if it is seen as a screen at all, is seen as white. It simplifies matters if the nonilluminated portions of the white screen are hidden by a diaphragm; then the observer sees the spots to be simply self-luminous areas whose colors can be described in terms of

brightness, hue, and saturation (see Table 2). This is called reducing the color to an aperture color, and the diaphragm is called a reduction screen.

We are now set up to make some very fundamental experiments on color vision, both qualitative and quantitative. First, we can find out whether the yellow color of an incandescent lamp is producible by a mixture of radiant energy from red, green, and blue lamps. The required proportion is found to be a preponderance of red, somewhat less of green, and still less of blue. And, in general, we find that colors perceived to have any hue whatever can be produced by this mixture of red, green, and blue lights. By turning out the blue light and altering the proportions of red and green, we can produce a series of colors passing through a yellow color perceived to be neither reddish nor greenish. By turning off the green light, and altering the proportions of red and blue, we can produce a series of purple colors passing through a purple perceived to be just as reddish as it is bluish. And by turning off the red light we can produce a series of greenish blues passing through a cyan color perceived to be just as greenish as it is bluish. Colors perceived to be less saturated than these border colors are produced when all three lights are shining at the same time, and there is a small range of proportions of the red, green, and blue lights producing mixture colors of such low saturation that they may be said to have no definite hue. These are called achromatic or neutral colors.

Further study shows that not all the colors that a self-luminous area may take on can be matched by this combination of red, green, and blue lights. Some are too bright to be matched. But we recognize this to be merely a limitation of the radiant flux at our disposal. These we could match by using higher-wattage bulbs in the projectors, or by applying more voltage to the bulbs, or by concentrating more flux on a given area of the screen by means of bigger lenses. However, and more important, some are too saturated to be matched. For example, if the incandescent-lamp spotlight used as the comparison source is covered with a yellow gelatine filter, or a yellow glass filter like those producing the amber traffic signal, or a yellow liquid filter such as is produced by a solution of potassium chromate, the resulting yellow color may be too saturated to be matched even if the blue spotlight is turned off. The colors producible by the mixture of the red, green, and blue lights chosen are spoken of as the color gamut of the system. Those that cannot be so matched are said to be out of the color gamut.

We can increase the gamut of a three-light mixture by choosing the three lights as parts of the spectrum itself. But experiments of this kind during the last 200 years (Newton, 1730; Helmholtz, 1853; Maxwell, 1860; König, 1892; Abney, 1900; Wright, 1928-1929; Guild, 1931) have shown that there is no choice of three lights that will yield color matches for all other lights. Considerable ranges of spectrum colors and near-spectrum colors always fall outside the gamut provided by any one choice of three lights.

It is evident that this simple experiment has put us in contact with some fundamental property of the eye. The usual interpretation of this finding is that the spectral sensitivities of the photopigments of the eye overlap (see Fig. 8); that is, nearly all parts of the spectrum serve to excite more than one of these pigments, hence, more than one of the three independent receptor systems comprising the visual mechanism. Conversely, if it were possible to find parts of the spectrum that yield excitation of each receptor system to the exclusion of the other two, then, and only then, could we expect to duplicate all colors with mixtures of these three spectrum lights.

Grassmann's Laws. The qualitative information revealed by these simple experiments with mixtures of three lights was summarized by Grassmann in 1853 as the first two of his principles or laws:

1. The eye can distinguish only three kinds of difference or variations (expressible as variations in dominant wavelength, luminance, and purity).

2. If, of a two-component mixture, one component is steadily changed (while the other remains constant), the color of the mixture steadily changes.

Grassmann then proceeded to state the most powerful principle of all, likewise verifiable with this simple experiment:

3. *Lights of the same color* (that is, same dominant wavelength, same luminance, and same purity) *produce identical effects in mixtures regardless of their spectral composition.*

All modern colorimetry is based upon this principle. It means that we can deal with lights on the basis of their colors alone, without regard for their spectral composition. It generates the following important corollaries:

- (a) Two lights of the same color, added to two other lights of the same color, produce mixtures that likewise have the same colors. That is, if light *a* matches light *b*, and light *c* matches light *d*, then the mixed light produced by *a* added to *c* color-matches the mixed light *b* added to *d*. This is equivalent to the arithmetical axiom: if equals are added to equals, results are equal.

(b) Two lights of the same color, each subtracted respectively from mixtures of equal color, leave remainders that color-match. That is, if light *a* matches light *b*, and mixture *c* matches mixture *d*, then the remainder produced by removing *a* from mixture *c*, color-matches the remainder produced by removing *b* from mixture *d*. This is equivalent to the arithmetical axiom: if equals are subtracted from equals, results are equal.

(c) If one unit of one light has the same color as one unit of another light, any number of units (or fraction of a unit) of the one light has the same color as the same number of units (or fraction of a unit) of the other. That is, increasing or decreasing by any factor the radiance of two lights of the same color but keeping their spectral compositions unchanged will not destroy the color match, regardless of the spectral composition of the two matching lights. This is equivalent to the arithmetical axiom: if equals are multiplied (or divided) by equals, results are equal.

That Grassmann's laws are crucial to color measurement was immediately recognized, and they have been subjected to and have passed repeated experimental tests (König, 1887). However, corollary (c) has been shown not to hold either for too great a diminution of radiance of both matching lights, or for too great an increase. Ladd-Franklin (1893) showed that the color match might break down if the field size, field luminance, and surrounding field luminance were such as to permit vision by the retinal rods to intrude. This may occur at luminances less than one footlambert. Wright showed (1934, 1936) that the color match might break down if the radiance of the two fields was increased unduly so as to approach the stimulation required to interfere with the usual nutritive processes of the retina. This may occur at luminances more than 1,000 footlamberts. Within this approximately thousandfold range, the range within which our eyes are most stable and precise, and only within this range, the methods of colorimetry apply strictly.

From the three laws, Grassmann was able to prove that the chromaticity of color can be represented in a plane diagram in which the results of mixture are found by the center of gravity principle with the amounts of the components in a mixture functioning as analogues of masses. This is Newton's law of color mixture. We may represent the chromaticity of any mixture color from the three primary lights, red, green, and blue, by a point in an equilateral triangle. The chromaticities of the primaries themselves (100 percent red, 100 percent green, or 100 percent blue) are represented by the corners of the triangle. The arbitrary scales of the amounts of red, green, and blue

are usually so adjusted that equal amounts of them correspond to some standard neutral color such as daylight or sunlight. In this diagram the results of mixtures are found by the center of gravity principle. For two-component mixtures the chromaticities are represented along the straight line connecting the points representing the chromaticities of the components. Furthermore, the center of gravity principle requires that the point representing the mixture be close to the point representing a component in direct proportion to the ratio of its amount to that of the other component. Figure 16 shows two

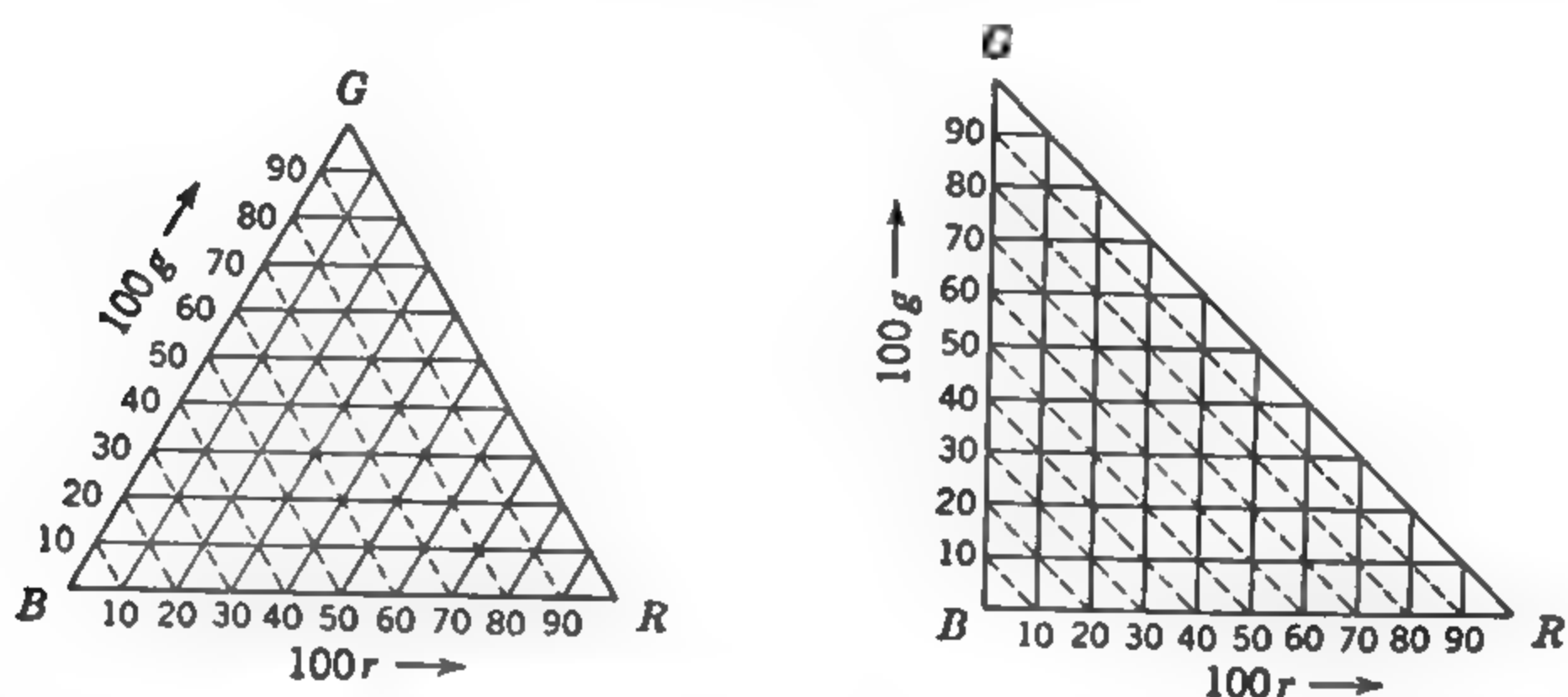


FIG. 16. Two forms of Maxwell triangle—the equilateral and the right-angled. On either form, chromaticities of additive color mixtures may be found by the center-of-gravity principle. The Maxwell triangle is now usually called a chromaticity diagram, sometimes a mixture diagram.

such triangles, called for many years Maxwell triangles, because of his introduction and practical use of them, but now called chromaticity diagrams. The equilateral form of the Maxwell triangle, the one originally used, has the advantage of treating the three primary lights in a symmetrical way. The right-angle form, now the more frequent, has the advantage of being adapted to ordinary coordinate paper. One percentage (such as $100g$) is plotted against another (such as $100r$), and, if the other percentage ($100b$) is required for any purpose, it is found simply by adding the first two and subtracting from 100. A little study will show that straight lines on the equilateral form of chromaticity diagram will plot as straight lines if transformed to the right-angle form; and indeed the center-of-gravity principle applies to each equally well.

The equipment sketched in Fig. 15 suggests a direct, practical way to measure color. For any color within the gamut of the system the amounts of the three lights, or primaries, required to produce it can serve as a measure of it. These amounts are called tristimulus values, and the equipment is, in fact, a crude tristimulus colorimeter. The

three stimuli are the three lights, and their amounts may be expressed in physical terms (such as radiant flux within the visible spectrum in terms of watts), or in formal psychophysical terms (such as luminous flux in lumens, or luminance in footlamberts), or, as is more usual, in arbitrary psychophysical terms (such as scales of red, green, and blue adjusted so that equal amounts of the three correspond to a neutral color). We could, for example, insert into the spot of incandescent-lamp light the deep red plastic for portable radio cases and find the proportions of the primaries required to produce a match. Then we could substitute the brownish red XG-12S, and we would find that a different proportion was required (less of the red primary and more of the green) to produce a match. These two proportions could be shown on the Maxwell triangle (Fig. 16) as two points whose separation would indicate amount and character (toward brown) of the chromatic difference.

To tell which of the two reds is the lighter requires another law; it cannot be shown on the Maxwell triangle which serves only to compare the chromaticities of two colors without regard to lightness. It is interesting to note that this law was also stated by Grassmann, as follows: *The luminance produced by the additive mixture of a number of lights is the sum of the luminances produced separately by each of the lights.* This law, stated as an assumption by Grassmann in 1853, is the basis for the modern photometric scale. With minor exceptions, too complicated to discuss here, this law of the additivity of luminance has been corroborated by repeated experiment. If a certain deep red is matched by three amounts (tristimulus values) of red, green, and blue lights, we may obtain the luminance of the deep red specimen by adding together the luminances produced by the red, green, and blue lights separately. A similar procedure serves for the brownish red specimen. We can then tell whether the deep red specimen is lighter or darker than the brownish red by comparing these two luminances.

The laws of color matching by mixtures of lights can be summarized in simple equations. The center-of-gravity principle means simply that, if amounts R_1, G_1, B_1 of any set of three lights produce one color, and amounts R_2, G_2, B_2 produce a second color, then the color produced by adding the first color to the second can be matched by amounts R, G, B of these three lights computed as follows:

$$\left. \begin{aligned} R &= R_1 + R_2 \\ G &= G_1 + G_2 \\ B &= B_1 + B_2 \end{aligned} \right\} \quad (2)$$

The additivity of luminances means simply that, if one light shining on a screen produces luminance L_1 and a second light shining on the same screen produces luminance L_2 , then the luminance, L , produced by both lights shining simultaneously onto the same spot of the screen can be computed by:

$$L = L_1 + L_2 \quad (3)$$

This additivity of luminances is the basis of the definition of the photometric scale adopted at the 1937 meeting of the International Committee on Weights and Measures in Paris. According to this definition, the luminance of any source is found by applying the standard spectral luminosity factors (see Fig. 2, cone). The luminance for each part of the spectrum of the unknown source is found separately, and the component luminances simply added to find the luminance of the unknown source itself. This is an application of equation 3 extended to the sum of the various parts (say 30) of the spectrum. It solves the vexing question of how to compare in brightness two lights of different chromaticity. If the chromaticity difference is great, no observer can tell precisely which of the two lights is brighter. Equation 3 gives a definite answer to this question, and this answer does not contradict what an observer of normal color vision sees except in rare cases (MacAdam, 1950).

That our eyes should work in a way that makes these simple additive laws apply is a marvel. No analogous laws can be stated for the other senses.

Additive combination of stimuli is very common in daily life. As you read this page light is being received on it from a number of sources—bulb of the reading lamp, shade, ceiling and walls of the room, and so forth. The color of the light reflected into your eyes from this page is the resultant of the colors produced by each of these sources separately and follows the above laws. When the backdrop of a stage is illuminated by chromatic wing lights, footlights, and spot light, the resulting color could be found by these laws. The color of a fluorescent lamp is the resultant of light from at least two phosphors plus some light diffused through the phosphor layer from the mercury arc; it can be computed from the separate colors of the phosphors and the arc by these laws. A prospective buyer viewing merchandise in a show window responds partly to light reflected by the goods through the window and partly to light reflected from the window. The resultant color of each area of his visual field can be computed as the sum of the effects of these two kinds of light, trans-

mitted and reflected. All of these are examples of the additive mixtures of lights. The stimuli are added together, and tristimulus values of the colors likewise can be computed by simple addition.

Negative Colors—an Interlude. The specification of colors by the tristimulus method has given rise to several useful concepts that engineers, physicists, physiologists, and psychologists have often found mysterious and puzzling. One of these concepts is the appearance of negative numbers for the specification of color by the tristimulus method. If a pale blue-green is specified by the tristimulus values: $R = 3, G = 4, B = 4$, nearly anyone can understand that the meaning is merely that 3 units of red light added to 4 units of green light added to 4 units of blue light are required to produce this particular pale blue-green. But, if the tristimulus values of a saturated blue-green turn out to be $R = -3, G = 4, B = 4$, it seems to mean that to produce this particular saturated blue-green it is necessary to add 4 units of green light to 4 units of blue light, and then take away 3 units of red light. The usual question is "How can you take away three units of red light that are not there?"

Negative specifications for color are unavoidable in color photography and lithography, and also in color television. Any color within the gamut of the particular primaries of the system (red, green, blue) can be specified by positive amounts of the primaries. At the edges of the triangular area representing this gamut one of the primaries is zero. Negative amounts are used to specify colors outside the gamut. No two tristimulus colorimeters are exactly alike. Some are made purposely different; others intended to have exactly the same triad of primaries still may yield consistently different results. It is a very natural desire to compare results from one tristimulus colorimeter with those of another. From Grassmann's third law stating that the colors of lights can be dealt with in mixtures entirely apart from their spectral compositions it is possible to write formulas showing how to compute the tristimulus values found on one tristimulus colorimeter for any color in terms of the tristimulus values of the same color found from any other tristimulus colorimeter of known primaries (Judd, 1949c).

If the tristimulus values of any color for the first triad of primaries are found to be R, G, B , the tristimulus values, X, Y, Z , of that same color for the second triad of primaries are given by:

$$\left. \begin{aligned} X &= X_r R + X_g G + X_b B \\ Y &= Y_r R + Y_g G + Y_b B \\ Z &= Z_r R + Z_g G + Z_b B \end{aligned} \right\} \quad (4)$$

where X_r , Y_r , Z_r are the amounts of the second triad of primaries required to match the color ($R = 1$, $G = 0$, $B = 0$), X_g , Y_g , Z_g are the amounts required to match the color ($R = 0$, $G = 1$, $B = 0$), and X_b , Y_b , Z_b are the amounts required to match the color ($R = 0$, $G = 0$, $B = 1$). You can see that the transformation equations are correct for any of these three colors. Take the first primary ($R = 1$, $G = 0$, $B = 0$). Substitute zero for B and G in equation 4, then unity for R , and you find $X = X_r$, $Y = Y_r$, and $Z = Z_r$, as intended.

Equation 4 comes simply from the idea that it is possible to take any sum of lights and treat it thereafter as a separate light. It applies to the mixture of any constituent with any other two constituents, if it is possible to take a sum of the constituents of a mixture and then deal with that sum as if it were a constituent in another system. In other words, this is the law of tripart mixtures of anything expressed in terms of two different sets of primaries. The corresponding law for two-part mixtures is simply:

$$\left. \begin{aligned} X &= X_r R + X_g G \\ Y &= Y_r R + Y_g G \end{aligned} \right\} \quad (4a)$$

The idea of negative amounts in mixtures enters just as inevitably in two-part mixtures as in three-part mixtures; so let us take a familiar example of two different ways of expressing a commercially important two-part mixture.

Let us deal with mixtures of alcohol and water. Let X be the amount in the mixture of the first primary (alcohol) in gallons, and let Y be the amount in the mixture of the second primary (water), likewise in gallons. The total number of gallons in the mixture is X plus Y , and the percentage of alcohol by volume is $100X/(X + Y)$. This is one common way of dealing commercially with mixtures of alcohol and water—specify it by the total number of gallons and the percentage of alcohol.

For obscure reasons the proportions of mixtures of alcohol and water flavored so as to be salable as a beverage are not always specified according to percentage of alcohol; they are often specified according to "percentage proof." In this latter system of specifying the same mixtures, one of the primaries is a fifty-fifty mixture of alcohol and water. Let R be the number of gallons of this fifty-fifty mixture. Let G be the number of gallons of water. Now X_r is the amount of the X primary for the R primary in unit amount ($R = 1$, $G = 0$), and, since 1 gallon of the fifty-fifty mixture contains only $\frac{1}{2}$ gallon of alcohol, $X_r = \frac{1}{2}$. Similarly $Y_r = \frac{1}{2}$. But 1 gallon of water in the

second system is the same as one unit of Y in the first system; so $X_0 = 0$ and $Y_0 = 1$. By substituting in equation 4a we find:

$$\left. \begin{aligned} X &= R/2 \\ Y &= R/2 + G \end{aligned} \right\} \quad (4b)$$

These are the transformation equations whereby the specification of alcohol-water mixtures on the percentage-alcohol (XY) system may be computed from the specification of the same mixtures on the percentage-proof (RG) system. We can test this out on the primaries of the (RG) system. One gallon of water is specified as $R = 0$, $G = 1$. From these transformation equations (4b) we see that $X = 0$, $Y = 1$, and, since Y is the number of gallons of water in the mixture, this checks. Similarly 1 gallon of the fifty-fifty mixture is specified on the RG system as $R = 1$, $G = 0$. From the transformation equations (4b) we find that both X and Y are equal to $\frac{1}{2}$. Since 1 gallon of the fifty-fifty mixture contains $\frac{1}{2}$ gallon of water and $\frac{1}{2}$ gallon of alcohol, this also checks.

Now the gamut of the XY system runs all the way from water to alcohol; but the gamut of the RG system runs only from water to a fifty-fifty mixture of alcohol and water. Any mixture having more than fifty per cent of alcohol cannot be expressed as a positive proportion of the primaries in the RG system. But we can compute numbers from the above transformation equations for R and G corresponding to these percentages above fifty. First we must solve for R and G explicitly, thus:

$$\left. \begin{aligned} R &= 2X \\ G &= Y - X \end{aligned} \right\} \quad (4c)$$

These are the reverse transformation equations. If we know the number of gallons of water (Y) in the mixture and the number of gallons of alcohol (X), we can compute R and G , and the percentage proof, $100R/(R + G)$, which is seen to be $200X/(X + Y)$. But this is just twice the percentage of alcohol in the mixture, $100X/(X + Y)$. So we have found, what most of us knew already, that the percentage proof is always twice the percentage alcohol. This is a demonstration that Grassmann's law for color mixtures applies to mixtures of constituents other than colors.

Mixtures with more alcohol than water fall outside the gamut of the RG system. Take as an example 1 gallon of alcohol. This is specified on the XY system as $X = 1$, $Y = 0$, and we see that the per-

centage proof is $200X/(X + Y)$, or 200. From the reverse transformation equation (4c) we find for the designation of 1 gallon of alcohol: $R = 2$, $G = -1$. This seems to mean: take 2 gallons of the fifty-fifty mixture, then subtract 1 gallon of water. This subtraction is obviously the only way to produce pure alcohol from a fifty-fifty mixture. The negative designation $G = -1$ is therefore correct. It is correct whether or not it is easy or possible to extract 1 gallon of water from 2 gallons of the fifty-fifty mixture.

We are now ready to return to the question posed earlier, "How can you take away three units of red light that are not there?" The primaries that are nominally green and blue must be thought of as containing also at least enough red to cancel out the most highly negative amount of red encountered in the specification of any blue-green light, just as one primary on the percentage-proof system really had enough water mixed with it to give meaning to the negative amount of the water primary. The tristimulus values obtained with any actual, convenient set of primary lights are convertible to other sets of primary lights for which no negative values appear, regardless of what light has to be duplicated in color. At least one of these sets refers to what actually goes on in the eye to permit us to see the color requiring the specification $R = -3$, $G = 4$, $B = 4$. Whenever a negative tristimulus value appears, it merely means that the color is outside the gamut of that particular system. A useful specification can still be obtained by adding one of the primaries to the light itself, instead of to the other two primaries. Thus, if the light combined with three units of the red primary matches four units of the green primary plus four units of the blue primary, we should write by Grassmann's corollary (b) as the tristimulus values: $R = -3$, $G = 4$, $B = 4$. This is analogous to the specification of 1 gallon of alcohol as the difference between 2 gallons of the fifty-fifty mixture and 1 gallon of water. By Grassmann's corollary (a) it really states that, if the negative portion of the specification (1 gallon of water) be added to the test specimen (1 gallon of alcohol), the mixture will be the same as the positive portion of the specification (2 gallons of the fifty-fifty mixture).

The psychophysical mystery of negative colors is thus neither mysterious nor uniquely psychophysical. We have seen by simple algebra that a negative number to specify proportions in mixture is not uniquely a property of the way our eyes judge color at all; it is simply a consequence of the general law of additive mixtures, to which our eyes fortunately conform.

There remains only to write down the general form of the reverse transformation equations for three-part mixtures. The equations are found by solving from equation 4 explicitly for R , G , and B , just as equations 4c were found from equations 4b.

$$\left. \begin{aligned} DR &= (Y_g Z_b - Y_b Z_g)X + (X_b Z_g - X_g Z_b)Y + (X_g Y_b - X_b Y_g)Z \\ DG &= (Y_b Z_r - Y_r Z_b)X + (X_r Z_b - X_b Z_r)Y + (X_b Y_r - X_r Y_b)Z \\ DB &= (Y_r Z_g - Y_g Z_r)X + (X_g Z_r - X_r Z_g)Y + (X_r Y_g - X_g Y_r)Z \end{aligned} \right\} \quad (4d)$$

where:

$$D = X_r Y_g Z_b + X_b Y_r Z_g + X_g Y_b Z_r - X_b Y_g Z_r - X_r Y_b Z_g - X_g Y_r Z_b$$

This equation is complicated because it must take account of the possibility that the second system may be based on three new primaries, each one of which may be a three-part mixture of the first set of primaries. The essential ideas are those exemplified in equations 4b and 4c. This equation is useful as a guide to the design of the color part of cameras that must produce three separate pictures to be combined additively to give a reproduction of the original scene in color, as in some systems of color photography and television in color. But this will come later.

By Rapid Succession of Lights. Another way to produce a color blend is by looking at a field illuminated alternately in rapid succession first by one light and then by a second light. If the change from one stimulus to the other is rapid enough, the colors of the individual lights cannot be perceived at all; nor can even a flicker be perceived. But a steady and uniform color is produced that is different from either of the colors of the two lights making up the intermittent stimulus. There is a very simple relation, however, between the mixture color and the two component colors.

Let us analyze what happens to the group of retinal cones receiving this intermittent stimulation. Each cone requires a fraction of a second to respond to any stimulus. If the frequency of alternation is sufficiently great, many short pulses of radiant flux of alternate colors will hit each cone within this fraction of a second. The cone will not be able to respond any differently to this pulsating stimulus than to a uniform stimulus of the same average radiant flux for each part of the spectrum. The law, therefore, is that the color of the time mixture of two lights is equal to the color of the time-weighted average of the two components. The eye will see the same color as it would if the radiant flux were uniformly distributed throughout the cycle.

From Grassmann's third law we can write the following law of time mixtures. If the tristimulus values of the first light are R_1, G_1, B_1 , and those of the second light relative to the same primaries are R_2, G_2, B_2 , then the tristimulus values, R, G, B , are likewise time-weighted averages:

$$\left. \begin{aligned} R &= f_1 R_1 + f_2 R_2 \\ G &= f_1 G_1 + f_2 G_2 \\ B &= f_1 B_1 + f_2 B_2 \end{aligned} \right\} \quad (5)$$

where f_1 is the fraction of the total time interval during which the first light acts on the retina, and f_2 is the fraction during which the second light acts. This law is somewhat like the law of color matching by

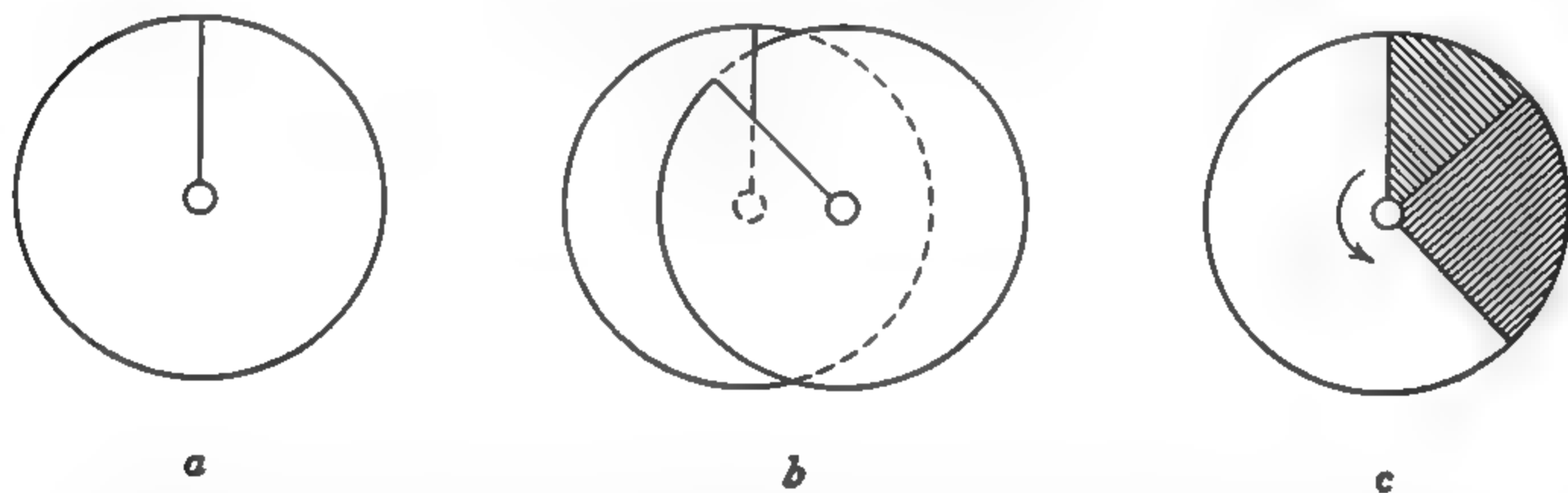


FIG. 17. Construction of a Maxwell disk for rotary color mixture by averaging.

addition of lights (equation 2), and because of this resemblance color mixture by rapid succession is often incorrectly called additive mixture. It differs from additive mixture, however, because f_1 and f_2 are fractions less than 1 ($f_1 + f_2 = 1$). Mixtures by rapid succession are thus more properly termed mixtures by averaging.

The easiest way to set up a color match with an unknown color by time-weighted averaging is to use a rotating sectored disk. Such a disk (see Fig. 17) is known as a Maxwell disk because of the important use made of rotating sectored disks by Maxwell (1860) in his studies of color. If the unknown color is mounted on a small disk, and if the three colors (such as red, green, and blue) chosen as primaries are mounted on disks slit along a radius and interlocked with a disk (perfect black) that does not reflect any light to the observer's eye, we have the essentials of a tristimulus colorimeter. The observer must vary the relative sizes of the red, green, blue, and black sectors until a color match is obtained for the unknown color of the central disk. If f_r, f_g, f_b are these relative sizes (each expressed, for example, as the ratio of the angle of the sector in degrees to 360), the unknown color may be specified by these numbers, which are, in fact, tristimulus values of the

unknown color relative to the colors of the sectors (red, green, blue) as primaries. Note, however, that because of the presence of the black sector these three fractions do not add up to unity.

In actual practical use of rotating sectored disks to color-match commercial products (paint samples, hay, cotton, tomatoes, and so on) the light reflected from all four disks has to be taken into account because all mat black surfaces reflect an appreciable fraction of incident light. Furthermore, a more precise measurement is obtainable if the colors of the sectors are not chosen to be as widely different as possible (such as black, red, green, blue) but as closely alike as possible and still include in its gamut the colors of the unknowns that are to be measured. In practical cases, therefore, the colors of the sectored disks are not taken to be the primary colors of the system, but they are themselves evaluated in terms of some generally used system. If the colors of the four sectors have tristimulus values: X_1, Y_1, Z_1 ; X_2, Y_2, Z_2 ; X_3, Y_3, Z_3 ; and X_4, Y_4, Z_4 , respectively, derived from their curves of spectral reflectance by means of the weighting functions of Fig. 14, then the tristimulus values, X, Y, Z , of any combination of the sectors can be computed as:

$$\left. \begin{aligned} X &= f_1 X_1 + f_2 X_2 + f_3 X_3 + f_4 X_4 \\ Y &= f_1 Y_1 + f_2 Y_2 + f_3 Y_3 + f_4 Y_4 \\ Z &= f_1 Z_1 + f_2 Z_2 + f_3 Z_3 + f_4 Z_4 \end{aligned} \right\} \quad (5a)$$

where f_1, f_2, f_3 , and f_4 are the relative sizes of the sectors expressed so that $f_1 + f_2 + f_3 + f_4 = 1$. Equation 5a says merely that the tristimulus values of the mixture by rapid rotation are time-weighted averages of the tristimulus values of the components. The tristimulus values, X, Y, Z , of the mixture apply also, of course, to the unknown color that matches the mixture.

This averaging law applies to any number of components in mixture by rapid succession; equation 5 gives the law for two-component mixtures, equation 5a for four-component mixtures. Sometimes the question is asked, "Why do you use four-component mixtures on a rotating sectored disk to specify color which, for the normal observer, is a tridimensional quantity?" The answer is that the variables (f_1, f_2, f_3, f_4) under the control of the observer are not independent. After the observer has chosen three of them, the other one is already fixed; it has to be what is left out of the 360 degrees of the circle. Therefore, four disks give you three degrees of freedom. Two disks, black and white, can produce all the colors seen by a totally color-blind observer; this is a one-dimensional array. Three disks, black, blue, and yellow, can

produce essentially the whole gamut of colors seen by a red-green confuser; this is a two-dimensional array. Four disks, black, blue, green, red, can produce essentially the whole gamut of colors seen by an observer of normal vision; this is a three-dimensional array.

Stimulation of the eye by rapid succession of lights occurs because of the small tremors of the eye whenever a distant detailed object is viewed. Each retinal element (cone) receives a succession of different stimuli and responds with the color corresponding to the average. A tree too distant to permit each individual leaf to be seen still reveals a number of patches of color, each patch being the average of a number of smaller patches, some bright corresponding to well-illuminated leaves, others dark corresponding to shadows. Even when the elements are so small as to fall entirely upon a single retinal cone of a perfectly steady eye and so produce a steady instead of an intermittent stimulus, the uniform color seen is the average of the colors of the unresolved elements; so the same law applies. Artists take advantage of this kind of color mixture in their mosaics and pointillistic paintings (paintings produced by small, differently colored, juxtaposed dots). Lithographers take advantage of it in half-tone printing.

In motion pictures, the eye is presented every second with 48 bright pictures separated by dark intervals about as long as the bright, the steady colors seen being the time-weighted average of the bright phase with the dark interval. The luminance of the motion picture is thus roughly one-half that which would be produced by stopping the film. In a black-and-white television picture each element is bright 30 times a second, the intervening dark interval being about 1,000 times as long as the bright interval. As a result the television picture has a luminance less than one-thousandth part of that of the portion of the cathode-ray tube actually being excited by the scanning electron beam. Production of colors by rapid succession of lights is thus a part of our daily lives, particularly when we are looking at scenes that we have constructed for our own pleasure. The purchase and sale of such scenes are an important part of our commerce.

By Mixture of Colorants. If a customer hands a color sample to a manufacturer and asks for an article (paper, plastic, fabric, painted article, and so on) to be made of the same color, the manufacturer will almost always have to resort to a mixture of colorants. He will have at his disposal a number of colorants, no one of which will produce the desired color; so he selects a colorant (pigment or dye) that is fairly close to the desired color, then adds small quantities of one or two others to produce it. If a paper or textile has to be dyed, he

will put two or more dyes into the bath and adjust the proportions to get the color asked for, or a reasonable approximation thereto. Similarly, if a plastic or a paint has to be supplied to match the customer's sample, the manufacturer will formulate a mixture of that color from the colorants at his disposal. In general, these articles do complicated things to an incident beam of light because some of the light is absorbed on striking a pigment particle, or dyed fiber, and some is scattered. This scattered light then strikes other pigment particles, the whole process is repeated, and then repeated again, and you lose track of what happens to a single element of the incident light.

Only one case is simple enough to be of interest at this stage; this is the case of transparent colorants. Suppose a customer wants to buy large sheets of plastic to market as a contrast improver for television receivers. He has a sheet of bluish green plastic that transmits relatively little incandescent-lamp light, but it transmits a relatively large amount of the kind of light emitted by the cathode-ray tubes used in television receivers. Since the incandescent-lamp light has to pass through the filter twice before it gets to the eye of the observer, and the light from the tube has to pass through only once, the plastic will protect the television-screen image from important degradation due to extraneous light falling on the screen from the room. Also it gives a rather pleasant bluish green color to the picture itself, and the customer thinks he can market the plastic filter to the public on the ground that it is easy on the eyes. The would-be purchaser of the plastic may know that color has little to do with eye strain in this application, but the important consideration is that his product should have ready acceptance by the public, and he knows that there is a (quite unproved) widespread conviction that green is easy on the eyes. So he wants to duplicate this particular bluish green color.

The plastic manufacturer does not happen to stock this particular bluish green color, however; so he shows the customer a laminated filter composed of two layers, one a green plastic, the other a pale blue. Since these plastics are colored with transparent dyes, the physics of the combination is relatively simple. Consider what happens to the part of the incident light that penetrates the first surface. For each part of the spectrum the action of the filter is to absorb a part of this penetrating flux and to transmit the remainder. The ratio of the transmitted flux to the penetrating flux of that wavelength is called internal spectral transmittance, T_1 . This fraction, T_1 , from the first layer then enters the second layer, and here a fraction T_2 is transmitted. It is evident that, if the first layer transmits 50 percent of the flux pene-

trating its surface, and the second layer transmits only 10 percent of this, the fraction of the penetrating flux incident on the back face of the double layer will be only 50 percent of 10 percent, or 5 percent, of that penetrating the front face. The law of this kind of colorant mixture is therefore:

$$T_i = T_1 T_2$$

or expressed in logarithms, greater than zero:

$$\log (1/T_i) = \log (1/T_1) + \log (1/T_2) \quad (6)$$

where T_1 is the spectral internal transmittance of the first layer, T_2 that of the second, and T_i that of the two-layer laminated filter. That is, the law is simply that of multiplying the internal transmittances of the components for each part of the spectrum to find the internal transmittance of the combination. By taking logarithms of the reciprocals, this law can be expressed as an addition of positive numbers, and the name absorbance is sometimes given to $\log (1/T_i)$. This law can be restated by saying that for any part of the spectrum the absorbance of a laminated filter is equal to the sum of the absorbances of the separate layers. If there are a number, n , of identical layers in the lamination, equation 6 gives the internal transmittance, T_i , of the lamination as:

$$\log (1/T_i) = n \log (1/T_n) \quad (6a)$$

where T_n is the internal transmittance of one of the n layers.

Figure 18 shows the application of equation 6 to a laminated filter composed of two layers, a green and a pale blue. The actual ratio between the transmitted and incident flux (transmittance) is less than is shown by about 10 percent because of reflection losses at the two faces of the lamination.

Because the action of each layer is to subtract from the incident beam some fraction of its original flux at each part of the spectrum, this kind of combination of colorants is called subtractive combination. It is also true that mixtures of light-scattering colorants behave much more closely like transparent layers through which the light must pass in succession than like small dots side by side, and these too are often called subtractive mixtures of colors. But the facts are much more complicated than either kind of combination. Only since 1930 by means of differential equations and hyperbolic functions have these refractory subjects been hammered into manageable shape. We

shall postpone consideration of these complicated subjects until a later chapter.

We have seen that in color matching by addition of lights, as well as by rapid succession of lights, the colors of the mixtures can be pre-

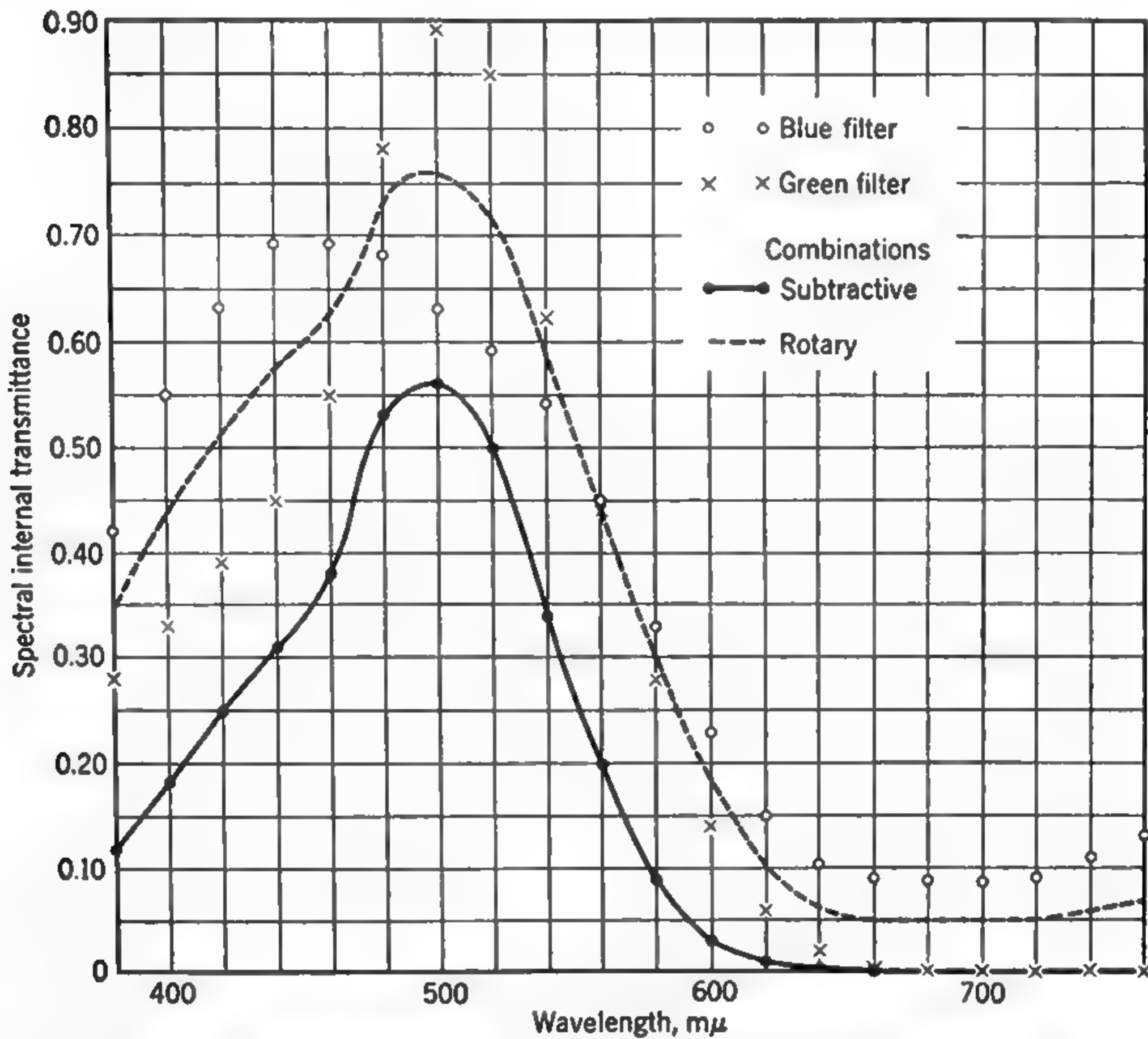


FIG. 18. Spectral transmittances of a blue and a green filter for which the pseudo-law of subtractive color mixture, "Blue plus green gives blue-green," holds.

dicted from the colors of the components. It is natural to inquire whether a similar relation holds for mixtures of colorants. That is, if the manufacturer were asked to duplicate a color (X, Y, Z) by a mixture of lights, he could do it from equation 2 merely by measuring the lights themselves. It is also true that a large gamut of colors can be produced by the lemon yellow, cyan, and magenta colorants used in color photography. The question is, then, can he compute the proportions of the mixture required to duplicate the desired color.

From equation 3 we have seen that the colors of additive mixtures can be predicted from the colors of the components. We do not need

to know the spectral compositions of the primaries. We can write qualitative laws for additive and rotary mixture thus:

Neighboring hues

Red plus yellow gives orange.
 Yellow plus green gives green-yellow.
 Green plus blue-green gives bluish green.
 Blue-green plus blue gives greenish blue.
 Blue plus purple gives purple-blue.
 Purple plus red gives red-purple.

Separated hues

Red plus green gives yellow.
 Green plus blue gives blue-green.
 Blue plus red gives purple.

Complementary hues

Yellow plus blue	} give neutral.
Blue-green plus red	
Purple plus green	

Not many years ago it was customary to write corresponding "laws" for subtractive mixtures of colorants, thus:

Neighboring hues

Red plus yellow gives orange.
 Yellow plus green gives green-yellow.
 Green plus blue gives blue-green.
 Blue plus purple gives purple-blue.
 Purple plus red gives red-purple.

Separated hues

Yellow plus blue-green (cyan) gives green.
 Blue-green plus purple (magenta) gives blue.
 Purple (magenta) plus yellow gives red.

Complementary hues

Red plus blue-green (minus red)	} give black.
Green plus purple (minus green)	
Blue plus yellow (minus blue)	

There are many combinations of colorants for which these "laws" of subtractive mixture hold, but there are also many combinations for which they are spectacularly wrong. The true law of subtractive mixture (equation 6) explains both why these pseudo-laws hold, whenever they do hold, and why they fail, when they fail. Figure 18 is an example in which the pseudo-law of subtractive mixtures, "Blue plus

green gives blue-green," holds. The dots represent the spectral internal transmittance of a blue plastic filter, such as might be considered for a contrast improver for a television receiver. The crosses refer to a green filter that might be combined with it. By equation 6 the spectral internal transmittance of a lamination of these two components is computed at each wavelength as the product of the transmittances of the components (solid curve). The dotted curve is the average of transmittances for the blue and green filters. It represents the spectral internal transmittance of a rotary filter composed half of the blue and half of the green filter. It is obvious that both of these mixture curves correspond to properties intermediate to those of the components. For both types of mixture in this case we may say, "Blue plus green gives blue-green." But the blue-green produced by subtractive combination is darker and purer than that by rotary mixture.

Figure 19 shows curves of spectral internal transmittance for a blue filter (circles) and a green filter (crosses), both filters having the fairly usual property of transmitting freely in the long-wave (far-red) region of the spectrum. It is not possible to tell from looking at these curves of transmittance against wavelength whether these filters will really have a blue or green color when viewed in daylight. We do not know how much weight to give to the far-red transmittance band. But, if the curves are averaged by means of the weighting functions of Fig. 14, or if such filters are viewed in daylight, it will be discovered that they are really blue and green filters, respectively. Note that the average of their transmittance curves (dotted) as in Fig. 19 corresponds to the transmittance of a fifty-fifty rotary mixture of them and indicates a blue-green color. Thus, the law of rotary mixtures, "Blue plus green gives blue-green," is verified again. But note also that the product of their transmittance curves (solid curve) corresponds to a dark red color. In this case the pseudo-law, "Blue plus green gives blue-green," in subtractive mixture has been shown to be false. The true statement for this case of subtractive combination is that blue plus green gives dark red. It is not possible to tell from the colors of the components what the color of a subtractive mixture will be.

We see, therefore, that even in the simple case of nonlight-scattering colorants like dyed plastics no straightforward explicit solution for the color of the subtractive mixture in terms of the colors of the components is possible. Subtractive mixture is essentially more difficult to predict than additive mixture of colors. What has to be done is to apply equation 6 wavelength by wavelength for a number of likely combinations of colorants, compute for each combination the resulting

tristimulus values, X , Y , Z ; then by interpolation (tridimensional, of course) the correct proportions may be read off.

The length and tedium of this trial-and-error solution are such that it is used only in exceptional cases (Forrest, Kreidl, and Pett, 1948). It is more usual to make up a series of different thicknesses of the

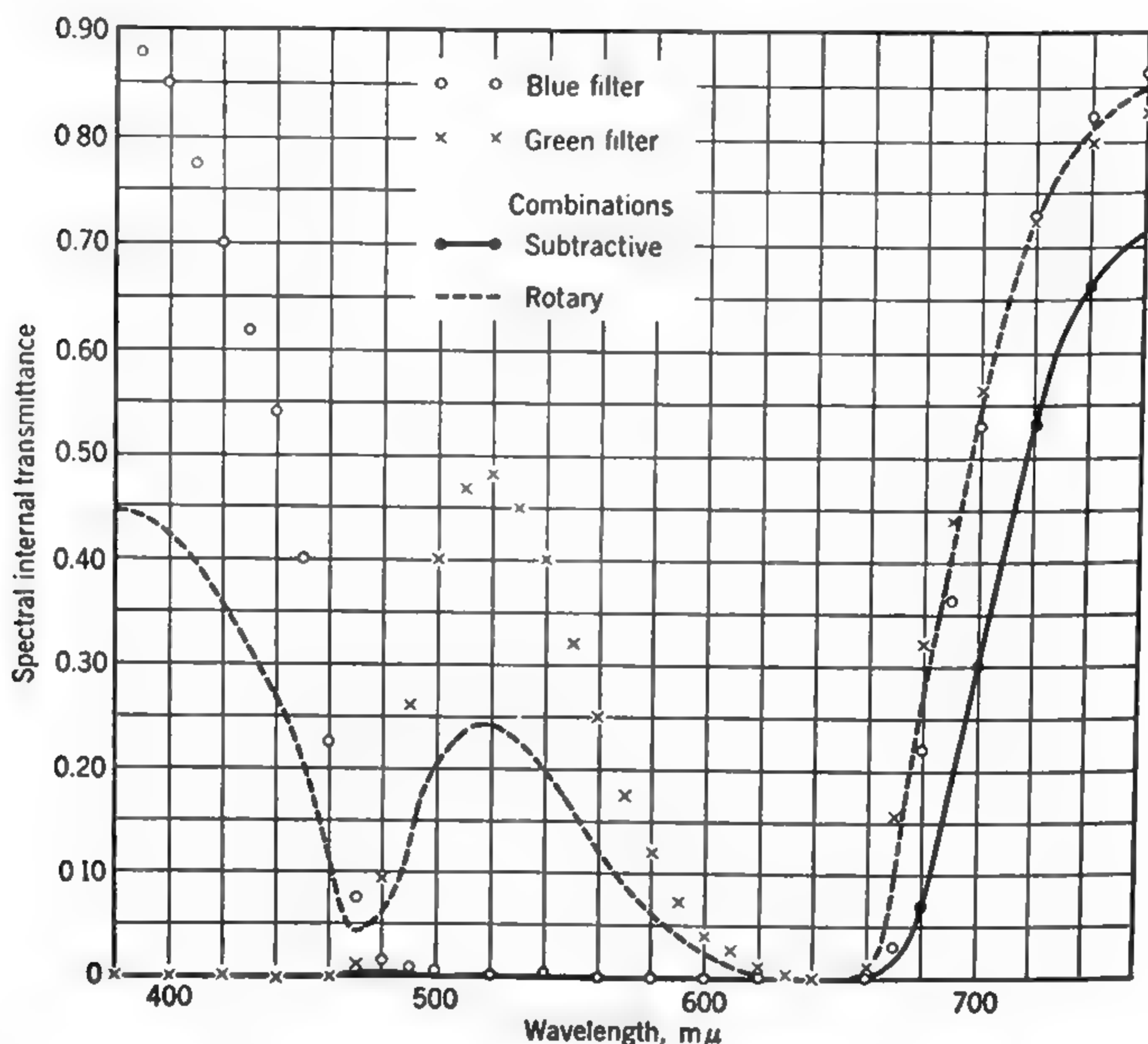


FIG. 19. Spectral transmittances of a blue and a green filter for which the pseudo-law of subtractive color mixture, "Blue plus green gives blue-green," fails to hold. In this case, blue plus green gives red.

various available colorants, and then try various combinations until visual comparison shows that a successful color match has been set up. Sometimes the colorants are put into wedge form so that variation in thickness can be made continuously as in a subtractive colorimeter. It is still more usual to choose in a nonsystematic way a convenient two-part combination of the colorants available in such a proportion as to come somewhere near what the customer asks for, and then present it to him as an improvement over what he originally submitted. If all he wants is a plastic for a contrast improver for tele-

vision receivers and he has no really good reason for wanting one particular blue-green instead of another, he will probably be easily sold on the substitute. But, if the choice has already cost the customer money, as for a survey of public preferences, and if the order is a large one, further work cannot be avoided. Either a trial-and-error formulation, yielding another color to stock, has to be carried out, or else a countersurvey has to be made in the hope of proving that the substitute is better in the first place. In any event it all comes under the heading of sales engineering and service.

COLOR DEFICIENCIES

So far the discussion has had to do chiefly with normal vision, and we have seen that normal color vision is tridimensional. For an observer of normal color vision a color may be specified by stating its equivalence to a tripart mixture, such as a mixture of three amounts of red, green, and blue light. An observer of normal vision is thus said to have trichromatic vision and is called a trichromat. Such observers may differ considerably in their ability to make fine color distinctions, but this is a difference in degree, not in kind of color discrimination. Others from birth fail utterly to distinguish colors that are easily distinguished by any observer of normal color vision. They are said to be congenitally color blind. Still others have their color vision impaired by inflammation of the retina or optic nerve. These are said to have acquired color blindness. If a color-blind observer can match all the colors that he is capable of experiencing with a mixture of but two kinds of light, he is said to be partially color blind. His visual system differs from the normal trichromatic system by being dichromatic, and he may be classed as a dichromat. It has been found experimentally that dichromats agree to color matches set up by a normal eye of the same lens and macular pigmentation, but they may set matches quite unacceptable to the normal eye. Still others can match all colors to a light of any color merely by adjustment of luminance. An observer of this sort has monochromatic vision and may be classed as a monochromat. Still others require three-light mixtures to match the colors that they experience, but most of the matches set up by an observer having normal color vision will fail to satisfy them even though allowance be made for abnormally heavy or light pigmentation of the lens and macula. These are called anomalous trichromats.

We know color-deviant observers primarily by the mistakes that they make in color matching; that is, mistakes from the point of view of the observer with normal vision. But, paradoxical as it may seem,

we also have a rather reliable idea of the colors seen by color-blind observers. This information comes chiefly from studies of a few observers found to have one normal eye and one color-deviant eye (Judd, 1948).

Classification of Visual Systems. Any type of visual system may be conveniently summarized by a description of an equal-energy spectrum as it appears to an observer of that type. The equal-energy spectrum is one having constant spectral radiance (see Table 2) throughout the visible spectrum. Every stimulus affecting the eye either is a portion of this spectrum or may be regarded as a combination of a number of such portions. A description of the appearance of the spectrum is therefore a fairly complete summary of the properties of the visual system yielding that appearance.

Normal Vision. To the observer of normal vision the spectrum for a wide range of spectral radiance appears as a series of chromatic colors varying from dim red through bright red, orange, yellow, bright yellow-green, green, blue, to dim violet. The brightest part of the equal-energy spectrum under usual observing conditions of daylight adaptation is between wavelengths 540 and 570 $m\mu$ (yellowish green), and from this point (average 555 $m\mu$) toward both greater and smaller wavelengths the brightness progressively diminishes. These brightness variations correlate with the luminosity function of the average eye whose maximum is at 555 $m\mu$ (see Fig. 2, cone; also Fig. 14, \bar{y} function). Since the normal eye has trichromatic vision it is capable of three kinds of color discrimination. It is convenient to classify the discriminations of the normal eye as light-dark, yellow-blue, and red-green; that is, a normal observer can tell light object colors from dark ones, yellowish colors from bluish ones, and greenish colors from reddish ones. For example, he can tell greenish grays from reddish grays, greenish yellows from reddish yellows, and greenish blues from reddish blues; all of these are examples of red-green discriminations.

Anomalous Trichromatism. Trichromats who will not accept normal color matches, regardless of how heavily or lightly pigmented the normal eye may be, are chiefly of two types called protanomalous and deuteranomalous. With regard to ability to make chromatic distinctions, observers of these types are intermediate to the normal trichromat and dichromats of the protanopic and deuteranopic observer types, respectively, to be discussed presently. The anomalous trichromat can make with more or less difficulty the distinctions that the corresponding dichromat cannot make at all. However, it is not possible to judge the impairment of sensibility to chromaticity differences

from the degree to which the matches of the anomalous trichromat differ from the normal matches, the correlation being low. This impairment must be measured separately. Furthermore, the luminosity function of protanomalous observers of all degrees of chromatic impairment like that of the protanope seems to be deficient at the long-wave

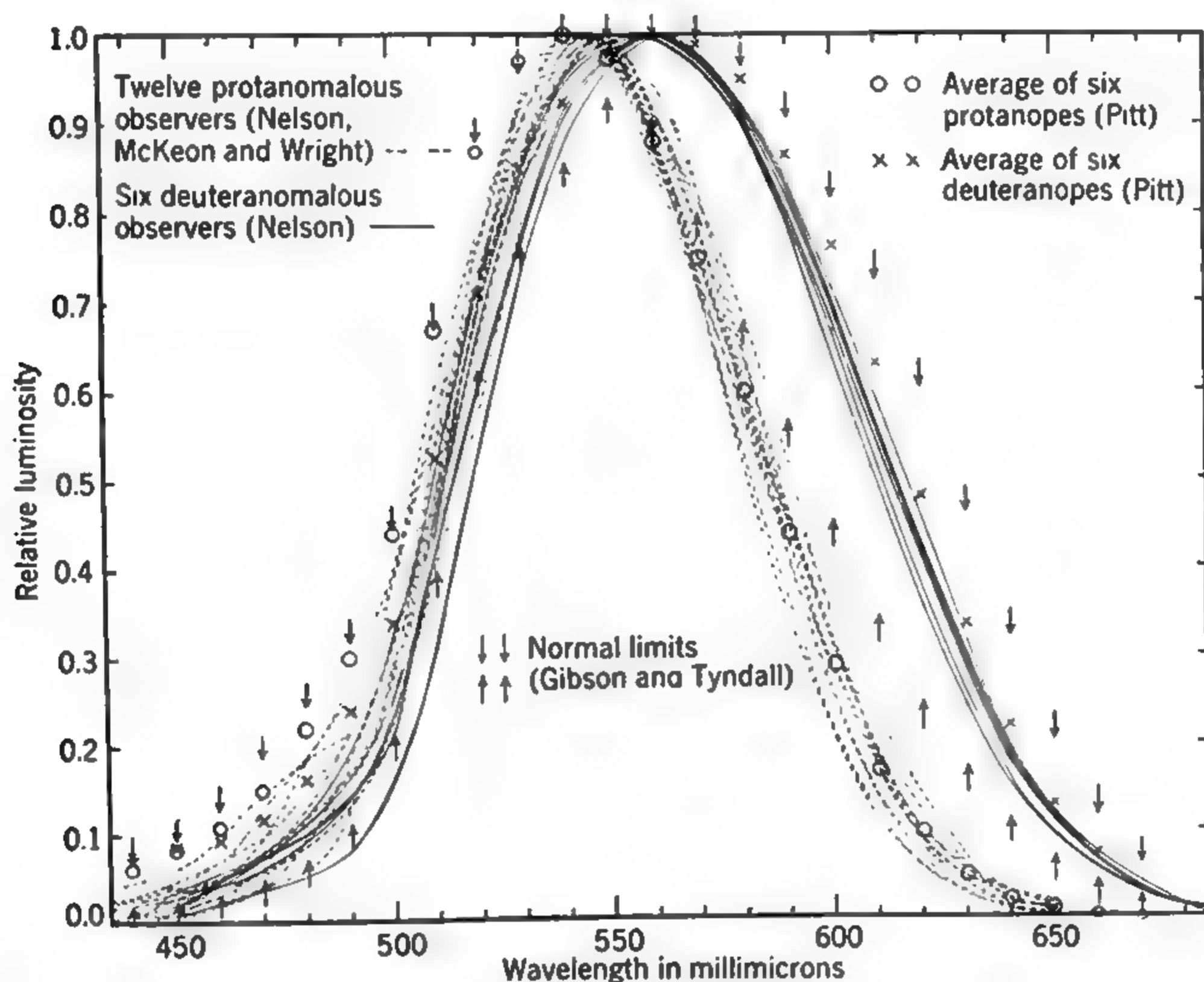


FIG. 20. Relative spectral luminosity for normal and color-deviant observers (after Judd, 1943). Note that deuteranopic and deuteranomalous luminosity functions are generally well within normal limits (arrows), but the protanopic and protanomalous luminosity functions are far lower than normal in the long-wave part of the spectrum.

end; see Fig. 20. The deuteranomalous observers, on the other hand, have luminosity functions well within the variations shown by observers with normal trichromatic vision.

Dichromatism. Dichromatic observers are capable of making color distinctions of only two kinds, one achromatic (light-dark), and one chromatic (either yellow-blue or red-green, usually the former).

The spectrum to an observer having red-green blindness appears in two hues only: the short-wave end of the spectrum appears blue; the long-wave, yellow. These two bands are separated by a region at about 495 mμ, which, like average daylight, has no hue at all and is called

the neutral point. From zero at the neutral point the saturations of the spectrum colors increase toward both the long-wave and the short-wave ends, and, of course, for these types of vision as for normal vision the equal-energy spectrum colors also become progressively less bright as the ends of the spectrum are approached. There are two subtypes of red-green blindness; one characterized by abnormally low luminosity of the long-wave portion of the spectrum, the other by a substantially normal luminosity function.

The name for red-green blindness with abnormally low luminosity of the long-wave end of the spectrum is protanopia. The wavelength of the equal-energy spectrum yielding the brightest color is about 540 $m\mu$, which is definitely, though only slightly, beyond normal limits. A protanope confuses red and bluish green with gray, and, indeed, with each other.

The name for red-green blindness with substantially normal luminosity function is deuteranopia. There is some indication, not amounting as yet to proof, that the deuteranopic luminosity function is slightly higher than normal at the long-wave end. The average location of the brightest spectrum color is about 560 $m\mu$ for the deuteranope. He confuses purplish red and green with gray, and with each other.

Dichromatic vision may also take the form of yellow-blue (more precisely greenish yellow and purplish blue) blindness, called tritanopia. The luminosity function is normal, or nearly so, a possible slight defect on the short-wave end being hard to separate from the effect of abnormally heavy lens and macular pigment. The spectrum appears red at the long-wave end, becoming more and more grayish as the neutral point (at about 570 $m\mu$) is approached. On the short-wave side of the neutral point the hue perceived is green or bluish green with saturation increasing up to about 470 $m\mu$, then decreasing toward zero as the short-wave end of the spectrum is approached. Some tritanopes have a second neutral point near the short-wave end (430 $m\mu$ and lower); others see green or bluish green of diminishing saturation as far down in wavelength as they can see anything at all. A tritanope confuses bluish purple and greenish yellow with gray, and with each other. Tritanopia occurs often as a result of inflammation or detachment of the retina but is rarely congenital.

Monochromatism. A totally color-blind observer can make only one kind of color discrimination—light-dark—and no chromatic discrimination whatsoever. His spectrum does not have merely one or two neutral points; all of it is neutral. On this account monochromatism is often referred to as achromatopsia or achromatopia. Mono-

chromatism can arise from failure of the retinal cones to develop properly. This kind of monochromatism is called either cone blindness or congenital total color blindness. Monochromats of this kind have at all times the central scotoma characteristic of normal twilight vision. They also tend to have photophobia, or fear of light, which is characteristic of owls and of the normal dark-adapted observer, and which comes from attempts to use rod vision for scenes too bright for it. Further common characteristics of such observers are low visual acuity and nystagmus, or side-to-side eye movements, as if in an attempt to improve deficient visual acuity by using retinal areas now to one side, now to the other side of a central blind area. The luminosity function of a cone-blind observer is the same as that of a normal dark-adapted observer. Compared to the normal luminosity function it is greatly deficient on the long-wave side and has a maximum at $510\text{ m}\mu$ (see Fig. 2, rod) instead of $555\text{ m}\mu$.

A less common form of monochromatism arises chiefly from inflammation of the optic nerve. Unlike cone blindness this form may show normal visual acuity, good foveal vision, and a normal luminosity function with maximum at 555 or $560\text{ m}\mu$.

Summary of Visual Systems. Table 3 repeats in tabular form the classification and characteristics of the various types of color vision.

Causes of Color Blindness. A person whose color vision deviates importantly from normal is more or less handicapped in the modern world. He cannot respond reliably to chromatic signals (traffic, railroad, marine, airplane); he does not get full benefit from color coding (office forms, telephone wires, safety markings, contents of pipes, identity of radio parts, and so forth); and certain jobs (chemist, surgeon, color matcher, interior decorator, air plane pilot, and even accountant) either are dangerous for him to undertake or place him at a severe disadvantage. He is both a problem and an opportunity in business ventures: a problem, because an employer must not assign him to tasks that are impossible for him to complete; an opportunity, because he has special needs and collectively constitutes a special consumer group so far not successfully tapped. Both to the employer and to the color deviant himself, an important question is, "Can color blindness be cured?" The answer is, "Yes and no."

Congenital Color Blindness. All forms of color vision listed in Table 3, except the last, have been found in people without previous history of disease in the eye or optic nerve. They are, therefore, considered to be congenital defects. Red-green confusion (protanopia and protanomaly, deuteranopia and deuteranomaly) are known to be sex-linked characteristics. A mother with normal color sense may transmit any of

Table 3. Classification and Characteristics of the Various Visual Systems

Designation of Type according to Number of Components	Discriminations Possible by This Type	Wave-length of the Maximum of Luminosity Function, $m\mu$	Wave-length of Neutral Points in Spectrum, $m\mu$	Preferred Designation of Type
Trichromatism	Light-dark Yellow-blue Red-green	555	None	Normal system
	Light-dark Yellow-blue Red-green (weak)	540	None	Protanomaly
	Light-dark Yellow-blue Red-green (weak)	560	None	Deuteranomaly
Dichromatism (partial color blindness)	Light-dark Yellow-blue	540	493	Protanopia
	Light-dark Yellow-blue	560	497	Deuteranopia
	Light-dark Red-green	560	572	Tritanopia
Monochromatism (total color blindness)	Light-dark	510	All	Congenital total color blindness (cone blindness)
	Light-dark	560	All	Acquired total color blindness

these defects to her sons, but her daughters by a father with normal color vision either are normal or become themselves carriers of the defect. Color-blind daughters result either from the union of a color-blind man with a woman who is a carrier of the same form of defect or from parents both of whom have the same form of defect (either proto-form or deutero-form). Congenital red-green confusion is therefore much more prevalent among males than among females. Table 4, based chiefly on the study of 5,000 school children by v. Planta (1928) and 18,000 school children by Waaler (1927), shows this clearly. Eight percent of the boys were found to possess a color-vision defect, whereas less than one-half of 1 percent of the girls had such a defect. In order to account for enough mothers for this 8 percent of red-green

Table 4. Incidence of Congenital Color Blindness

Types	Percentage of the Population That Have These Types	
	Male	Female
Protanomaly	1.0	0.02
Deuteranomaly	4.9	.38
Protanopia	1.0	.02
Deuteranopia	1.1	.01
Tritanopia	0.00001	.00000
Congenital total color blindness	0.003	.002

confusing sons, however, it follows that about 16 percent of all mothers are carriers. There are more than 10 million red-green confusers in this country, many of whom are unaware that their vision differs from normal.

Congenital total color blindness is only slightly more prevalent among males than females. It is regarded as a simple recessive characteristic; total color blinds are frequently found to have parents of close blood relationship. Table 4 shows, however, that total color blindness is much rarer than partial color blindness. There are probably less than half a million totally color-blind people in this country.

Congenital tritanopia is so much rarer even than this that it is probably misleading to express the incidence as a percentage in Table 4 at all. There are less than a score of such cases known (Judd, 1943).

Attempts to cure congenital color defects have been frequent since 1870. Warming of the eyes by hot applications and diathermy, administration of massive doses of various vitamins, exposure of the eyes to red and green lights, and color training in identification of colored patterns and in color naming are some of the treatments that have been tried. A number of "cures" have been reported, and these will be discussed further in connection with tests of color perception. But not one single authentic cure has ever been established. It seems to be just as hard to persuade the eyes to develop a color sense whose pattern is not laid down by the chromosomes at the moment of conception as it is to persuade a dog by suitable treatments to change into a cat. The answer to the question, "Can congenital color deficiency be cured?" is therefore a categorical "No!"

*Acquired Color Blindness.** A defect in color perception in the center or the periphery of the visual field may occur in any disease

* Prepared (Judd, 1943) in collaboration with Dr. LeGrand H. Hardy, Institute of Ophthalmology, New York.

affecting the retina, the optic nerve, or the optic cortex in the occipital lobe of the brain. The most common disease of the central nervous system that causes central color defects is multiple sclerosis. This is associated with defective perception of form. Several generalized non-infectious diseases cause optic neuritis with attendant loss of color and form discrimination. The most common are blood dyscrasias (pernicious anemia, secondary anemia, or leukemia) and deficiency diseases (vitamin B₁ deficiency, so-called "diabetic amblyopia," and optic neuritis of pregnancy). The defects in color perception sometimes reported to follow a blow on the head are probably due either to a similar optic neuritis or to damage to the optic cortex. The most common occurrences of a defect of color perception in the center of the retina are in connection with toxic amblyopia (loss of visual acuity), and the poisons that most commonly depress the color sensitivity of the center of the retina (as opposed to the more usual defects of the peripheral retina) are:

Carbon disulfide. Used in preparation of rayon, rubber, explosives, hides, insecticides, and wall paper.

Lead poisoning. Found in painters, plumbers, riveters, storage battery workers, and compositors.

Spinal anesthesia.

Sulfanilamide. May cause optic neuritis or hemorrhages into the optic nerve.

Snuff, iodoform, and stramonium. Rare cases.

Thallium. Used in industry as a rat poison and a depilatory.

Tobacco—alcohol. The most common of all the agents producing impairment of vision.

Toxic agents and disease affecting the conducting (transmissive) elements of the visual mechanism (nerve fiber layer of the retina, optic nerve, and tract) are classically supposed to cause red-green weakness. This is not anomalous trichromatism (deuteranomaly) in the sense that normal color matches fail to hold, but the chromatic mistakes are similar. Acquired red-green weakness is also accompanied by less than normal visual acuity and ability to distinguish yellow from blue and light from dark. The defect becomes progressively worse until the deuteranopic form of red-green blindness results. This defect differs from congenital deuteranopia, however, by being associated with lowered acuity and lowered yellow-blue and light-dark discrimination. If the poisoning or disease continues, total color blindness results.

The precise characteristics of acquired red-green weakness leading through deuteranopia to total color blindness are not known very well because the visual system of the patient is changing from day to day. On this account the name, progressive red-green blindness, is often used to denote the symptoms of inflammation of the optic nerve.

On the other hand, injuries (such as those produced by syphilitic infections) that primarily affect the receptive (rod-cone) mechanism show first a depression of the ability to make yellow-blue discriminations. Detachments of the retina from the choroid coat always show this. Acquired tritanopia, unlike acquired red-green blindness, may become stabilized and be distinguished from congenital tritanopia chiefly by the memory possessed by the subject of his former yellow and blue sensations. It is also often localized so that only a small retinal area will be tritanopic, the remainder being normal. Acquired tritanopia leads to total color blindness only when progressive red-green blindness ensues. The latter, however, may of itself lead to total color blindness as noted above.

There is no therapy for acquired color blindness since this is merely a symptom. The therapy is directed towards the underlying cause. The return of normal vision is usually by stages, the reverse of those that characterized the disease. The answer to the question, "Can acquired color blindness be cured?" is therefore a qualified "Yes." That is, sometimes the damage done is slight and temporary; in other cases it is permanent.

Types of Color Vision in the Normal Eye. Lest it be forgotten that the normal eye yields trichromatic vision only under usual conditions of observation, the following circumstances are listed under which normal eyes are color blind.

Indirect Vision. Vision in the retinal regions somewhat removed from the fovea resembles deuteranomaly just as much as do the early stages of progressive red-green blindness. There is a markedly reduced ability to distinguish red-green differences compared to yellow-blue and light-dark differences, though these, too, are considerably reduced compared to central vision. Vision in a more peripheral zone (roughly 30 to 50° from the fovea) is deuteranopic. Vision in the periphery (more than 50° from the fovea) is very much like acquired total color blindness. The boundaries between the trichromatic, dichromatic, and monochromatic zones of the normal retina, however, are not sharp. Their exact location depends upon luminance of the test field and the surround, and on the size of the test field used for the exploration.

Insufficient Size. An area of given luminance perceived by the region in and around the fovea (foveal and parafoveal region) may, by sufficient reduction of its area, be made to fail to show any yellow-blue distinctions, although red-green and light-dark distinctions can still be made easily. Measurements (Wright, 1949; Middleton, 1949) have shown that for such field sizes (about 15 minutes of arc) the normal

retina responds like the tritanopic eye and is, in fact, tritanopic. Further reduction of the area can cause the area to fail even to show red-green distinction, although light-dark distinction can still be made. For such areas the normally trichromatic foveal and parafoveal regions become monochromatic, indistinguishable from acquired total color blindness. This emphasizes the difficulty of making chromatic signals perceptible at great distances, and it also accounts for the predominant use of red and green for signal lights. At usual signaling distances blue and green lights of low intensity look very much alike, and violet and greenish yellow exactly alike. This is a sufficient explanation for the nonuse of blue and violet lights for signaling at a distance.

Insufficient Luminance. The normal dark-adapted eye responds to weak stimuli in every way exactly like that of the congenitally total color-blind. Of course, if the test field has a luminance high enough to correspond to cone vision, previous adaptation to darkness does not prevent perception of chromatic differences.

Insufficient Time. A given luminous area may be perceived by the light-adapted eye as a bright flash even though exposed for so short a time that no chromatic color is experienced. Under these conditions, as in flicker photometry, the normal eye makes light-dark discriminations only, the type of vision being closely like acquired total color blindness.

The ability to overcome these handicaps to chromatic vision renders one observer of normal vision much more valuable than another. For many tasks that observer is more valuable who can distinguish correctly the colors of small, dim, briefly visible objects even though he "catches them only in the corner of his eye." It has been found that, on the average, anomalous trichromats are much more susceptible to impairment of their chromatic vision by these circumstances than are normal observers. Because of this average susceptibility to handicaps, there has been a disposition to design color-perception tests so as to detect not only dichromats but also anomalous trichromats, and to disqualify the entire group for tasks to which dichromats are unfitted. This has sometimes resulted in anomalous trichromats being excluded from jobs that they can do quite satisfactorily.

Tests of Color Blindness. Tests of color perception are designed chiefly to detect protanopes, deuteranopes, and anomalous trichromats having extremely weak red-green discrimination. Detection of other forms of color blindness is relatively unimportant first because only a few such cases exist in the population, second because a test adequate to detect

all types is much more difficult to design, and third because the red-green type of color confusion is particularly unsuited to speedy discrimination of traffic, railway, marine, and aviation signal lights. All the successful tests have the following properties in common: (1) they present reddish and greenish varieties of colors that the subject must distinguish, (2) they present them in small areas so that some anomalous trichromats as well as dichromats will be discovered, (3) they present them intermingled with other colors in such a way as to discourage attempts upon the part of the subject to conceal either defective vision by guessing or normal vision by making mistakes on purpose, and (4) they present reds, greens, confusion and background colors of a variety of lightnesses and degrees of yellowness so that the subject cannot get any clue about the colors from his ability to discriminate yellow from blue or light from dark. The various successful tests differ essentially only as to the form of response required of the subject.

Holmgren Wool Test. A number of small varicolored skeins of wool are spread out before the subject who is required to select those resembling three larger skeins, a red, a green, and a rose.

Nagel Charts. A number of cards are shown to the subject in succession. Each card has a series of small circular colored spots arranged in a circular ring. All the spots on some of the cards are weak green, some light and some dark; spots on others are combinations of green with gray, pink, and other confusion colors, and so on. The subject is required to say which cards bear spots of but one hue and which have more than one hue.

Stilling Charts. The irregular and varicolored spots on these charts are so arranged and colored that a digit can be read on each chart by an observer of normal color vision. Some of the charts cannot be read by dichromats and anomalous trichromats having extremely weak red-green discrimination; and some even for an observer with normal color vision are hard to read.

Ishihara Charts. This improved form of Stilling chart supplies both cards that can be read by the observer with normal color vision but not by dichromats and cards that show one number to the normal-eyed observer and another to the red-green confuser. These latter are called double-number cards. Another innovation introduced by Ishihara is a card showing a bluish figure on a brownish ground to a red-green confuser, but which shows a bold red-green pattern to the observer of normal color vision that usually prevents him from discovering the figure appearing in weak bluish colors. Finally there are

cards for testing the vision of observers who cannot read numbers on which, instead of a digit to be read, there is a continuous path to be traced.

American Optical Company Test. The American Optical Company test is a compilation of copies made for war-time use of 46 of the Stilling and Ishihara charts. About half of these 46 charts have little or no diagnostic value, being read by about equal percentages of color-blind observers and those having normal color vision.

American Optical Company Test (revised selection). A selection of 18 of the successful charts of the first AO test with instructions has been approved by the Inter-Society Color Council Subcommittee on Color Blindness Studies.

Other Chart Tests. Other chart tests based on the principles developed by Stilling and Ishihara, or minor variations thereof, are the Rabkin and the Böstrom-Kugelberg charts.

The chart tests based on the requirement that the subject must try to read numbers from them are convenient, easy to administer, and yield for most subjects a definite diagnosis. They have the following disadvantages: (1) the spectral character of the light by which they are read affects what numbers a given observer will be able to read, (2) the amount of lens and macular pigment likewise affects it, (3) the color of the figure intended to be exactly the same as the background for the type of subject to be detected is, because of technical difficulties impossible to avoid, not exactly that color. Because of the first two disadvantages, some red-green confusers pass these tests on the first trial. Because of the third it is possible to teach a red-green confuser to read any given chart test from the nonred-green clues unintentionally and unavoidably present. This accounts for the mistaken belief that congenital color deficiency can be cured.

Anomaloscope. Protanomaly and deuteranomaly are detected by an instrument, known as the Rayleigh anomaloscope, that provides a mixture of two parts of the spectrum (red and green) that the observer must adjust to match a third part (yellow). Forms of anomaloscope in which the colors are produced by means of glass or gelatine filters have also been successfully used. Deviation from the normal proportion of red to green to match the yellow indicates anomalous vision, and the degree of precision of the settings indicates how well the observer can distinguish red from green.

The employer should know about the color vision of those working for him. These tests serve to single out quickly those whose color vision is sufficiently abnormal to be a handicap. These people should

work where critical color judgments do not have to be made. They should be encouraged to find out the nature of color defects of vision in general, and their own deviations from normal color vision in particular. It is usually impossible to improve defective color vision; but those who have this handicap can learn to avoid taking on responsibilities likely to lead to costly mistakes.

PART II

Tools and Technics

Our prime source of energy is the sun. Our raw materials come from the earth—from our mines and farms. Our wealth comes from using the sun's energy to process these raw materials so as to make them into things we can use—food, drink, clothes, shelter, medicines, cosmetics, automobiles, trains, boats, airplanes, telephones, radios, newspapers, books, motion pictures, television receivers, and so forth. Nearly every step of the process of changing raw materials into consumer goods and putting them into the hands of consumers is guided somewhat by the color of the raw material or product. It is not surprising that nearly every business man at one time or another encounters a color problem of some sort. This may be in the inspection of the materials that he buys, in the color control of his product, or in the finishing or packaging of his product for sale. In most cases the problem can be solved easily and cheaply without the use of color standards or measurement. But in thousands of color problems it pays to supplement the trained eye of the inspector with the special technics and tools of color measurement. In the discussion that follows, the emphasis will be, not on the technical details, but on what the tools and technics can do. New tools are constantly being devised, and old ones sharpened. It is just as important to recognize a simple job of color measurement and apply a simple tool to do it, as it is to recognize when a considerable outlay for colorimetric equipment and testing will pay off.

SPECTROPHOTOMETRY

It has already been pointed out that the fundamental way to measure the color belonging to an area is to break down into its component spectral parts the radiant flux entering the eye from that area and to measure each of these parts of the flux separately. A prism or diffraction grating serves to disperse the radiant flux into a spectrum, and such a dispersing element in combination with slits and lenses forms a spectrometer; see Fig. 21. The radiant flux to be analyzed enters the

slit at the left, is rendered nearly parallel by the lens, is refracted and dispersed by the prism, the resulting spectrum being brought to a focus by the second lens in the plane of slit *A*, which serves to isolate one or another part of the spectrum, depending on the angle to which the receptor arm is set. The sensitive receptor or radiometer (thermopile, thermocouple, photoelectric cell) is placed exterior to slit *A*, and the combination of spectrometer and radiometer is called a spectroradiometer. It is calibrated to read directly the density of spectral radiant flux of the beam falling on the entrance slit in some such units as microwatts per square millimeter per millimicron of spectrum.

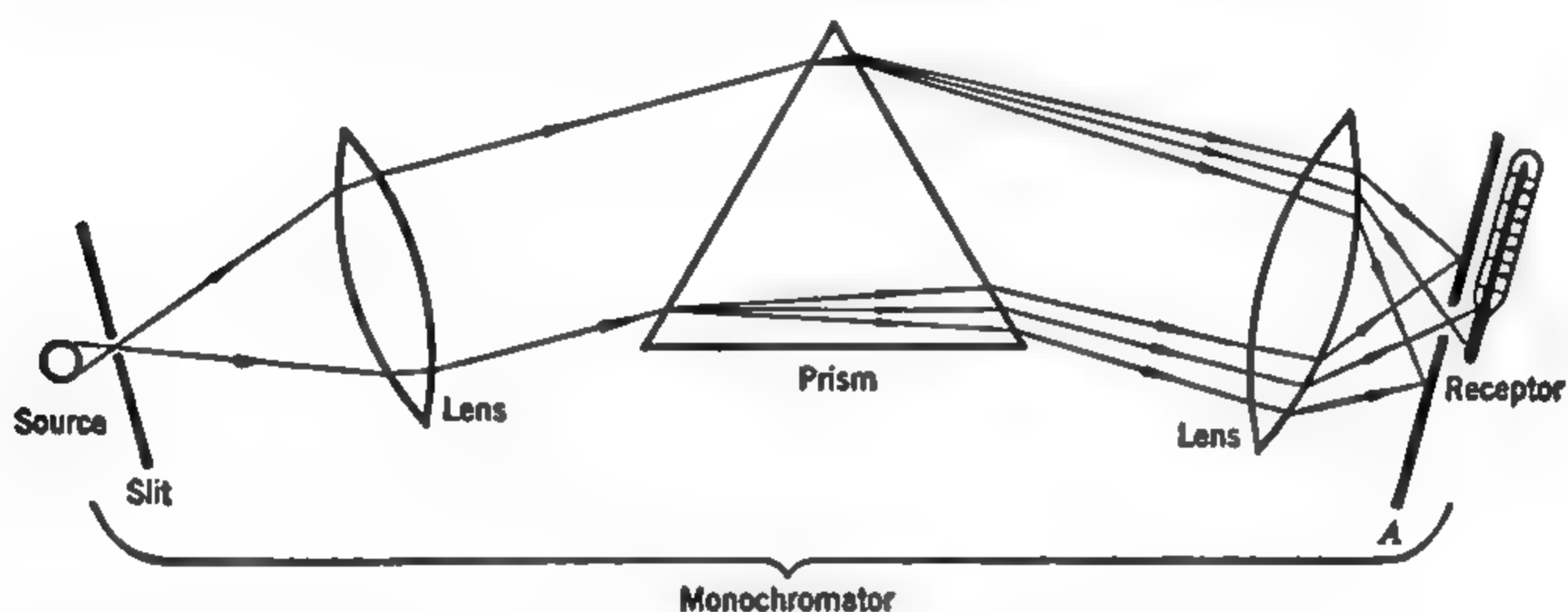


FIG. 21. Diagram of a prism spectrometer.

Spectroradiometers calibrated to read spectral radiant flux in absolute units are useful in research work in color of television tubes, in the color and color-rendering properties of light sources (gaseous discharge tubes, fluorescent lamps), and, in general, in the colors of self-luminous areas. To build a spectroradiometer is not too difficult. To calibrate it is more difficult. And to keep it in calibration day after day is almost a continuous research project in itself. Spectroradiometers can be used for nonself-luminous objects, but this is almost never done, because there is an easier way in which to do it.

The color of an object is perceived about the same even though the kind of light under which it is seen varies a great deal. That this is so to a surprising extent is a tribute to the success of the visual mechanism in taking into account the kind of light falling on the object as a part of the process of perceiving the color belonging to the object. What the eye perceives is really a double thing, the color of the light itself, judged by the colors and arrangement of parts in the whole scene, and the change in this color produced by the object. It is this change that is ascribed to the object itself. Thus, if we wish to obtain a physical measurement of the property giving rise to our perception of object color, we must not measure merely the spectral distribution

of radiant flux reaching our eyes from that object. This distribution depends both on the object and the light source. What we must measure is the change produced on the incident light by the object.

The instrument used to measure the fundamental properties of an object responsible for its color is the spectrophotometer. The spectrophotometer is not calibrated to read spectral distribution of radiant flux in absolute units, but instead merely compares at each wavelength of the visible spectrum the radiant flux leaving the object with some other flux such as that incident on it. Since it requires merely the comparison of two amounts of radiant flux of the same wavelength range, the spectrophotometer may use (in addition to thermopiles, photocells, and so forth) the photographic plate, or the human eye. A spectrophotometer applicable to objects must also have a built-in illuminating device. The quantities measured by the spectrophotometer depend upon the illuminating and viewing conditions. For transparent objects the quantity is spectral transmittance—ratio of transmitted to incident flux for one part of the spectrum; and the incident flux is usually taken along the perpendicular with the angles of view confined to those close to an extension of the same perpendicular. For opaque mat objects the quantity is spectral reflectance—ratio of reflected to incident flux for one part of the spectrum; and the incident flux is again usually along the perpendicular. Sometimes all the reflected flux is collected for measurement by an integrating sphere, regardless of the angle of its reflection; sometimes this flux is sampled in a few directions only, such as those near 45° .

All diffusing materials and articles of commerce fail to be perfectly mat; that is, they exhibit more or less gloss. Part of the light incident on the surface is reflected in some more or less good approximation of an image-forming state, the remainder penetrates the surface, and, after suffering absorption, scattering, and multiple reflection beneath the surface, is re-emitted in a nearly diffuse state. Now, in our judgment of object color, we customarily view an object from several angles and so perceive how glossy it is as well as what color belongs to the part beneath the glossy surface. To measure the property of the object responsible for the color perceived beneath a more or less glossy surface requires a more complicated measurement than simply spectral reflectance. We have to separate, at least approximately, the surface-reflected flux from the flux re-emitted from the body of the object. This may be done by illuminating the surface with essentially unidirectional light (at 45°) and viewing it along directions (nearly perpendicular to the surface) separated by many degrees from the angle

of mirror reflection (-45°), as is recommended by the International Commission on Illumination (ICI, 1931). It does not matter if the illuminating and viewing directions are interchanged; the same numerical results are obtained. The standard in this method is not the incident flux, but instead is the flux reflected by the ideal perfectly diffusing, perfectly reflecting surface illuminated and viewed along the same directions and to the same extent as the specimen. The ratio of these two fluxes is called directional reflectance. A magnesium oxide reflectance standard (Keegan, 1939; TAPPI Spec. T 633m-47; ASTM Method D 986-48T) is customarily used; the accepted 45° normal directional reflectance for daylight of such a standard is 1.00 (Preston, 1929-1930). Directional reflectance for the 45° normal condition is a fairly representative measure of the flux diffusely reflected by the body of an opaque specimen; sometimes, with this sampling of the reflected flux, the result is called diffuse reflectance. Spectral directional reflectance for 45° illumination and perpendicular viewing leads to good correlation with judgments of body color of semiglossy and glossy objects, better than can be derived from spectral reflectance itself, part of which refers to surface-reflected light.

The two spectrophotometers most used for evaluating the color of industrial products are the General Electric Recording Spectrophotometer and the Beckman Quartz (nonrecording) Spectrophotometer. These two instruments are the most important basic tools commercially available for fundamental color measurements today (1952) and deserve some detailed consideration.

General Electric Recording Spectrophotometer. The following description is taken from *Circular 484* of the National Bureau of Standards (Spectrophotometry, by Kasson S. Gibson):

The present model of the General Electric Recording Spectrophotometer (Michaelson, 1938), originally designed by Hardy (1935, 1938), gave great impetus to colorimetric analysis and specification because (1) there was great increase in speed over previous nonrecording spectrophotometers, (2) it could make spectral-reflection measurements just as readily as spectral-transmission measurements, and (3) its precision and reliability were adequate for most types of measurement. The recording feature is also of great importance in many kinds of work.

A schematic diagram of the instrument is shown in Fig. 22 which is a reproduction (with slight changes) of Fig. 1 of Michaelson's paper (1938). Radiant energy from the lamp forms a prismatic spectrum in the plane of the mirror at the left of the figure. A second slit here isolates a small part of the spectrum and passes it on through prism 2. This second dispersion spreads the stray energy into a secondary spectrum in the plane of slit 3.

The third slit obstructs most of this stray energy but transmits most of the spectral band isolated by the second slit. Thus there passes into the photometer a spectral band of width determined by the slits used, and almost wholly free of stray energy from other spectral regions. As the spectrum is traversed the mechanical widths of the slits are continually changed by cams so as to transmit a constant amount of spectrum expressed in terms of millimicrons. The usual nominal width is $10\text{ m}\mu$ over the range from 400 to $700\text{ m}\mu$ but instruments with 4 , 5 , and $8\text{ m}\mu$ widths have been made, and the wavelength

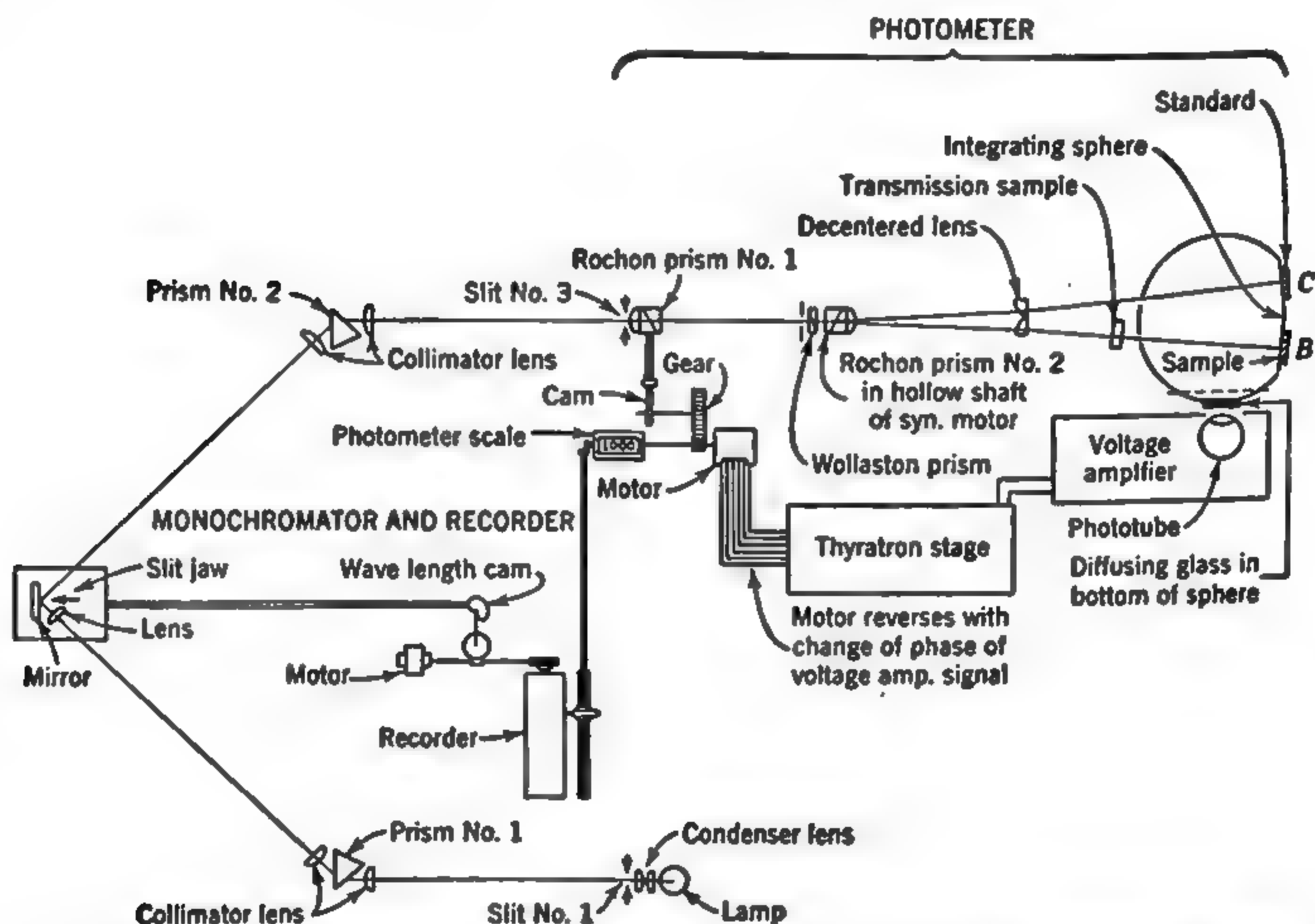


FIG. 22. Schematic diagram of the General Electric Recording Spectrophotometer (after Michaelson, 1938).

range has occasionally been extended to $750\text{ m}\mu$ (and with wider slits to above $1,000\text{ m}\mu$), as on the National Bureau of Standards instruments.

The radiant energy transmitted by slit 3 passes in turn through (1) the balancing rochon prism 1, (2) the wollaston prism, where it is split into two beams polarized at right angles to each other, (3) the rotating rochon prism 2, mounted in the hollow shaft of a synchronous motor, (4) the separating decentered lenses, (5) the transmission compartment, and (6) the sphere openings, and is finally incident upon the reflecting samples at *B* and *C*.

As rochon 2 rotates, the respective radiant energies incident upon *B* and *C* alternately go through successive maxima and minima, each maximum on *B* coinciding in time with the minimum on *C*. (The expressions for these energies as functions of time are given, and it is shown that the alternating component of the photocell current that results when the balancing rochon prism is set to a position of imbalance has a frequency of 60 cycles per second.)

A photoelectric cell of the cesium oxide type is placed just beneath a ground-glass window in the bottom of the sphere. This phototube is connected, through an a-c voltage amplifier and a "thyatron stage," so that any alternating current from the photocell will in effect drive the balance motor, the direction of turn of the motor depending on the phase of the current. This phase will, in turn, depend on which of the two radiant energies reflected from *B* and *C* is the greater.

The motor is geared through a cam arrangement so that, as the armature turns, it turns rochon prism 1 and always in the direction that will balance the two radiant energies reflected from *B* and *C*. When these two reflected radiant energies are equal, there is no torque on the motor and the motor and rochon are at rest.

In operation, another motor (at the left) runs steadily; through cams it changes the wavelength of the radiant energy passing slit 3 by moving the second slit through the spectrum, and at the same time it turns the wavelength drum on which the recording paper is placed. The final connection is made by translating through levers and cams the amount that rochon 1 turns so as to operate a recording pen that moves up and down on the paper. This pen moves linearly with the transmittance or reflectance of the sample and thus the respective curve is steadily traced through the spectrum. In addition to the normal cam, cams giving " $\times 5$," "log," and "loglog" curves can be used.

A typical sheet of such recordings as regularly run at the National Bureau of Standards is shown in Fig. 23. The slits for this recording were equivalent to approximately 10 m μ of spectrum. In this figure will be noted first the three calibration curves: (1) the "zero curve" is run with the sample beam blocked off; (2) the "Vitrolite curve" is run to enable all the reflectance curves to be corrected so as to be truly relative to freshly prepared MgO; (3) the "didymium curve" is run with a calibrated didymium glass in the sample beam; this curve is for wavelength calibration of the sheet. After the three calibration curves, the curves for the samples tested are successively run; then the whole series of sample curves and calibration curves is recorded in reverse order. This repetition ensures against undetected erratic or gradual changes in conditions during the run. Each curve takes 2½ or 5 minutes, at the choice of the operator, depending on the spectral selectivity of the specimen.

In transmission measurements it does not matter what the reflecting materials *B* and *C* (Fig. 22) are, provided they will give a "100-percent" curve entirely on the network part of the sheet. (The cams are reliable for extrapolation above the network to only about 100.5 percent.) They can be fresh or old MgO, MgCO₃, white glass, white paper, and so forth. However, it is a matter of importance that the reflectance be high, to avoid loss of sensitivity;

and it is a matter of convenience that B and C be essentially identical, so that the 100-percent curve will lie closely if not exactly on the 100-percent coordi-

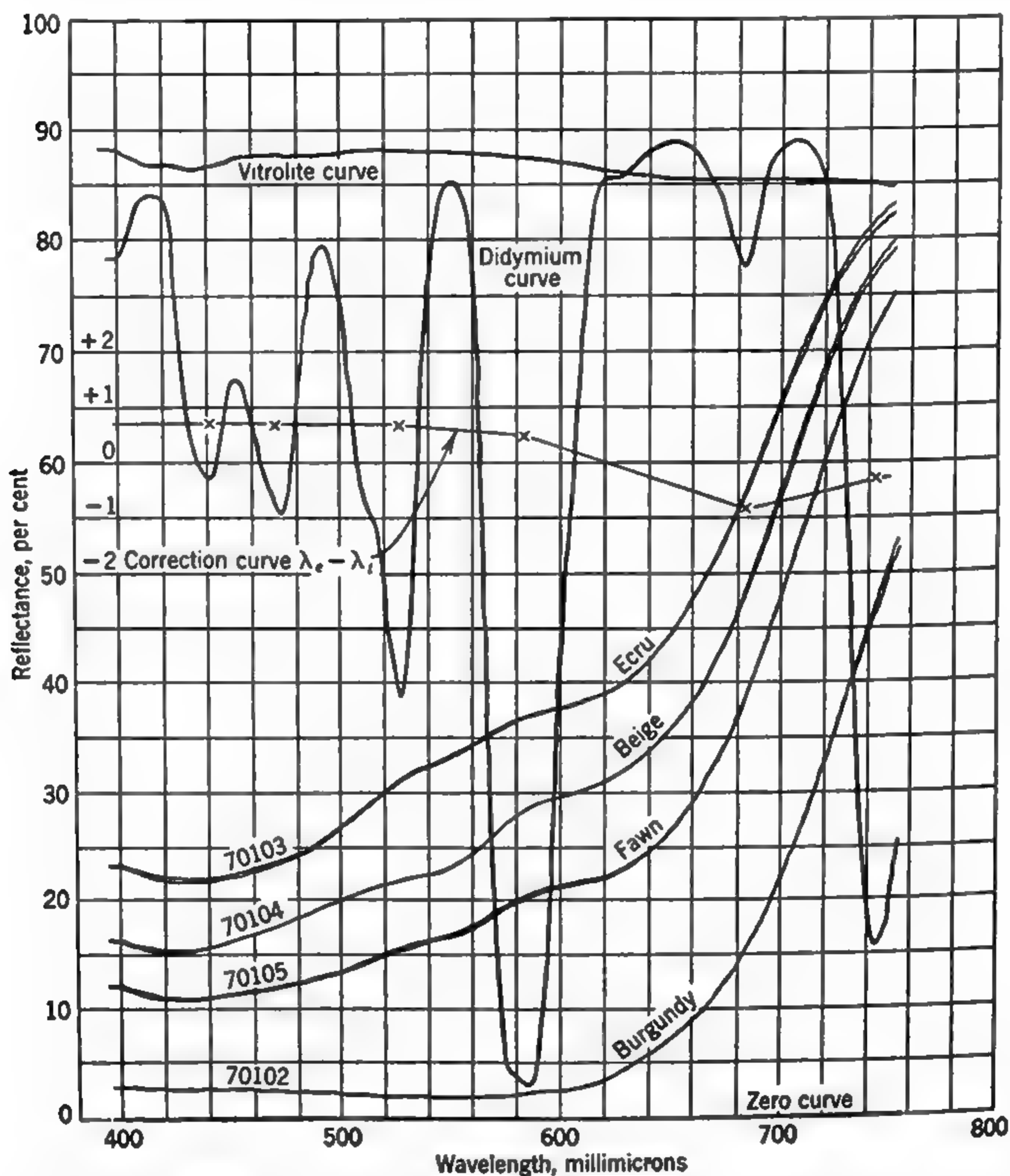


FIG. 23. Recordings of the curves of spectral reflectance of the colors of four silk swatches taken from the Standard Color Card of America (70103, Ecrú; 70104, Beige; 70105, Fawn; and 70102, Burgundy). The Vitrolite and zero curves serving to calibrate the photometric scale are also shown as well as the didymium curve and the correction curve derived from it to calibrate the wavelength scale (after Gibson, 1949).

nate line. The substitution principle is fully complied with when at any wavelength the transmittance is computed from the ratio of ordinates of the transmission sample curve to the 100-percent curve (correcting each for the zero curve if necessary).

For spectral reflection measurements (transmission sample removed), if one is willing to prepare identical fresh MgO standard surfaces each day, the operation is similar to that for transmission measurements, except that by the substitution principle the MgO surface at *B* is *the* standard surface; the other MgO at *C*, identical or not, is merely the comparison sample. Because of the slight decrease in reflectance of the MgO, it is necessary that each MgO sample be newly prepared each day. If *B* is not newly prepared it is no longer standard; if *C* is not newly prepared (and *B* is), the 100 percent curve will rise off the network below 500 $m\mu$. Of course if it is known by experience that the 100 percent curve is "on the line" and one wishes to take the chance, he can omit the 100 percent curve and simply prepare sample *C* each morning for the day's operation.

This is risky, however, for certain kinds of work, for experience has indicated that newly prepared MgO samples may vary slightly (0.1 or 0.2 percent) for unknown reasons. At the National Bureau of Standards it has been considered better to calibrate a working standard of directional reflectance (placed at *B*) by comparison with several different new preparations of MgO and to repeat this check occasionally to detect any possible change in the working standard.

The "Vitrolite curve" in Fig. 23 represents such a calibrated standard of directional reflectance. By its use, comparison sample *C* can be any sample of high enough reflectance to keep the recording within the network; however, in order to keep the recordings close to their true values (which has several advantages), it is well for sample *C* to be a reasonably fresh surface of MgO. At any wavelength, the correct value of the directional reflectance of the sample (placed at *B*), relative to that of fresh MgO, is obtained by dividing the recorded ordinate for the sample by the ratio of the recorded ordinate of the Vitrolite to the correct value for the Vitrolite relative to fresh MgO.

The type of directional reflectance measured is perpendicular incidence and nearly diffuse viewing with the type of sphere illustrated in Fig. 22; part of the specular component of the energy reflected from a plane polished surface at *B* goes back through the entrance aperture. With the newer design of sphere, shown in Fig. 24, the radiant energy is incident upon samples *B* and *C* at approximately 6° and the specular component of the reflected energy is returned to the sphere apertures at *B'* and *C'* as shown. If the surface of the test sample placed at *B* is plane and polished, this specular component can be wholly excluded from or included in the measurements by either placing a completely absorbing "light trap" at *B'* or filling the aperture with a sample of fresh MgO. . . . Two things of importance should be noted in this connection:

1. *Specular component included.* If the sample at *B* is 96 percent diffusing and only 4 percent specularly reflecting (approximated by a plane, polished white glass), the error involved if the reflectance of *B'* does not quite equal that of fresh MgO is of second order, a small fraction of the 4 percent. How-

ever, if the sample at B is a polished mirror, so that the whole of the reflected beam is first incident upon B' before being diffused throughout the sphere, the error is of first-order importance and should be carefully considered.

2. *Specular component excluded.* If the surface of the sample is not plane but is mottled or irregular in flatness, such, for example, as a piece of enameled iron may be, some of the specularly reflected energy may not be caught in the light trap at B' , and the specular component is thereby not completely excluded from the measurements.

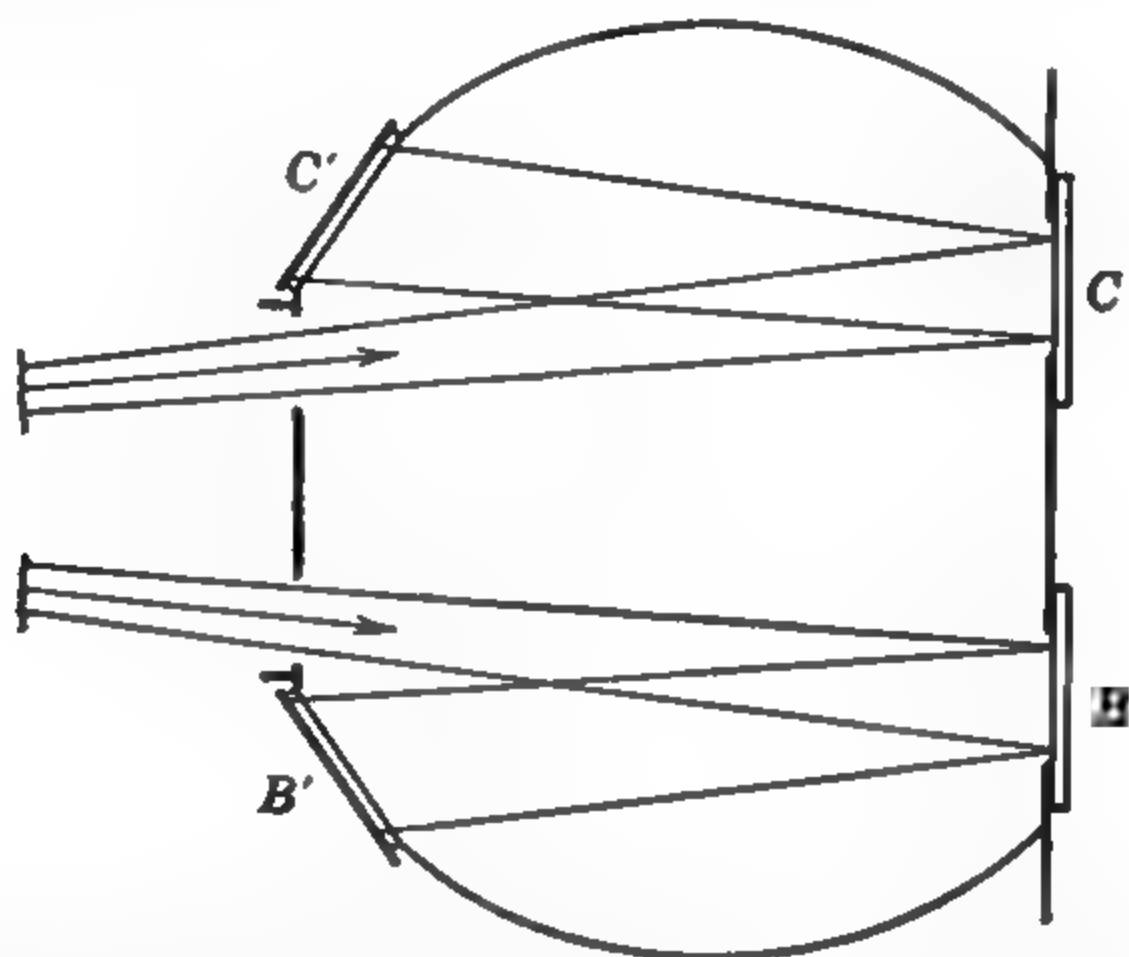


FIG. 24. Design of integrating sphere supplied with the General Electric Recording Spectrophotometer (after Gibson, 1949). The illumination is at about 6° from the perpendicular, and removable ports are provided so that the specular component of the energy reflected from samples with plane surfaces may be included in the measurement or excluded at the choice of the operator.

The wavelength correction curve for the graphs of Fig. 23 is also shown in Fig. 23. It is the difference between the wavelengths of the minima of the didymium curve read from Fig. 23 and the known, true wavelengths. The method of applying this correction is simply to read the values of transmittance or directional reflectance at $\lambda + (\lambda_e - \lambda_i)$ instead of at the nominal wavelength, λ .

The introduction of the General Electric Recording Spectrophotometer even at rather high prices (\$5,000 to \$15,000) had a pronounced, almost revolutionary, influence on the color measurement of industrial products in this country. It gave speed combined with reliability. For the first time a check on the color of a commercial article or material could be obtained in a fundamental way quickly enough to be of some use. Spot checks of material from the production line could be made without expensive delays. Large bodies of information could be ground out in a few weeks, whereas the older methods would require much more expense and many months, after which time the information might be of little interest. For example,

Fig. 23 shows a part of the calibration of 238 color standards used in the textile industry (TCCA Standard Color Cards); without the recording spectrophotometer the cost of this work would probably have been prohibitive and, if undertaken at all, would probably not have been finished before new color standards had been adopted. Figure 23 refers to 4 samples (Reimann, Judd, and Keegan, 1946) and required between 1 and 2 hours to record and about 4 hours to read, apply corrections, and check. With spectrophotometers available before the GE recording model, equally reliable work took about 1 man-day for each specimen. This is a gain by more than a factor of 5. For spot checks not requiring application of the corrections, the time is reduced to about 3 minutes instead of the previous 30 to 60 minutes. This is a gain by a factor of more than 10.

Before a business man can decide whether it would pay him to invest a good many thousands of dollars in a GE Recording Spectrophotometer he should know its principal limitations.

Semiglossy Specimens. To obtain a measurement of the color of semiglossy specimens that will correlate with the judgment of an inspector requires, as previously pointed out, the elimination of surface-reflected light from the measurement. A part of the surface-reflected light is eliminated by passing out through the illuminating hole (Fig. 22) or into the light trap (Fig. 24), but not all. The exact fraction counted in the measurement depends upon the gloss of the specimen. Two semiglossy specimens judged to be identical in color though slightly different in gloss may give different curves. Conversely, two semiglossy specimens may measure the same but be judged to have different colors. To obtain good data on semiglossy specimens on the GE Recording Spectrophotometer and interpret them correctly will require a specially trained operator. It must not be thought that because the instrument always draws the same curve for a given specimen, the meaning of the curve will be apparent to any high-school graduate.

Polarizing Specimens. From Fig. 22 it will be noted that the last polarizing element in the beam before it strikes surface *B* and *C* is rochon prism 2 rotating at 1,800 rpm. The beams striking surfaces *B* and *C* are therefore plane polarized, but the plane of polarization is rotating continuously during each cycle. If the transmission sample or the reflection sample at *B* is polarizing, the fraction of the incident flux reaching the walls of the integrating sphere is not constant but is a function of time during the cycle. Now the condition of balance is that, as rochon prism 2 rotates, the rise and fall of flux from surface *B* is exactly compensated for by a corresponding fall and rise from sur-

face *C*, giving an illumination of the integrating sphere that is constant in time. But, if the fraction of flux reflected by surface *B* varies throughout the cycle while that reflected from surface *C* (nonpolarizing) does not, there is no setting of rochon prism 1 that can result in an illumination of the sphere wall constant in time. The result is that there is a change of phase twice in each cycle, the balance motor receives contradictory impulses in quick succession (120 times a second), and the instrument "blocks." The GE Recording Spectrophotometer cannot be used without special modification to measure polarizing materials such as mica, cellophane, and satin-finish textiles.

Nonuniform Specimens. Some commercial products requiring color measurement and control exhibit nonuniform color from spot to spot, either regular (textile fabrics, half-tone prints) or irregular (brush-marked paint films, finished wood, marbled wall tile). If the grain of the nonuniformity is appreciable compared to the size of the illuminated spot (about 1 inch diameter), a single curve may not be a reliable indication of the average color of the nonuniform specimen. This difficulty can be overcome by running curves for more than one area of the specimen and taking an average. However, specimens having a contrasty fine-grained texture often cause the GE Recording Spectrophotometer to "block" just as polarizing specimens do. This is ascribable to failure of the plane front and back faces of rochon prism 2 (Fig. 22) to be exactly parallel. A slight wedge shape of this rotating prism can cause the illuminating beam to strike somewhat different portions of the specimen during the cycle. If the instrument has this defect, and it seems to be almost unavoidable, there must necessarily be by chance a certain percentage of specimens nonuniform in such a way that they reflect into the sphere different fractions of the incident flux at different parts of the cycle, thus preventing a balance from being obtained.

Fluorescent Specimens. Some materials have the property of absorbing radiant flux of one wavelength and re-emitting it at another (almost always longer). The process by which this wavelength change is produced is called fluorescence. More than 25 percent of the TCCA color standards were found to be strongly fluorescent (Reimann, Judd, and Keegan, 1946). It will be noted from Fig. 22 that the radiant energy from the lamp is dispersed into a spectrum before it strikes the specimen at *B*. The flux incident on the specimen is therefore spectrally homogeneous, or nearly so; that is, it consists of radiant flux having wavelengths varying only over a small range (20 m μ or less). However, any radiant flux leaving the specimen, whether it be of the same wave band by reflection, or of a longer wave band by fluorescence,

reaches the integrating sphere and is sampled by the cesium oxide photocell behind the diffusing glass in the bottom of the sphere; that is, the GE Recording Spectrophotometer cannot distinguish between reflected flux and fluorescent flux. A material that absorbs middle-wave energy (green) and re-emits it by fluorescence as long-wave (red) energy may have a reddish color in daylight, but any spectrophotometer that fails to analyze the flux emitted by the specimen will indicate an erroneously high reflectance in the middle (green) part of the spectrum and will fail to record the fluorescent flux in the long-wave part (red) of the spectrum.

We may summarize what the GE Recording Spectrophotometer does as follows: It measures and records accurately and speedily the physical quantities (spectral transmittance, spectral directional reflectance) responsible for the colors of uniform, nonsemiglossy, nonpolarizing, nonfluorescent specimens. Because it does analyze the basic physical cause of these colors, it is particularly valuable for industrial research; and, because of its speed, it is also applicable to routine inspection and color control (Hardy and Young, 1949). The initial high cost limits its economical application to establishments requiring several hundreds of color determinations yearly and makes it a virtual necessity to assign to the spectrophotometer a full-time specially trained operator.

Beckman Quartz (Nonrecording) Spectrophotometer. The following description is also taken from *Circular 484* of the National Bureau of Standards (Spectrophotometry, by Kasson S. Gibson):

The Beckman Quartz Photoelectric Spectrophotometer, model DU, has several features not found in other commercial spectrophotometers: (1) it covers the wavelength range from 210 to 1,000 $m\mu$ approximately, (2) transmission measurements in the ultraviolet from 210 to 400 $m\mu$ are made as readily as in the visible spectrum, (3) directional reflectance measurements over the range from 350 to 1,000 $m\mu$ can be made under conditions closely equivalent to those recommended by the International Commission on Illumination in 1931 for colorimetric work.

The optics of the apparatus for transmission measurements are as shown in Fig. 25, taken from the paper by Cary and Beckman (1941). Radiant energy from an incandescent lamp or other source *A* is focused on the slit *D* by means of the concave mirror *B* and the plane mirror *C*. The beam entering the slit is collimated by the mirror *E* and passes through the quartz prism to the reflecting surface *F*. After reflection at *F* the beam returns along nearly the same path to the same slit *D*, where it emerges slightly above the entrance beam and the mirror *C*. After passage through the sample or sample compartment *G*, the beam is incident on the phototube *H*.

Two phototubes are supplied installed in the instrument, either of which may be inserted in the beam at will. One is a "cesium oxide" phototube ("red

sensitive") for use primarily above $620\text{ m}\mu$. The other is a special "cesium antimony" phototube ("blue sensitive") for use primarily below $620\text{ m}\mu$

The photometric scale is based on electrical rather than optical principles. The photoelectric current from the irradiated phototube produces across the load resistor a voltage drop that is balanced by a potentiometer. While this null setting is being made, any imbalance is amplified electronically and is indicated by a milliammeter on the instrument. Accuracy of the photometric scale depends among other things on conformity of the phototube load resistor to Ohm's law and on the linearity of the irradiance-current relationship. Since vacuum phototubes are used, the relation should hold closely. Linearity is still further assured by the use of a frame-type anode in the phototube. . . .

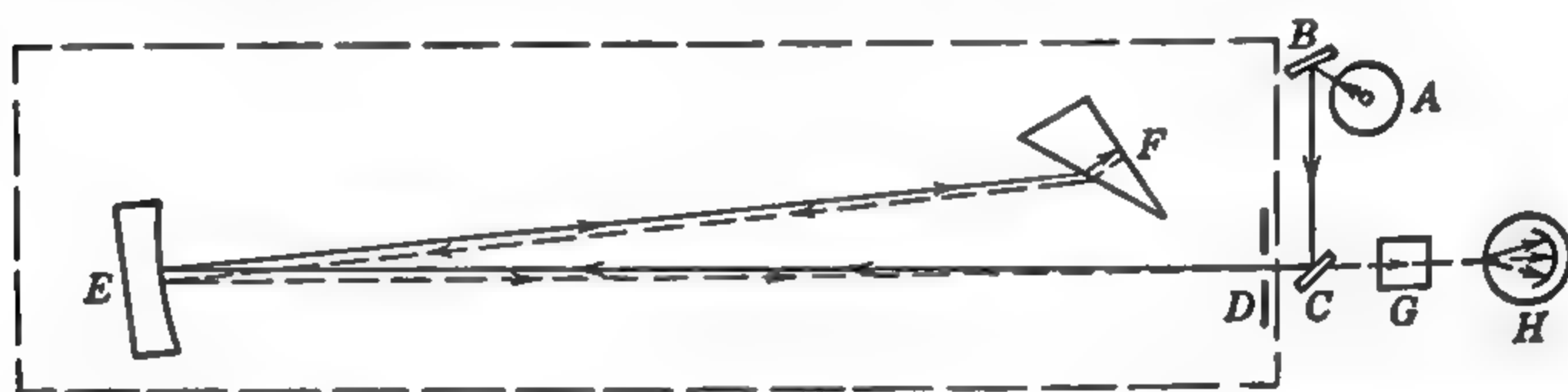


FIG. 25. Schematic diagram of the Beckman photoelectric spectrophotometer (after Cary and Beckman, 1941).

The slits are narrower than usual on commercial spectrophotometers (for transmission measurements), normally isolating spectral regions of only 0.5 to $1.5\text{ m}\mu$ within the range from 220 to $950\text{ m}\mu$. This feature of the instrument serves three useful purposes. It enables the spectral transmittance of samples having high selectivity to be determined with an accuracy unattainable with instruments having wide slits. It makes possible the precise and accurate comparison of nearly identical samples, even those having high absorption, without the use of slits unduly wide. It also makes possible the reliable determination of very low transmittances by a "step-down" procedure using samples of intermediate transmittance.

No error was detected in the photometric scale on an instrument obtained at the National Bureau of Standards in 1945. These tests were made by measuring and comparing the spectral transmittances of certain glass standards on the Beckman, the König-Martens, and the Gibson spectrophotometers. . . .

The quartz mercury arc supplied with the instrument serves not only for wavelength calibration but can be used at certain wavelengths for photometric measurements. . . .

Trouble has been experienced in the measurement of polarizing materials if the quartz plate over the entrance slit is not cut with faces perpendicular to the axis (Gibson and Balcom, 1947). At the National Bureau of Standards this plate has been removed.

The instrument has the usual amount of stray energy present for single-dispersion spectrophotometers. The purple filter supplied with the instrument should always be used with incandescent source below $400\text{ m}\mu$, and other

stray-energy filters should be obtained and used under certain circumstances.

Any deviation of the beam by the sample (if wedge-shaped or lens-shaped, for example) may cause error in the measurements.

Directional reflectance measurements can readily be made on the Beckman spectrophotometer by means of special attachments obtained from the manufacturer. The optics of such measurements are shown in Fig. 26. The radiant energy transmitted through the exit slit *D* is directed downward by means of

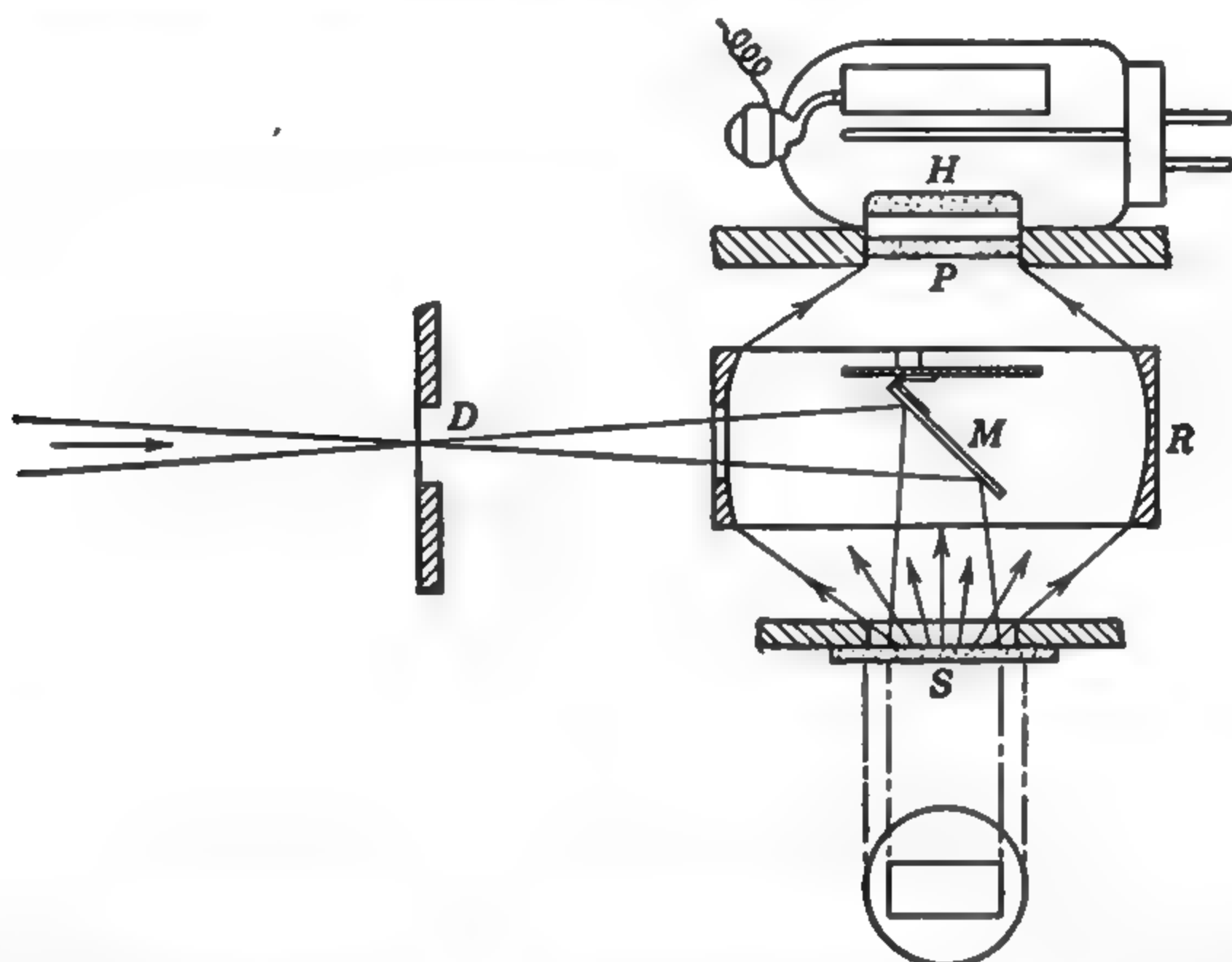


FIG. 26. Schematic diagram of the reflectance attachment for the Beckman photoelectric spectrophotometer (after Gibson, 1949).

the mirror *M* and is incident approximately at right angles upon the sample or standard at *S*. The irradiated area is about 8 by 17 mm. A carrier enables either the sample or the reflectance standard with which it is being compared to be placed in turn in the beam.

The energy reflected from the sample or standard at approximately 45° in all directions is reflected and focused by the ellipsoidal metallic mirror ring at *R* onto the diffusing plate at *P*. The phototube compartment is mounted on top of the attachment so that the phototube *H* is just above the diffusing element.

The operation of the instrument for reflection measurements is similar in all respects to that for transmission measurements. The following points may be noted however:

1. The slit widths necessary for normal operation are much wider than for transmission measurements. They vary from 3 to 4 $m\mu$ in the region near 530 $m\mu$ to about 8 $m\mu$ at 350 $m\mu$, and 15 $m\mu$ at 650 $m\mu$. These widths may be made to vary considerably by adjustment of the sensitivity dial, however, or by operation solely at the "0.1" position of the selector switch.

2. While the axis of the reflected energy is closely at 45° , the angular spread of the beam is approximately 20° (35° to 55°). The directional reflectance $R_{0,45}$ thus measured is (usually) numerically equivalent to that, $R_{45,0}$, recommended by the International Commission on Illumination, as noted above, except that the CIE recommendation implies that the spread of the reflected beam should be zero.

3. In reflection measurements of fluorescent materials the values obtained are liable to the same sort of error as with the General Electric spectrophotometer.

We may summarize what the Beckman Quartz Spectrophotometer does as follows: It measures (but does not record) accurately and with considerable speed the physical quantities (spectral transmittance, spectral directional reflectance) responsible for the colors of non-fluorescent specimens. Because it does provide at moderate expense a reliable analysis of the basic physical cause of the colors of these specimens, it is particularly valuable for industrial research in moderate to small operations; but because of the simplicity and flexibility of its operation it is also applicable to routine inspection, color control, and the short-range research that comes under the head of trouble shooting. Since it is a single-dispersion instrument often requiring filters for the elimination of stray energy, and since it has slits somewhat under the control of the operator, considerable knowledge and judgment on the part of the operator are required if the instrument is to yield the reliability of which it is capable.

The National Bureau of Standards has developed and regularly issues permanent glass standards of spectral transmittance for checking the performance of spectrophotometers (see *NBS Letter Circular* LC929). It also issues Vitrolite working standards of directional reflectance calibrated in terms of freshly prepared magnesium oxide either for the General Electric Recording Spectrophotometer or for the Beckman spectrophotometer.

FUNDAMENTAL STANDARDS IN COLORIMETRY

The physical basis for the colors of nonfluorescent objects is given by the spectrophotometric curve. If two such objects have identical curves for given angular conditions of illumination and view we may be sure that under these conditions they will be perceived to have the same colors, no matter what light is used to see them by, and no matter what observer looks at them. No further reduction of the spectrophotometric data is necessary to support this conclusion. It is also true that, by direct inspection of two spectrophotometric curves differing in a simple way, it is possible to judge qualitatively how the

colors of the specimens will be perceived to differ. Thus, on Fig. 13 from inspection of the curves for white and tan we can see immediately that the tan specimen has a darker and more reddish yellow color than the white specimen. This kind of difference is called a nonmetameric difference.

But if the two curves differ in a complicated way, such as with three or more crossing points, it may not be possible to tell by mere inspection of them in what way the colors of the specimens differ. Indeed, it may be that, under some lights and judged by some observers, the two will be seen to have the same color. Such specimens are said to have metameric colors, that is, the light beams leaving the two specimens are alike in color although different in spectral composition. Near-metamers are said to exhibit metameric difference. The only information to be obtained by inspection of the spectrophotometric curves of metamers is that whether they will or will not yield the same colors will depend on what light they are viewed by, and who looks at them.

If a standard of color is made of the same material and colorants as the manufactured product, it is certain that the product either will color-match the standard or will show a nonmetameric difference. No reduction of such spectrophotometric data is required because mere inspection of the two curves will show whether and in what way the product differs in color from the standard. As a matter of fact, no spectrophotometric curve is required. Conformity of product to standard can safely be judged by direct visual comparison under any light source giving enough incident luminous flux density, or illuminance, for the inspector to work by.

Oftentimes, however, it is necessary to have a color standard more permanent than the manufactured product itself. Glass and ceramic products are a frequent choice for color standards because of their generally superior permanence. In such cases, the colorants and materials used for the product are usually different from those used for the color standard, and it is almost certain that the color match will be metameric to some degree. When the spectrophotometric curve of the product is compared with that for the standard, there will necessarily be differences of complicated character. Interpretation of such curves requires a knowledge of the light source to be used for the product and the person who is to view it.

From the spectral directional reflectance of any object viewed by reflected light we can determine by computation whether it has the same color as another object of known spectral directional reflectance similarly illuminated and viewed. This is a perfectly general statement and holds for any illuminant of known spectral character and

any observer whose spectral responses are known. The computation takes the form of finding the tristimulus values for the illuminant-object-observer combination and is based on Grassmann's laws (equations 2 and 3). These tristimulus values may be expressed for any known light source and for any known observer with respect to any of a triple infinity of primary colors (see equation 4), each set of primaries yielding a separate colorimetric coordinate system.

Standard Sources. Although a manufacturer may be primarily interested in the color of his product viewed by daylight, it serves no useful purpose to check it against the standard color for all phases of daylight. These phases might include light from the clear sky, light from sun plus clear sky, light from the overcast sky, direct sunlight, or mixtures of these with light reflected from a brick wall, and so forth. If it color-matches the standard for any of these phases of daylight, a commercial product will usually also match it under any other, or, at least, will nearly match. A similar statement holds for products, like subway wall tile, always viewed by artificial light. If the product is right for illumination by incandescent-lamp light of high color temperature (movie flood light) it is usually acceptable for one of low (25-watt vacuum lamp). To permit color measurements to be directly comparable as often as possible it has become customary to use one of three standard sources unless there is a compelling reason to use some other. These standard sources were recommended in 1931 by the International Commission on Illumination * and are:

Source A, representative of gas-filled incandescent lamps.

Source B, representative of noon sunlight.

Source C, representative of average daylight such as that from a completely overcast sky.

Source A is an incandescent lamp operated at a color temperature of $2,854^{\circ}\text{K}$ on the international temperature scale (Judd, 1950) of 1948 ($C_2 = 14,380$).

Source B is obtained by using the same lamp in combination with a two-cell Davis-Gibson liquid filter giving a correlated color temperature of about $4,870^{\circ}\text{K}$.

Source C is obtained similarly and results in a source of correlated color temperature of about $6,740^{\circ}\text{K}$.

Incandescent-lamp standards representing source A may be purchased from the National Bureau of Standards. Detailed instructions

* The initials ICI have often been used as a brief designation of this Commission. The official abbreviation, CIE, adopted at the 1951 meeting, refers to the French name, Commission Internationale de l'Éclairage, and is used in this book.

Table 5. Relative Spectral Irradiance of CIE Standard Sources A, B, and C

Wave-length, mμ	<i>H_A</i>	<i>H_B</i>	<i>H_C</i>	Wave-length, mμ	<i>H_A</i>	<i>H_B</i>	<i>H_C</i>
380	9.79	22.40	33.00	580	114.44	101.00	97.80
385	10.90	26.85	39.92	585	118.08	100.07	95.43
390	12.09	31.30	47.40	590	121.73	99.20	93.20
395	13.36	36.18	55.17	595	125.39	98.44	91.22
400	14.71	41.30	63.30	600	129.04	98.00	89.70
405	16.15	46.62	71.81	605	132.70	98.08	88.83
410	17.68	52.10	80.60	610	136.34	98.50	88.40
415	19.29	57.70	89.53	615	139.99	99.06	88.19
420	21.00	63.20	98.10	620	143.62	99.70	88.10
425	22.79	68.37	105.80	625	147.23	100.36	88.06
430	24.67	73.10	112.40	630	150.83	101.00	88.00
435	26.64	77.31	117.75	635	154.42	101.56	87.86
440	28.70	80.80	121.50	640	157.98	102.20	87.80
445	30.85	83.44	123.45	645	161.51	103.05	87.99
450	33.09	85.40	124.00	650	165.03	103.90	88.20
455	35.41	86.88	123.60	655	168.51	104.59	88.20
460	37.82	88.30	123.10	660	171.96	105.00	87.90
465	40.30	90.08	123.30	665	175.38	105.08	87.22
470	42.87	92.00	123.80	670	178.77	104.90	86.30
475	45.52	93.75	124.09	675	182.12	104.55	85.30
480	48.25	95.20	123.90	680	184.43	103.90	84.00
485	51.04	96.23	122.92	685	188.70	102.84	82.21
490	53.91	96.50	120.70	690	191.93	101.60	80.20
495	56.85	95.71	116.90	695	195.12	100.38	78.24
500	59.86	94.20	112.10	700	198.26	99.10	76.30
505	62.93	92.37	106.98	705	201.36	97.70	74.36
510	66.06	90.70	102.30	710	204.41	96.20	72.40
515	69.25	89.65	98.81	715	207.41	94.60	70.40
520	72.50	89.50	96.90	720	210.36	92.90	68.30
525	75.79	90.43	96.78	725	213.26	91.10	66.30
530	79.13	92.20	98.00	730	216.12	89.40	64.40
535	82.52	94.46	99.94	735	218.92	88.00	62.80
540	85.95	96.90	102.10	740	221.66	86.90	61.50
545	89.41	99.16	103.95	745	224.36	85.90	60.20
550	92.91	101.00	105.20	750	227.00	85.20	59.20
555	96.44	102.20	105.67	755	229.58	84.80	58.50
560	100.00	102.80	105.30	760	232.11	84.70	58.10
565	103.58	102.92	104.11	765	234.59	84.90	58.00
570	107.18	102.60	102.30	770	237.01	85.40	58.20
575	110.80	101.90	100.15	775	239.37	86.10	58.50
580	114.44	101.00	97.80	780	241.67	87.00	59.10

for making the filters for sources B and C are given by Davis and Gibson (1931), and the composition of the solutions is specified in many accessible publications (Smith and Guild, 1931-1932; Judd, 1933; Hardy, 1936; OSA Committee on Colorimetry, 1944).

For the reduction of spectrophotometric data, however, what is needed is the relative spectral irradiance (radiant flux incident per unit area) from each source. The spectral irradiance from source A is found

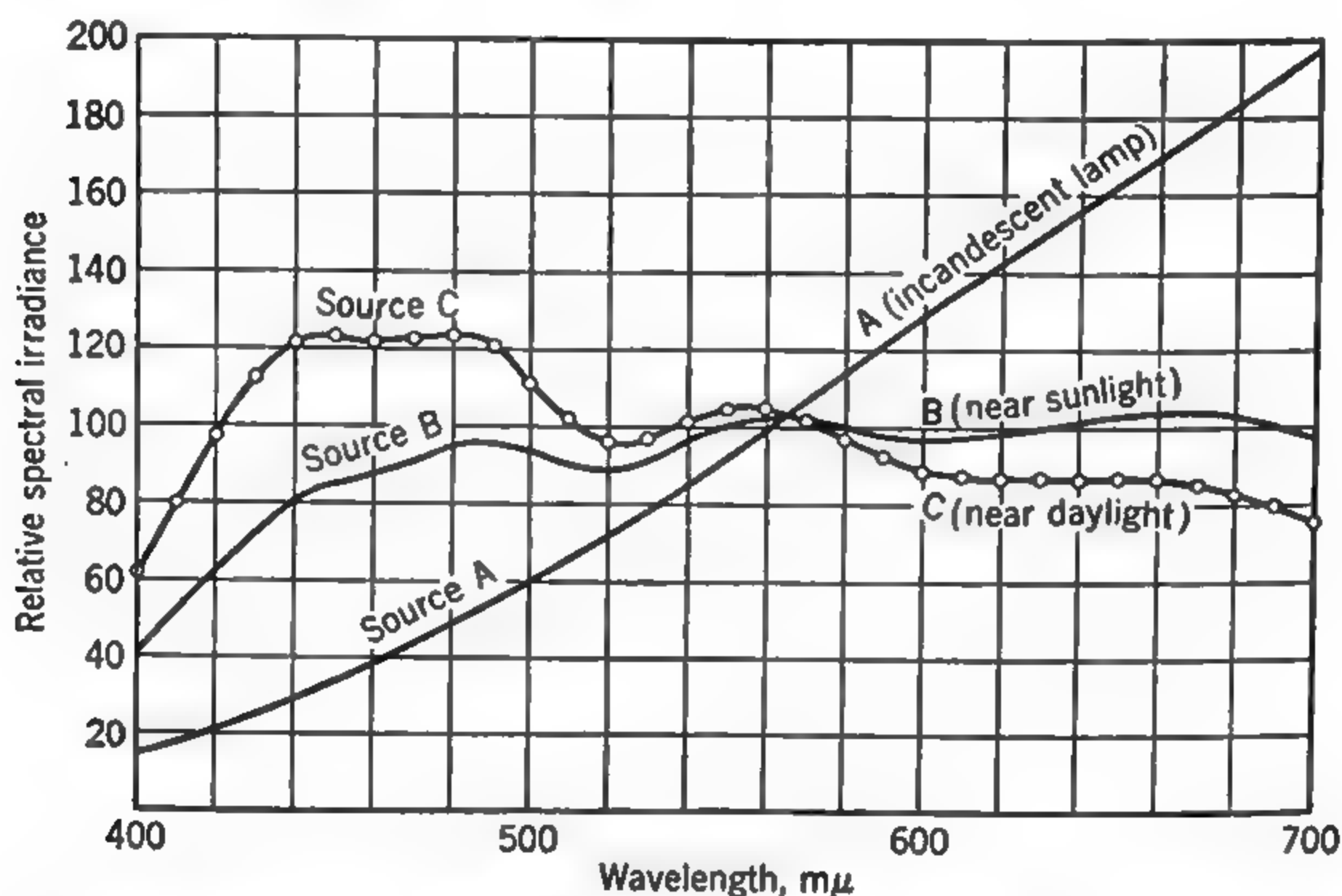


FIG. 27. Relative spectral irradiances from standard sources A, B, and C. Source A is typical of the gas-filled incandescent lamp; source B, of noon sunlight; and source C, of average daylight.

from the Planck radiation law (to be discussed later) on the well-supported assumption that radiant energy from a tungsten-filament, clear-bulb, gas-filled lamp has closely the same spectral distribution as that from a small hole in a furnace whose walls are heated to a temperature equal to the color temperature of the lamp. The relative spectral irradiances from sources B and C are found by measuring spectrophotometrically the spectral transmittances of the filters required to produce them, and then multiplying, wavelength by wavelength, by the spectral irradiance from source A. Table 5 gives the relative spectral irradiances of the three standard sources, A, B, and C. Figure 27 is a plot of these relative spectral irradiances. Use of source B is prevalent in Great Britain; use of source C is prevalent in this country.

Standard Observer and Coordinate System. When the manufactured product is compared to the color standard by each of a number of

people, it will probably be found that there are almost as many descriptions of the color difference as there are people. About one man in twenty will have anomalous vision and if product and standard form a metameric pair may obtain from his comparison a result quite different from those of normal vision. Other discrepancies will arise from the varying degrees of ocular pigmentation among people, from their differing ideas as to what constitutes a good color match, and from their different color languages arising from their different backgrounds and training. The businessman will immediately recognize the need for some assurance that the purchaser will judge the goodness of match the way he does. In many cases it has been found worth while to write into contracts a specific test method for color. Resort to a standard observer by means of which to interpret the spectrophotometric result is one of these methods.

The standard observer widely used today was recommended in 1931 by the International Commission on Illumination. The spectral responses of this standard observer are defined by the tristimulus values of the spectrum given in Table 6 and plotted as Fig. 14. At any wavelength the values of the three functions \bar{x}_λ , \bar{y}_λ , \bar{z}_λ give the amount of the X (red) primary, the amount of the Y (green) primary, and the amount of the Z (blue) primary, respectively, required to produce for the standard observer the color of the spectrum at that wavelength; that is \bar{x}_λ , \bar{y}_λ , \bar{z}_λ are the tristimulus values of the colors produced by unit density of radiant flux of very narrow wavelength range, so narrow that further reduction in band width would produce no change in the tristimulus values. These tristimulus values were derived from measurements with a 2° field by Guild (1931) on 7 observers at the National Physical Laboratory in Teddington, combined with similar measurements by Wright (1928–1929) at the University of London on 10 observers, supplemented by Wright's measurements on 35 observers to obtain a more reliable indication of the effect of ocular pigmentation. Note that each part of the spectrum can be duplicated by positive amounts (\bar{x}_λ , \bar{y}_λ , \bar{z}_λ) of the primaries. Recall, however, that experiments on additive mixture of lights prove that there are no three lights whose mixture duplicates all the spectrum colors. It follows, therefore, that the primaries of the system recommended by the International Commission on Illumination (the CIE system) are unreal, or imaginary.

We are now in position to show in detail why these primaries are unreal, and also to show other facts about normal color vision. It is only necessary to compute the chromaticity coordinates (x , y , z) of

Table 6. 1931 CIE Standard Observer for Colorimetry

Wave-length, mμ	Tristimulus Values of Equal-Energy Spectrum			Wave-length, mμ	Tristimulus Values of Equal-Energy Spectrum		
	\bar{x}_λ	\bar{y}_λ	\bar{z}_λ		\bar{x}_λ	\bar{y}_λ	\bar{z}_λ
380	0.0014	0.0000	0.0065	580	0.9163	0.8700	0.0017
385	.0022	0.0001	0.0105	585	0.9786	.8163	.0014
390	.0042	0.0001	0.0201	590	1.0263	.7570	.0011
395	.0076	0.0002	0.0362	595	1.0567	.6949	.0010
400	.0143	0.0004	0.0679	600	1.0622	.6310	.0008
405	.0232	0.0006	0.1102	605	1.0456	.5668	.0006
410	.0435	0.0012	0.2074	610	1.0026	.5030	.0003
415	.0776	0.0022	0.3713	615	0.9384	.4412	.0002
420	.1344	0.0040	0.6456	620	0.8544	.3810	.0002
425	.2148	0.0073	1.0391	625	0.7514	.3210	.0001
430	.2839	0.0116	1.3856	630	0.6424	.2650	.0000
435	.3285	0.0168	1.6230	635	0.5419	.2170	.0000
440	.3483	0.0230	1.7471	640	0.4479	.1750	.0000
445	.3481	0.0298	1.7826	645	0.3608	.1382	.0000
450	.3362	0.0380	1.7721	650	0.2835	.1070	.0000
455	.3187	0.0480	1.7441	655	0.2187	.0816	.0000
460	.2908	0.0600	1.6692	660	0.1649	.0610	.0000
465	.2511	0.0739	1.5281	665	0.1212	.0446	.0000
470	.1954	0.0910	1.2876	670	0.0874	.0320	.0000
475	.1421	0.1126	1.0419	675	0.0636	.0232	.0000
480	.0956	0.1390	0.8130	680	0.0468	.0170	.0000
485	.0580	0.1693	0.6162	685	0.0329	.0119	.0000
490	.0320	0.2080	0.4652	690	0.0227	.0082	.0000
495	.0147	0.2586	0.3533	695	0.0158	.0057	.0000
500	.0049	0.3230	0.2720	700	0.0114	.0041	.0000
505	.0024	0.4073	0.2123	705	0.0081	.0029	.0000
510	.0093	0.5030	0.1582	710	0.0058	.0021	.0000
515	.0291	0.6082	0.1117	715	0.0041	.0015	.0000
520	.0633	0.7100	0.0782	720	0.0029	.0010	.0000
525	.1096	0.7932	0.0573	725	0.0020	.0007	.0000
530	.1655	0.8620	0.0422	730	0.0014	.0005	.0000
535	.2257	0.9149	0.0298	735	0.0010	.0004	.0000
540	.2904	0.9540	0.0203	740	0.0007	.0003	.0000
545	.3597	0.9803	0.0134	745	0.0005	.0002	.0000
550	.4334	0.9950	0.0087	750	0.0003	.0001	.0000
555	.5121	1.0002	0.0057	755	0.0002	.0001	.0000
560	.5945	0.9950	0.0039	760	0.0002	.0001	.0000
565	.6784	0.9786	0.0027	765	0.0001	.0000	.0000
570	.7621	0.9520	0.0021	770	0.0001	.0000	.0000
575	.8425	0.9154	0.0018	775	0.0000	.0000	.0000
580	.9163	0.8700	0.0017	780	0.0000	.0000	.0000
Totals					21.3713	21.3714	21.3715

Table 7. Chromaticity Coordinates of the Spectrum Colors

Wave-length, mμ	Chromaticity Coordinates			Wave-length, mμ	Chromaticity Coordinates		
	<i>x</i>	<i>y</i>	<i>z</i>		<i>x</i>	<i>y</i>	<i>z</i>
380	0.1741	0.0050	0.8209	580	0.5125	0.4866	0.0009
385	.1740	.0050	.8210	585	.5448	.4544	.0008
390	.1738	.0049	.8213	590	.5752	.4242	.0006
395	.1736	.0049	.8215	595	.6029	.3965	.0006
400	.1733	.0048	.8219	600	.6270	.3725	.0005
405	.1730	.0048	.8222	605	.6482	.3514	.0004
410	.1726	.0048	.8226	610	.6658	.3340	.0002
415	.1721	.0048	.8231	615	.6801	.3197	.0002
420	.1714	.0051	.8235	620	.6915	.3083	.0002
425	.1703	.0058	.8239	625	.7006	.2993	.0001
430	.1689	.0069	.8242	630	.7079	.2920	.0001
435	.1669	.0086	.8245	635	.7140	.2859	.0001
440	.1644	.0109	.8247	640	.7190	.2809	.0001
445	.1611	.0138	.8251	645	.7230	.2770	.0000
450	.1566	.0177	.8257	650	.7260	.2740	.0000
455	.1510	.0227	.8263	655	.7283	.2717	.0000
460	.1440	.0297	.8263	660	.7300	.2700	.0000
465	.1355	.0399	.8246	665	.7311	.2689	.0000
470	.1241	.0578	.8181	670	.7320	.2680	.0000
475	.1096	.0868	.8036	675	.7327	.2673	.0000
480	.0913	.1327	.7760	680	.7334	.2666	.0000
485	.0687	.2007	.7306	685	.7340	.2660	.0000
490	.0454	.2950	.6596	690	.7344	.2656	.0000
495	.0235	.4127	.5638	695	.7346	.2654	.0000
500	.0082	.5384	.4534	700	.7347	.2653	.0000
505	.0039	.6548	.3413	705	.7347	.2653	.0000
510	.0139	.7502	.2359	710	.7347	.2653	.0000
515	.0389	.8120	.1491	715	.7347	.2653	.0000
520	.0743	.8338	.0919	720	.7347	.2653	.0000
525	.1142	.8262	.0596	725	.7347	.2653	.0000
530	.1547	.8059	.0394	730	.7347	.2653	.0000
535	.1929	.7816	.0255	735	.7347	.2653	.0000
540	.2296	.7543	.0161	740	.7347	.2653	.0000
545	.2658	.7243	.0099	745	.7347	.2653	.0000
550	.3016	.6923	.0061	750	.7347	.2653	.0000
555	.3373	.6589	.0038	755	.7347	.2653	.0000
560	.3731	.6245	.0024	760	.7347	.2653	.0000
565	.4087	.5896	.0017	765	.7347	.2653	.0000
570	.4441	.5547	.0012	770	.7347	.2653	.0000
575	.4788	.5202	.0010	775	.7347	.2653	.0000
580	.5125	.4866	.0009	780	.7347	.2653	.0000

the spectrum colors. These are the tristimulus values expressed as fractions of their total:

$$x \equiv X/(X + Y + Z) \quad y \equiv Y/(X + Y + Z) \quad z \equiv Z/(X + Y + Z) \quad (7)$$

Table 7 gives these chromaticity coordinates, and Fig. 28 shows a plot of the y coordinate against the x coordinate.

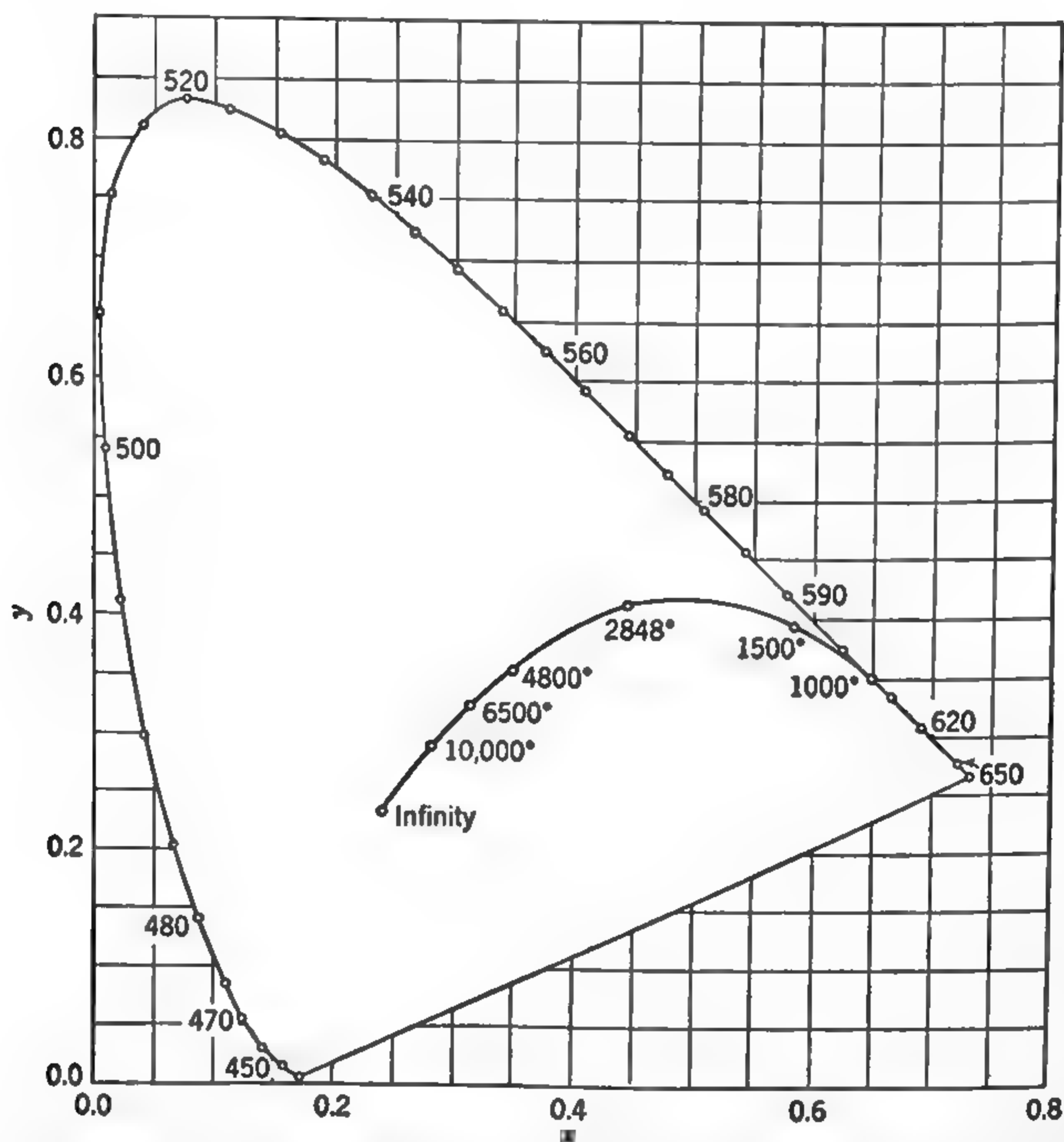


Fig. 28. The (x, y) -chromaticity diagram, showing the spectrum locus and the purple boundary. Wavelength is indicated in millimicrons.

The chromaticity diagram shown in Fig. 28 is the right-angle type of Maxwell triangle; see Fig. 16. The chromaticities of the primaries are represented at the apices of the triangle: $x = 1, y = 0$ (red); $x = 0, y = 1$ (green); and $x = y = 0$ (blue). The small circles represent the chromaticities of the spectrum identified by wavelength in

millimicrons; these circles define the spectrum locus. As in all Maxwell triangles, the chromaticities of additive combinations of lights are found by the center-of-gravity principle (Newton's law of mixture), the weights being the sums of the tristimulus values, $X + Y + Z$; and, in particular, the chromaticity of two-component mixtures are represented by points on the straight line connecting the two points representing the chromaticities of the component lights. Thus, the straight line connecting the extremes of the spectrum locus is, itself, the locus of chromaticities producible by additive combination of the extremes of the spectrum. Furthermore, the area inclosed by the spectrum locus and this straight line is the locus of all chromaticities producible by additive combination of any spectrum lights. Since all actual lights are such combinations, this area is the locus of all real chromaticities. Points on the (x, y) -chromaticity diagram outside of this area represent imaginary chromaticities, that is, chromaticities not producible by real lights. It will be noted that all the chromaticities of the primaries are represented by such points; that is, they are all imaginary. Furthermore, it will be noted that any triangle inclosing the spectrum locus must have at least two apices outside the locus of real chromaticities. Thus we see why no set of real primaries can yield all positive tristimulus values of the spectrum.

From the spectrum locus itself may be seen a number of the properties of normal color vision. Note (Table 7) that the spectrum near the long-wave extreme (700 to 770 $m\mu$) is represented by a single point. That is, this part of the spectrum has a constant chromaticity and is often referred to as the "long-wave end stretch." Any part of this end stretch may be completely color-matched with any other part merely by adjusting the two parts to the same luminance. Near the short-wave extreme of the spectrum (430 to 380 $m\mu$) there is no such precise constancy of chromaticity, but there is an approach to it.

Note, secondly, that the spectrum locus from about 540 to 700 $m\mu$ closely approaches a straight line on the Maxwell triangle. This means that the chromaticity of any spectrum color within this wavelength range may be closely matched by a combination in proper proportions of the extremes of this range (540 and 700 $m\mu$). This range is sometimes called the "between stretch." This is the range used in the Nagel anomaloscope to test for anomalous color vision by means of the Rayleigh equation. The remainder of the spectrum locus (380 to 540 $m\mu$) is significantly curved. This means that additive combinations of pairs of spectrum lights within this range fail to match the chromaticities of the intermediate parts of the spectrum, the amount of the failure increasing with the wavelength separation. The kind

of discrepancy between the chromaticity of the two-part mixture and that of the intermediate part of the spectrum may also be seen from Fig. 28. Since the spectrum locus is nowhere concave outward, the mixture chromaticity, if it departs at all from the chromaticity of the intermediate parts of the spectrum, must depart in the direction of the chromaticities represented near the center of the mixture diagram. If (as is usual) the observer is so adapted that the central chromaticities yield nearly achromatic color perceptions, the mixture chromaticity will appear less saturated (more neutral) than the spectrum chromaticity yielding the same hue.

To push this study of two-component mixtures to an extreme and include not only the curved portion of the spectrum locus but also the "between stretch" and "end stretch," note that it is possible to increase the wavelength separation of the components until the mixture matches stimuli (such as the equal-energy stimulus) ordinarily perceived as having no hue at all. In such a case the two spectrum lights are said to be complementary with respect to the stimulus appearing neutral. The wavelengths of pairs of complementary spectrum lights may be found from Fig. 28 by drawing a straight line through the point representing the chromaticity of the stimulus that appears neutral and reading the wavelengths of the two intersections with the spectrum locus. It will be noted that the complements of spectrum colors between 380 and 494 $m\mu$ relative to the equal-energy stimulus are to be found between 570 and 700 $m\mu$, and, of course, the reverse is true. But the complements of the spectrum colors between 494 and 570 $m\mu$ cannot be formed from any one part of the spectrum but must be made up of mixtures of at least two spectrum lights, one from the short-wave, the other from the long-wave end. Such colors, not producible by mixtures of a neutral light with some part of the spectrum, are sometimes called nonspectral, or purple, colors.

The fundamental nature of the chromaticity diagram is strikingly proved by the simplicity with which it shows the chromaticities confused by color-blind observers. Figure 29a shows in complete detail the chromaticities confused by observers having the protanopic form of dichromatic vision. All chromaticities represented on each of the solid straight lines of Fig. 29a are seen alike by the protanope. Figures 29b and 29c give similar information for the deuteranopic and tritanopic forms of dichromatic vision. These three figures, taken together, supplement the summary of the properties of dichromatic vision given in Table 3 and make it more precise. For example, from the chromaticity-confusion lines passing through the point representing standard source C (average daylight), it may be seen that protanopes,

roughly described as red-green confusers, are, to be more precise, confusers of red with bluish green. Similarly, deuteranopes confuse green with purplish red, and tritanopes confuse bluish purple with greenish yellow. The straight lines on each of Figs. 29a, 29b, and 29c form a simple family. All the lines in each family intersect in a common point outside the area representing real chromaticities. For protanopes this copunctal point is $x = 0.747$, $y = 0.253$; for deuteranopes, $x = 1.000$, $y = 0.000$; and for tritanopes, $x = 0.180$, $y = 0.000$. If we wish to set up a tristimulus system describing the relationship between normal and dichromatic vision even more simply than the standard CIE system, we have merely to build it on these chromaticities as primaries. By setting the units of these new primaries equal for the equal-energy spectrum we obtain (Judd, 1944) the transformation equations:

$$\left. \begin{aligned} W_d &= 1.000Y \\ W_p &= -0.460X + 1.359Y + 0.101Z \\ K &= 1.000Z \end{aligned} \right\} \quad (7a)$$

Figure 29d shows the tristimulus values of the spectrum colors expressed relative to these new primaries which may be called the failure colors of dichromatic vision. These tristimulus values are found by substituting the tristimulus values of the spectrum (\bar{x}_λ , \bar{y}_λ , \bar{z}_λ) from the standard CIE system (Table 6, Fig. 14) for X , Y , and Z , respectively, in equation 7a. According to the three-components theory of vision developed by Young (1807), Helmholtz (1852), and König (1897), these tristimulus values are the spectral sensitivities of the three types of cones in the normal retina, but that is a matter of conjecture.

The standard observer serves to group lights together on the basis of color match. Each triad of tristimulus values identifies a single color but may correspond to an infinity of lights of different spectral compositions, as witness the color of an equal-energy stimulus and the infinite series of complementary spectrum pairs equivalent to it. A word of caution is necessary at this point. The standard observer alone does not provide the basis for predicting what an observer of normal color vision will see. We believe that, by adding to the tristimulus values of a color a specification of the real observer's state of chromatic adaptation, such a valid prediction will ultimately be possible (Judd, 1940); but that is still somewhat in the future.

The acceptance of this standard observer and coordinate system recommended by international action in 1931 has been very extensive. The chromaticity diagram by far most often used is the (x, y) -chro-

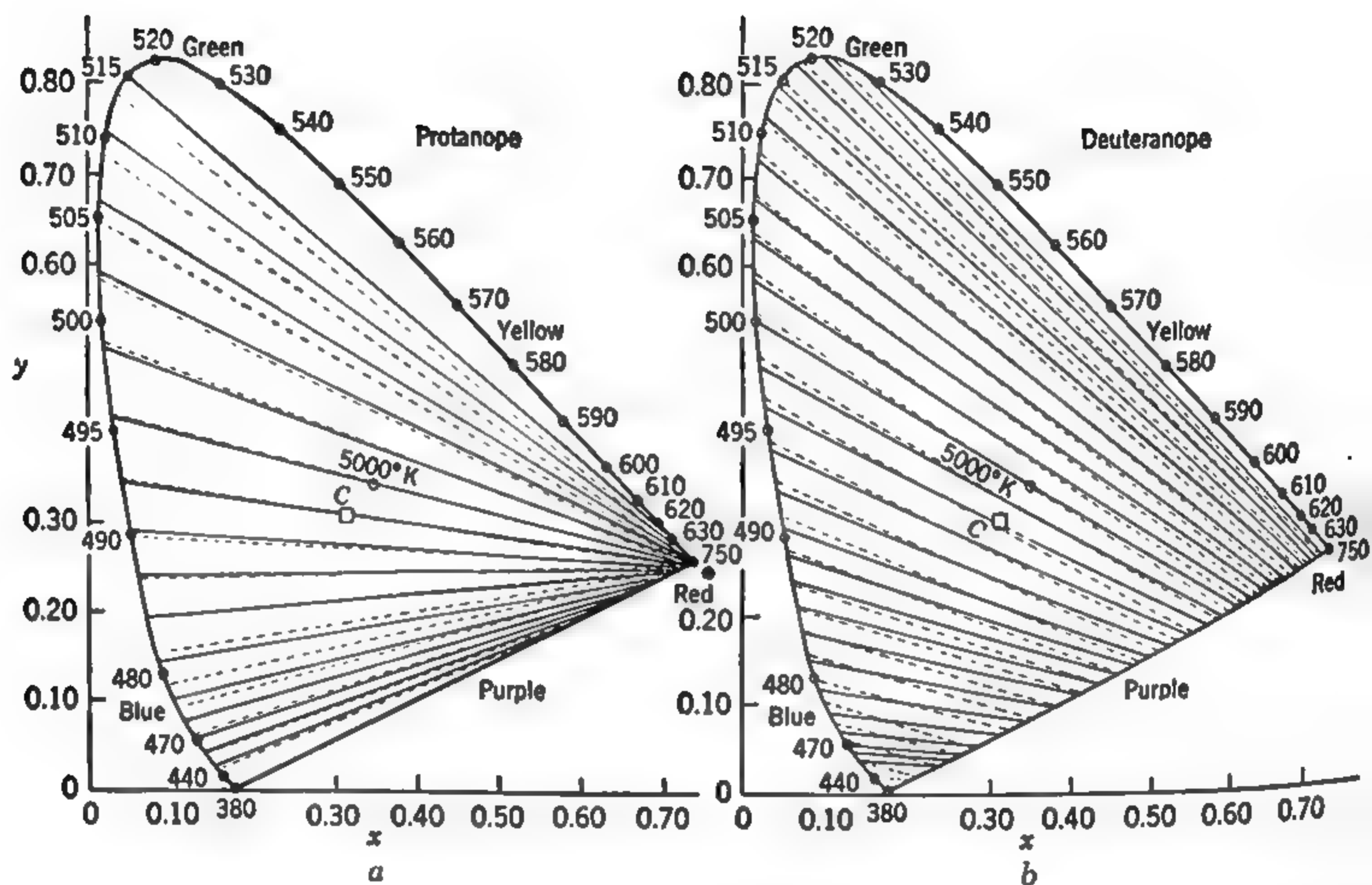


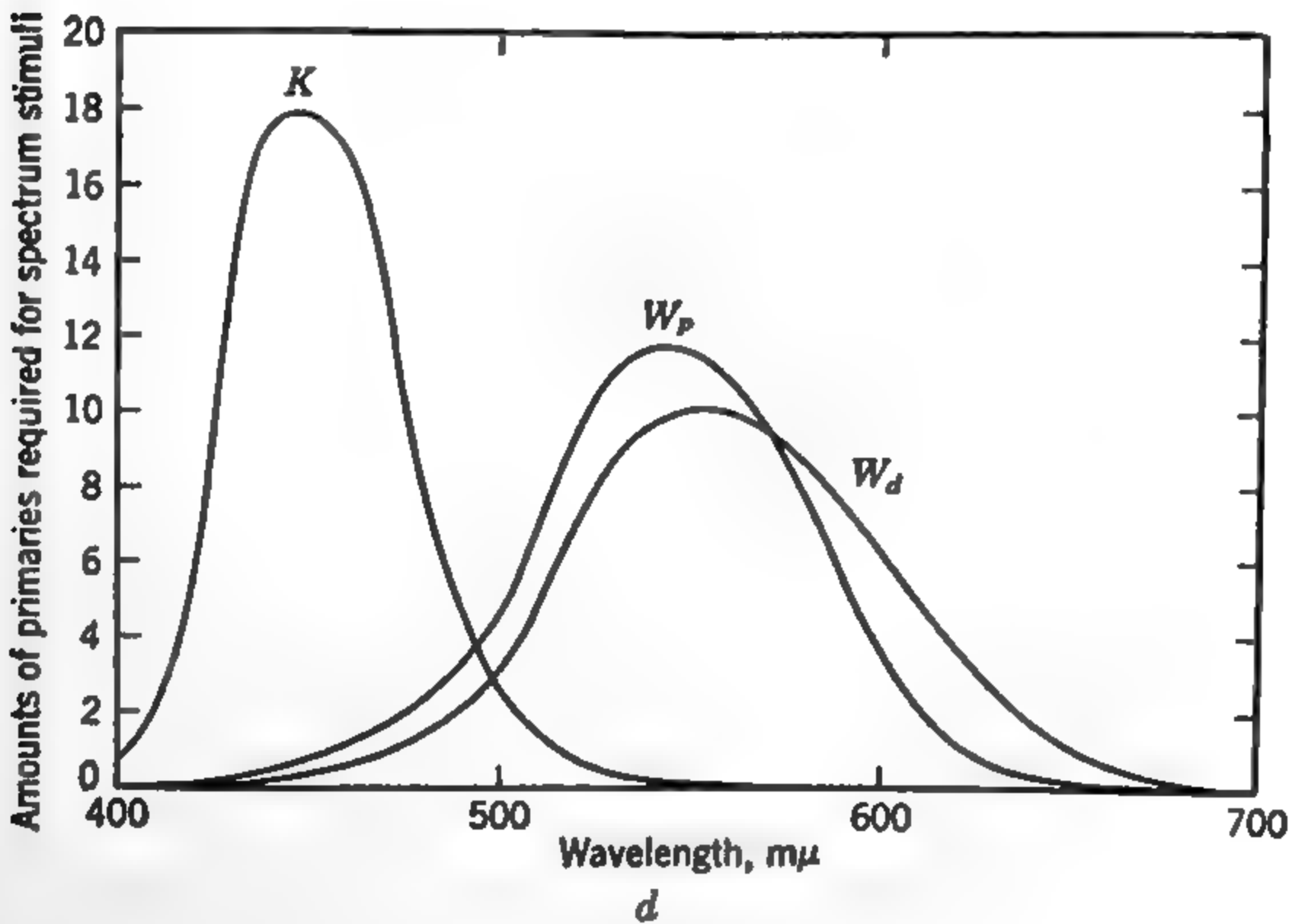
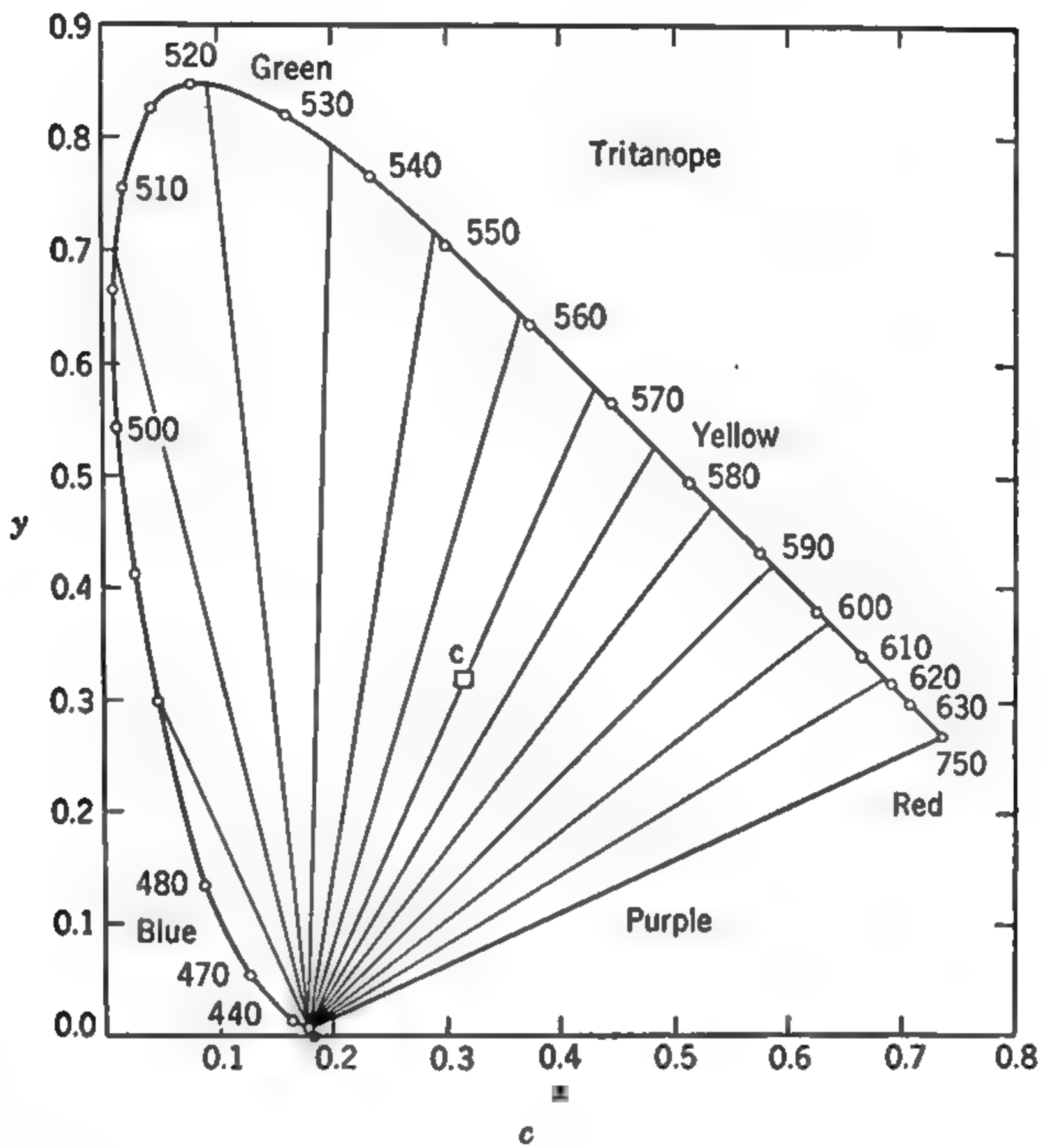
FIG. 29. Relation between normal trichromatic vision and the three most common types of dichromatic vision. Each straight line shown corresponds to a series of chromaticities impossible for the dichromatic observer to distinguish, although easy for the normal trichromat.

(a) Protanopic chromaticity confusions shown on the (x, y) -chromaticity diagram.

(b) Deuteranopic chromaticity confusions shown on the (x, y) -chromaticity diagram.

(c) Tritanopic chromaticity confusions shown on the (x, y) -chromaticity diagram.

(d) Tristimulus values of the spectrum colors expressed relative to the failure colors of dichromatic vision as primaries. According to the Young-Helmholtz-König three-components theory of vision, these curves correspond to the spectral sensitivities of the three types of cone in the normal retina.



maticity diagram shown in Fig. 28.* This diagram has been used to interpret thousands of routine determinations and in hundreds of industrial research projects and has served as a satisfactory map of chromaticities. Large sheets of coordinate paper (18 by 22 inches) showing the spectrum locus are available in lots of 50 through the Munsell Color Company.†

The standard observer and coordinate system was recommended in 1931, not because it had been proved to be a statistically reliable average of normal color vision, which it had not, but because it was believed to be intermediate to actual observers with normal color vision. Now after 20 years of extensive use only a few isolated cases have been found (Jacobsen, 1948; Judd, 1949, *b*) suggesting that improved commercial practices could be set up, if a revision of the standard observer were made. The 1931 CIE standard observer will continue to be used at the National Bureau of Standards, and probably elsewhere, until more extensive studies of color vision supply a significantly better basis for a standard resulting in a new international agreement.

Reduction of Spectrophotometric Data to Tristimulus Values. Reduction of spectrophotometric data to tristimulus values X , Y , Z is based on Grassmann's laws (equations 2 and 3). The spectrophotometer shows how the radiant flux entering the observer's eye is distributed throughout the visible spectrum. For each part of the spectrum the tristimulus values are given in Table 6 expressed in terms of the CIE standard observer and coordinate system. We have, therefore, merely to write down the tristimulus values corresponding to the spectral irradiance of the observer's eye for each part in succession and add them. The question arises, of course, how finely the spectrum must be divided to give a valid result. The answer is that it must be so finely divided that further splitting up into smaller wavelength intervals would make no difference in the computed results. The definitions, therefore, of the tristimulus values, X , Y , Z , corresponding to a stimulus of spectral irradiance, H_λ , are:

* This particular choice of rectangular Maxwell triangle (out of the three possible) is apparently a matter of pure chance. The use of the letters X , Y , Z to designate the three scales of the CIE system recommended for practical use came about from a need to distinguish them from the R , G , B scales applying to real red, green, and blue colors in terms of which the standard observer was also defined in the same document. It seems to have fallen to the author (Judd, 1933) to plot the first (x, y) diagram, and the choice was made solely because y is the traditional designation of the ordinate, and x that of the abscissa.

† 10 East Franklin Street, Baltimore 2, Md.

$$\left. \begin{aligned} X &\equiv \int_0^\infty H_\lambda \bar{x}_\lambda d\lambda \\ Y &\equiv \int_0^\infty H_\lambda \bar{y}_\lambda d\lambda \\ Z &\equiv \int_0^\infty H_\lambda \bar{z}_\lambda d\lambda \end{aligned} \right\} \quad (8)$$

where $d\lambda$ is an infinitesimal wavelength interval.

To evaluate X , Y , Z strictly requires the continuous multiplication of H_λ by \bar{x}_λ , \bar{y}_λ , and \bar{z}_λ , separately, the plotting of these products against wavelength, and the measurement of the areas beneath the three curves as by a planimeter. Devices to accomplish this multiplication and integration simultaneously by electrical and mechanical means have been designed and built by Razek (1934), Van den Akker (1939), Davidson and Imm (1949,) Peterson et al. (1949), and Middleton (1950). The practical importance of such devices (Davidson and Godlove, 1950) becomes considerable if they can be adapted to form an attachment to a recording spectrophotometer. The Davidson-Imm integrator has been adapted in this way to the recording spectrophotometer built by the General Electric Company, and the integrator is commercially available as an attachment to that spectrophotometer. A limitation of such attachments is that they cannot easily be modified to take account of errors in the wavelength scale that may arise from various causes.

Most specimens (lights or objects) do not require continuous multiplication throughout the spectrum for accurate calculation of their tristimulus values. If a gaseous discharge tube is the specimen, the products need be taken only for each of the finite number of emission lines in the spectrum. A mercury-vapor lamp can be reasonably well evaluated, for example, by taking only the four most prominent lines or doublets in the visible spectrum of mercury (about 405, 436, 546, and 578 $m\mu$). If the source is an incandescent lamp, or a transparent or opaque object illuminated by such a lamp, it is usually true that the wavelength distribution of spectral irradiance (radiant-flux density) incident on the eye of the observer is so smooth that it is sufficient to sample the distribution at a relatively small number of points (10 to 100). The approximate methods to be described are known as the weighted-ordinate and the selected-ordinate methods.

Weighted-Ordinate Method. In the weighted-ordinate method the spectrum is broken down into a finite number of equal wavelength intervals, and Grassmann's law (equations 2 and 3) is applied directly:

$$\left. \begin{aligned} X &= \sum_{380}^{760} H_{\lambda} \bar{x}_{\lambda} \Delta\lambda \\ Y &= \sum_{380}^{760} H_{\lambda} \bar{y}_{\lambda} \Delta\lambda \\ Z &= \sum_{380}^{760} H_{\lambda} \bar{z}_{\lambda} \Delta\lambda \end{aligned} \right\} \quad (9)$$

It is called the weighted-ordinate method because each ordinate of the curve of spectral irradiance H_{λ} plotted against wavelength λ is weighted by the tristimulus values, \bar{x}_{λ} , \bar{y}_{λ} , and \bar{z}_{λ} , for that wavelength. Some wavelength distributions of spectral irradiance exhibit such irregular variation with wavelength that the wavelength interval, $\Delta\lambda$, must be taken as 5 m μ . Usually $\Delta\lambda = 10$ m μ gives a sufficient approximation, and this wavelength interval is used for routine computation.

Table 8 shows all the details of the calculation of tristimulus values X , Y , Z , both for a self-luminous area and for a combination of this source with a light-transmitting specimen. These calculations are by summation over wavelength intervals of 10m μ . The source taken in this example is standard source A (incandescent lamp at a color temperature of 2,854° K), and the relative spectral irradiance, H_A , of this source, taken from Table 5 and multiplied by the constant factor, 92.685, is shown in the fifth column. The preceding three columns give the tristimulus values, \bar{x}_{λ} , \bar{y}_{λ} , \bar{z}_{λ} , per millimicron of the equal-energy spectrum of unit irradiance taken from Table 6. The next three columns give the tristimulus values of the spectrum of source A computed as $(H_A)_{\lambda} \bar{x}_{\lambda}$, $(H_A)_{\lambda} \bar{y}_{\lambda}$, and $(H_A)_{\lambda} \bar{z}_{\lambda}$, respectively. By Grassmann's laws (equation 2) the sums of these columns multiplied by the wavelength interval in m μ (= 10) are the tristimulus values, X_A , Y_A , Z_A , of source A itself, except for the slight errors introduced by using the finite wavelength interval, $\Delta\lambda = 10$ m μ . It will be noted from the sum of the column of Table 8 headed $\bar{y}H_A$ that the density of incident luminous flux, or illuminance, represented by Y_A , is found in arbitrary units as 1,000,000, the constant factor (92.685) being adjusted to obtain this convenient number. Similarly, X_A is found to be 1,098,280, and Z_A is 355,470, by this method. From the definition (equation 7) we then find the chromaticity coordinates:

$$x = 1,098,280 / (1,098,280 + 1,000,000 + 355,470) = 0.4476$$

$$y = 1,000,000 / (1,098,280 + 1,000,000 + 355,470) = 0.4075$$

Table 9 shows the chromaticity coordinates, x , y , for radiant energy emitted from furnaces maintained at temperatures other than 2,854°

Table 8. Calculation of Tristimulus Values, X, Y, Z, for Source A, Then for the Liquid Filter Used with It to Obtain Source B

Wave-length, λ , m μ	Equal-Energy Spectrum			Source A			Source A Spectrum			Filter Transmittance	Source B (Source A plus Filter) Spectrum		
	\bar{x}_λ	\bar{y}_λ	\bar{z}_λ	$(H_A)\lambda$	$\bar{x}H_A$	$\bar{y}H_A$	$\bar{z}H_A$	T_λ	$\bar{x}H_AT$	$\bar{y}H_AT$	$\bar{z}H_AT$		
380	0.0014	0.0000	0.0065	907	I		6	0.588	1				
390	0.0042	.0001	0.0201	1,121	II		23	.666	II				15
400	0.0143	.0004	0.0679	1,363	III	I	93	.721	14	II			67
410	0.0435	.0012	0.2074	1,639	71	2	340	.757	54	2			257
420	0.1344	.0040	0.6456	1,946	262	8	1,256	.774	203	6			972
430	0.2839	.0116	1.3856	2,287	649	27	3,167	.761	494	21			2,410
440	0.3483	.0230	1.7471	2,660	926	81	4,647	.723	669	III			3,360
450	0.3362	.0380	1.7721	3,067	1,031	117	5,435	.663	684	III			3,603
460	0.2908	.0600	1.6692	3,505	1,019	210	5,851	.601	612	126			3,516
470	0.1954	.0910	1.2876	3,973	776	362	5,116	.551	428	199			2,819
480	0.0956	.1390	0.8130	4,472	428	622	3,636	.507	217	315			1,843
490	0.0320	.2080	0.4652	4,997	160	1,039	2,324	.460	74	478			1,069
500	0.0049	.3230	0.2720	5,548	27	1,792	1,509	.404	11	724			610
510	0.0093	.5030	0.1582	6,123	57	3,080	969	.353	20	1,087			342
520	0.0633	.7100	0.0782	6,720	425	4,771	522	.317	135	1,512			166
530	0.1655	.8620	0.0422	7,334	1,214	6,322	309	.300	364	1,897			93
540	0.2004	.9540	0.0203	7,966	2,313	7,600	162	.290	671	2,204			47
550	0.4334	.9950	0.0087	8,611	3,732	8,568	78	.279	1,041	2,390			21
560	0.5945	.9950	0.0039	9,268	5,510	9,222	36	.264	1,455	2,435			10
570	0.7821	.9520	0.0021	9,934	7,571	9,457	21	.246	1,862	2,326			5
580	0.9163	.8700	0.0017	10,607	9,719	9,228	11	.227	2,206	2,095			1
590	1.0263	.7570	0.0011	11,283	11,579	8,540	12	.209	2,420	1,785			3
600	1.0622	.6310	0.0008	11,960	12,704	7,547	10	.195	2,477	1,472			2
610	1.0026	.5030	0.0003	12,637	12,669	6,356	4	.186	2,356	1,182			1
620	0.8544	.3810	0.0002	13,311	11,373	5,071	1	.178	2,024	903			1
630	0.6424	.2650	0.0000	13,980	8,980	3,704		.172	1,545	637			
640	0.4479	.1750	0.0000	14,642	6,558	2,562		.166	1,089	425			
650	0.2833	.1070	0.0000	15,296	4,336	1,637		.162	702	265			
660	0.1649	.0610	0.0000	15,938	2,628	972		.157	413	153			
670	0.0874	.0320	0.0000	16,569	1,448	530		.151	219	80			
680	0.0468	.0170	0.0000	17,187	804	292		.144	116	42			
690	0.0227	.0082	0.0000	17,789	404	146		.136	55	20			
700	0.0114	.0041	0.0000	18,376	209	78		.128	27	10			
710	0.0058	.0021	0.0000	18,946	110	40		.121	13	5			
720	0.0029	.0010	0.0000	19,497	57	19		.113	6	2			
730	0.0014	.0005	0.0000	20,031	28	10		.106	3	1			
740	0.0007	.0003	0.0000	20,545	14	6		.101	1	1			
750	0.0003	.0001	0.0000	21,039	6	2		.096	1				
760	0.0002	.0001	0.0000	21,513	4	1		.094					
770	0.0001	.0000	0.0000	21,967	2			.093					
Sum (X, Y, Z):					109,828	100,000	35,547		24,685	24,923	21,240		

Table 9. Chromaticity Coordinates of the Ideal Radiator Based on the Planck Formula:

$$H_{\lambda} = C_1/\lambda^5(e^{-C_2/\lambda T} - 1), C_2 = 14,380 \text{ micron degrees}$$

Temperature, T, Degrees K	Chromaticity Coordinates		Temperature, T, Degrees K	Chromaticity Coordinates	
	x	y		x	y
1,500	0.5856	0.3932	4,000	0.3804	0.3767
1,600	.5731	.3993	4,500	.3607	.3635
1,700	.5609	.4043	5,000	.3450	.3516
1,800	.5491	.4083	5,500	.3324	.3410
1,900	.5377	.4112	6,000	.3220	.3317
2,000	.5266	.4133	6,500	.3135	.3236
2,100	.5158	.4146	7,000	.3063	.3165
2,200	.5055	.4152	7,500	.3003	.3103
2,300	.4956	.4152	8,000	.2952	.3048
2,400	.4860	.4147	8,500	.2907	.2999
2,500	.4769	.4137	9,000	.2869	.2956
2,600	.4681	.4123	9,500	.2836	.2918
2,700	.4597	.4106	10,000	.2806	.2883
2,800	.4517	.4086	10,500	.2780	.2852
2,900	.4441	.4064	11,000	.2757	.2824
3,000	.4368	.4041	13,333	.2676	.2724
3,200	.4232	.3989	20,000	.2565	.2577
3,400	.4109	.3935	40,000	.2472	.2448
3,600	.3997	.3879	Infinity	.2399	.2341
3,800	.3896	.3823			

on the Kelvin scale computed in the same way (Harding, 1944). The energy distributions were computed from the Planck formula and have been extended far above temperatures of furnaces actually measured. The chromaticities of these theoretical furnaces are of interest, however, because they are much like the chromaticities of the various phases of daylight (blue sky, overcast sky, noon sun, and so on). Figure 28 shows a plot of these chromaticity coordinates identified by the temperature of the radiator on the Kelvin scale. Fluorescent lamps, artificial daylight lamps for color inspections, and cathode-ray tubes for television receivers are often color graded by reference to this plot, sometimes called the Planckian locus.

Returning now to the last four columns of Table 8 we note that the light-transmitting specimen referred to is the double-cell liquid filter used with standard source A (incandescent-lamp light) to produce standard source B (representative of noon sunlight). The spectral transmittances, T_{λ} , for this filter are shown in the ninth column, and

the last three columns show the tristimulus values of the spectrum of the radiant energy transmitted by this filter so illuminated. These tristimulus values are computed as $(H_A)_\lambda \bar{x}_\lambda T_\lambda$, $(H_A)_\lambda \bar{y}_\lambda T_\lambda$, and $(H_A)_\lambda \bar{z}_\lambda T_\lambda$, respectively. As before, the sums of these columns give the tristimulus values of source B by Grassmann's laws (equation 2), except for the slight errors introduced by using the finite wavelength interval, $\Delta\lambda = 10 \text{ m}\mu$.

The same values of chromaticity coordinates as found in Table 8 for standard source B are, of course, obtainable by calculation directly from the values of spectral irradiance given in Table 5 by using the tristimulus values of the equal-energy spectrum given in Table 6. The calculation of tristimulus values of the colors of objects illuminated by one of the standard sources and seen either by transmitted light (as in the case of the liquid filter producing standard source B, see Table 8), or by reflected light, is so frequent that it saves a significant amount of time to compute first the tristimulus values for the spectrum of the standard source itself. Table 8 has already given these for source A. Table 10 gives them for sources B and C. Similar values have been published for many nonstandard sources by the OSA Committee on Colorimetry (1944); they are a valuable part of the kit of tools available for color measurement.

The products of spectral transmittance or spectral directional reflectance with these tristimulus values of the source are usually found by means of a computing machine, slide-rule computation being not quite precise enough. For extensive computation, Kelly, Gibson, and Nickerson (1943) and MacAdam (1950) have applied the progressive digitation technic of computing on punch-card business machines suggested by Knudsen (1942).

Selected-Ordinate Method. In the selected-ordinate method also the spectrum is broken into a finite number of wavelength intervals, and Grassmann's laws are applied directly. However, instead of intervals of the spectrum uniform in wavelength, intervals of the spectrum are selected that correspond to uniform increments of each primary in the spectrum of the source. If, for example, the Y value of a specimen illuminated by source A is required, the wavelength intervals $\Delta\lambda_1$, $\Delta\lambda_2$, and so forth are chosen so that:

$$H_A \bar{y}(\Delta\lambda_1) = H_A \bar{y}(\Delta\lambda_2) = H_A \bar{y}(\Delta\lambda_3) = \dots$$

The spectral transmittance, T_λ , of the specimen within each such interval is as important as that for each other such interval; so in computing an approximate value of Y for the whole spectrum no further

Table 10. Tristimulus Values for the Spectra of Standard Source B (Representative of Noon Sunlight) and Source C (Representative of Average Daylight)

λ	Source B			Source C		
	$(\bar{x}H)\lambda$	$(\bar{y}H)\lambda$	$(\bar{z}H)\lambda$	$(\bar{x}H)\lambda$	$(\bar{y}H)\lambda$	$(\bar{z}H)\lambda$
380	3		14	4		20
90	13		60	19		89
400	56	2	268	85	2	404
10	217	6	1,033	329	9	1,570
20	812	24	3,899	1,238	37	5,949
30	1,983	81	9,678	2,997	122	14,628
40	2,689	178	13,489	3,975	262	19,938
450	2,744	310	14,462	3,915	443	20,638
60	2,454	506	14,085	3,362	694	19,299
70	1,718	800	11,319	2,272	1,058	14,972
80	870	1,265	7,396	1,112	1,618	9,461
90	295	1,918	4,290	363	2,358	5,274
500	44	2,908	2,449	52	3,401	2,864
10	81	4,360	1,371	89	4,833	1,520
20	541	6,072	669	576	6,462	712
30	1,458	7,594	372	1,523	7,934	388
40	2,689	8,834	188	2,785	9,149	195
550	4,183	9,603	84	4,282	9,832	86
60	5,840	9,774	38	5,880	9,841	39
70	7,472	9,334	21	7,322	9,147	20
80	8,843	8,396	16	8,417	7,992	16
90	9,728	7,176	10	8,984	6,627	10
600	9,948	5,909	7	8,949	5,316	7
10	9,436	4,734	3	8,325	4,176	2
20	8,140	3,630	1	7,070	3,153	1
30	6,200	2,558		5,309	2,190	
40	4,374	1,709		3,693	1,443	
650	2,815	1,062		2,349	880	
60	1,655	612		1,361	504	
70	876	321		708	259	
80	465	169		369	134	
90	220	80		171	62	
700	108	39		82	29	
10	53	19		39	14	
20	26	9		19	6	
30	12	4		8	3	
40	6	2		4	2	
750	2	1		2	1	
60	2	1		1	1	
70	1			1		
Sums	99,072	100,000	85,223	98,041	100,000	118,103
$\bar{x}_w, \bar{y}_w, \bar{z}_w$	0.3485	0.3518	0.3000	0.3101	0.3163	0.3736

Table 11. Selected Ordinates (in Millimicrons) for Computing Tristimulus Values, X, Y, Z, for Specimens under Standard Sources

Number	Standard Source A			Standard Source B			Standard Source C		
	X	Y	Z	X	Y	Z	X	Y	Z
1	444.0	487.8	416.4	428.1	472.3	414.8	424.4	465.9	414.1
2 *	516.9	507.7	424.9	442.1	494.5	422.9	435.5	489.4	422.2
3	544.0	517.3	429.4	454.1	505.7	427.1	443.9	500.4	426.3
4	554.2	524.1	432.9	468.1	513.5	430.3	452.1	508.7	429.4
5 *	561.4	529.8	436.0	527.8	519.6	433.0	461.2	515.1	432.0
II	567.1	534.8	438.7	543.3	524.8	435.4	474.0	520.6	434.3
7	572.0	539.4	441.3	551.9	529.4	437.7	531.2	525.4	436.5
8 *	576.3	543.7	443.7	558.5	533.7	439.9	544.3	529.8	438.6
9	580.2	547.8	446.0	564.0	537.7	442.0	552.4	533.9	440.6
10	583.9	551.7	448.3	568.8	541.5	444.0	558.7	537.7	442.5
11 *	587.2	555.4	450.5	573.1	545.1	446.0	564.1	541.4	444.4
12	590.5	559.1	452.6	577.1	548.7	448.0	568.9	544.9	446.3
13	593.5	562.7	454.7	580.9	552.1	450.0	573.2	548.4	448.2
14 *	596.5	566.3	456.8	584.5	555.5	451.9	577.3	551.8	450.1
III	599.4	569.8	458.8	588.0	559.0	453.9	581.3	555.1	452.1
16	602.3	573.3	460.8	591.4	562.4	455.8	585.0	558.5	454.0
17 *	605.2	576.9	462.9	594.7	565.8	457.8	588.7	561.9	455.9
IV	608.0	580.5	464.9	598.1	569.3	459.8	592.4	565.3	457.9
19	610.9	584.1	467.0	601.4	572.9	461.8	596.0	568.9	459.9
20 *	613.8	587.9	469.2	604.7	576.7	463.9	599.6	572.5	462.0
21	616.9	591.8	471.6	608.1	580.6	466.1	603.3	576.4	464.1
22	620.0	595.9	474.1	611.6	584.7	468.4	607.0	580.5	466.3
23 *	623.3	600.1	476.8	615.3	589.1	470.8	610.9	584.8	468.7
24	626.9	604.7	479.9	619.1	593.9	473.6	615.0	589.6	471.4
25	630.8	609.7	483.4	623.3	599.1	476.6	619.4	594.8	474.3
26 *	635.3	615.2	487.5	628.0	605.0	480.2	624.2	600.8	477.7
27	640.0	621.5	492.7	633.4	611.8	484.5	629.8	607.7	481.8
28	640.0	629.2	499.3	640.1	619.9	490.2	636.6	616.1	487.2
29 *	655.9	639.7	508.4	649.2	630.9	498.6	645.9	627.3	495.2
30	673.5	659.0	526.7	666.3	650.7	515.2	663.0	647.4	511.2
Multiplying factors:									
30 ordinates	0.03661	0.03333	0.01185	0.03303	0.03333	0.02842	0.03268	0.03333	0.03938
10 ordinates	.10984	.10000	.03555	.09909	.10000	.08526	.09804	.10000	.11812

* For an abridged method use only these 10 ordinates.

weighting of each wave band is needed. No tedious multiplications of transmittances by weighting factors are required. To apply Grassmann's laws (equations 2 and 3) we need only to add together the values of transmittance applying to each wave band. The curve of spectral transmittance is thus sampled more frequently in the wavelength regions of greater importance; in this case, the curve is sampled often in the middle of the visible spectrum but is sparsely sampled near the ends.

Table 11 gives (Hardy, 1936; OSA Committee on Colorimetry, 1944 *b*) selected ordinates for computing the tristimulus values, X , Y , Z , of specimens illuminated either by source A, or by source B, or by source C. This table shows 30 ordinates for each source (A , B , C) and each primary (X , Y , Z). By taking every third ordinate (second, fifth, eighth, and so on) indicated by asterisks, the thirty-selected-ordinate method can be abridged to the ten-selected-ordinate method. After the sums of the spectral transmittances at these wavelengths have been found, it is necessary to apply multiplying factors to them to obtain the tristimulus values, X , Y , Z . The need for these factors may be seen by assuming no specimen at all in the beam, in which case the tristimulus values, X , Y , Z , should be those of the source. For no specimen at all in the beam the spectral transmittance would be identically 1.000 for all wavelengths, and the sum of these values for the thirty selected ordinates would be 30 for all three columns, the X , the Y , and the Z columns. The multiplying factors for source A are thus:

$$X_A/30 \doteq 1.0983/30 = 0.03661$$

$$Y_A/30 \doteq 1.0000/30 = 0.03333$$

$$Z_A/30 \doteq 0.3555/30 = 0.01185$$

Note that the arbitrary units used for the tristimulus values obtained from these multiplying factors differ from those obtained in Table 8 by a factor of 1 million. This change in scale factor has been generally adopted for convenience.

Table 12 gives (Bowditch and Null, 1938; OSA Committee on Colorimetry, 1944) selected ordinates for computing the chromaticity coordinates, x , y , of lights of known spectral energy distribution. The sums of the spectral energies at these wavelengths give the tristimulus values, X , Y , Z , directly in arbitrary units; no multiplying factors different for X , Y , and Z are needed. The chromaticity coordinates, x , y , are then found from equation 7.

Table 12. Selected Ordinates for Evaluation of Radiant Energy

Number	X	Y	Z
1	423.7	469.7	410.4
2 *	436.5	493.1	419.6
3	446.8	504.6	424.1
4	457.7	512.1	427.4
5 *	472.1	518.0	430.1
6	531.6	522.9	432.5
7	544.9	527.4	434.8
8 *	553.4	531.6	437.0
9	560.0	535.5	439.1
10	565.5	539.3	441.1
11 *	570.4	543.1	443.1
12	574.8	546.7	445.1
13	578.9	550.3	447.1
14 *	582.8	553.9	449.1
15	586.4	557.4	451.1
16	589.9	561.0	453.2
17 *	593.4	564.6	455.2
18	596.7	568.3	457.2
19	600.1	572.0	459.3
20 *	603.5	575.9	461.5
21	606.9	579.9	463.7
22	610.4	584.1	466.0
23 *	614.0	588.5	468.5
24	617.9	593.3	471.3
25	622.1	598.5	474.3
26 *	626.7	604.3	477.9
27	632.2	611.0	482.4
28	638.8	619.1	488.2
29 *	648.0	629.9	496.9
30	665.4	649.7	513.7

* For an abridged method use only these 10 ordinates.

Table 13 shows a sample calculation by means of both the 30-selected-ordinate method and the 10-selected-ordinate method. The specimen taken is the double-cell liquid filter required to produce source B when illuminated by source A—the same example taken in Table 8. Figure 30 shows a plot (circles) of the values of spectral transmittance of this specimen taken from Table 8. The entries in Table 13 were read from the smooth curve of this plot at the wavelengths indicated in Table 11 for source A.

Table 13. Spectral Transmittances, T_{λ} , of the Double-Cell Liquid Filter Converting Source A into Source B (See Fig. 30) Read for the Selected Ordinates for Source A (See Table 11)

Number	X		Y		Z	
1	0.701		0.473		0.773	
2 *	.324	0.324	.363	0.363	.773	0.773
3	.287		.323		.765	
4	.271		.317		.754	
5 *	.266	.266	.300	.300	.742	.742
6	.251		.296		.732	
7	.241		.292		.718	
8 *	.231	.231	.287	.287	.702	.702
9	.225		.282		.688	
10	.219		.276		.672	
11 *	.213	.213	.270	.270	.657	.657
12	.207		.266		.643	
13	.203		.260		.630	
14 *	.199	.199	.253	.253	.618	.618
15	.196		.246		.607	
16	.193		.238		.598	
17 *	.189	.189	.231	.231	.586	.586
18	.187		.224		.576	
19	.184		.219		.567	
20 *	.182	.182	.212	.212	.557	.557
21	.179		.206		.545	
22	.178		.200		.532	
23 *	.175	.175	.195	.195	.519	.519
24	.172		.190		.506	
25	.170		.185		.487	
26 *	.168	.168	.181	.181	.474	.474
27	.166		.176		.446	
28	.163		.171		.406	
29 *	.160	.160	.165	.165	.360	.360
30	.150		.158		.303	
Totals	6.650	2.107	7.455	2.457	17.936	5.988
Times factor	0.244	0.231	0.248	0.246	0.212	0.213

* For an abridged method use only these 10 ordinates.

The tristimulus values of the light transmitted by this specimen are seen from Table 8 to be $X_B = 0.247$, $Y_B = 0.249$, and $Z_B = 0.212$, correct to less than 1 in the third decimal. The approximate values found in Table 13 by the 30-selected-ordinate method agree within 3 in the third decimal; and those by the 10-selected-ordinate method agree within 2 in the second. This single comparison suggests that the 30-selected-ordinate method is uncertain by about 1 in the third decimal, and that the 10-selected-ordinate method is uncertain by 1 in the

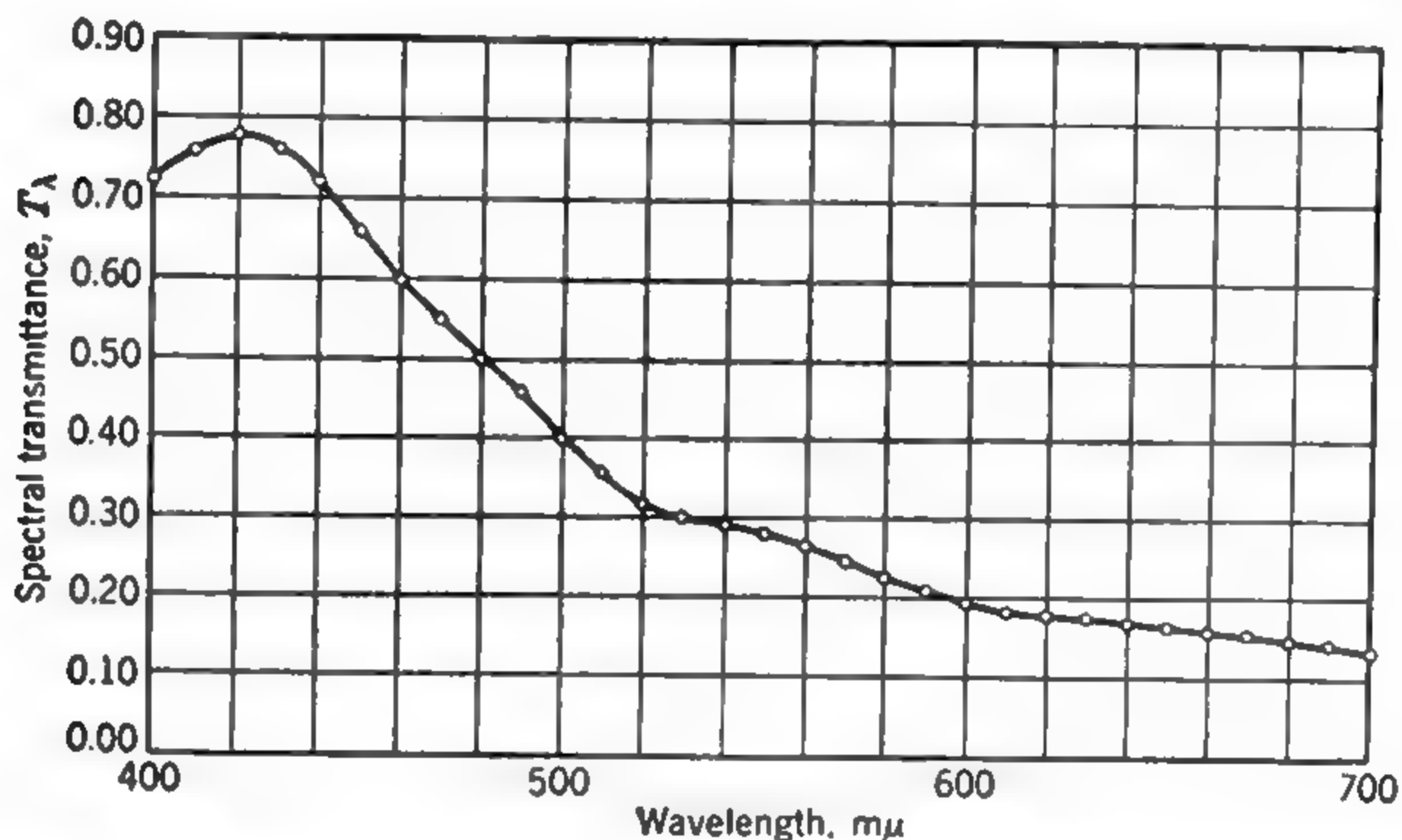


FIG. 30. Spectral transmittance of Davis-Gibson (1931) filter for converting standard source A to standard source B.

second decimal. A more thorough study by Nickerson (1935) indicates "that summation for intervals of 10 mμ gives x and y with an average uncertainty of 0.0004, that the 30-selected-ordinate method has an average uncertainty of 0.0010, and that the 10-selected-ordinate method has an average uncertainty for samples such as these of 0.0035."

The chief reason for the uncertainties of the selected-ordinate method is that near the ends of the spectrum the sampling is so sparse that even a moderate irregularity in the curve of spectral transmittance (or reflectance, as the case may be) makes the sampling not representative of the curve as a whole. Refer, for example, to the discrepancy in X value shown in Table 13 between the 30- and the 10-selected-ordinate methods which amounts to $0.244 - 0.231 = 0.013$. Now, if for each triad of wavelengths the average value of the spectral transmittance was equal to that for the central wavelength of the triad, the 10-ordinate method would agree exactly with the 30-ordinate method. This holds fairly well for all but the first triad; there

the average is 0.437 which is higher by 0.113 than the value, 0.324, at ordinate 2. If we substitute for this nonrepresentative value, 0.324, the average of the triad, 0.437, the total becomes 2.220, and the tristimulus value becomes $2.220 \times 1.0983 = 0.244$, which is the same as was found by complete application of the 30-selected-ordinate method. By using supplementary ordinates in one or two wavelength regions the uncertainty of the selected-ordinate method may be considerably reduced. The sets of 30 selected ordinates given in Table 11 can be supplemented to advantage with partial lists of sets of 90 and 270 ordinates for these same illuminants. These supplementary ordinates for standard sources A, B, and C have been published with complete instructions for their use by the Committee on Colorimetry of the Optical Society of America (1944). In the same publication are also given selected ordinates for Planckian radiators at 2,000° K and up, for 5 phases of natural daylight, and for 3 sources of artificial daylight. These data are convenient and valuable tools for the technician whose job it is to interpret the curves drawn by a recording spectrophotometer with correctly adjusted wavelength scale. But, if the wavelength scale is significantly in error, the unique convenience of the selected-ordinate method is lost, and resort must usually be had to the weighted-ordinate method. MacAdam (1949) has applied the 30-selected-ordinate method in a comprehensive study of the color gamuts of dye mixtures. This study makes use of the punched-card equipment of the International Business Machines Corporation.

VISUAL COLORIMETERS

A visual colorimeter is a device whereby the unknown light fills one part of the field of view and another near-by part, the comparison field, may be filled with one known color after another. The operator looks at these two fields and adjusts the color of the comparison field until it matches the unknown color. This known adjustment is then taken as the specification of the unknown color.

Visual colorimeters may be classified according to the manner of producing the known colors of the comparison field. This may be by combinations, in different amounts, of lights of three fixed chromaticities, by combinations of two lights, one of fixed, the other of continuously adjustable chromaticity, or by a single light whose chromaticity is controlled by interposition of three color filters of adjustable thickness. Since color is a tridimensional quantity, there must always be three independent controls on the color of the comparison field of a visual colorimeter.

If the comparison field is illuminated either simultaneously or in quick succession by three lights of different fixed colors, the amounts being independently adjustable, we have a *tristimulus colorimeter*.

If the comparison field is illuminated simultaneously by a two-part mixture, one part having a neutral color, the other a spectrum color of adjustable wavelength, the amounts of these two parts being independently adjustable, we have a colorimeter giving dominant wavelength and purity directly. The wavelength of the spectrum color required to give a color match for the unknown is called the dominant wavelength of the unknown color. The purity of the color is determined by the ratio of the amounts of the two parts making up the combination.

The purpose of visual colorimeters is to obtain a three-number specification of color in a direct and simple way. These three-number specifications are at least potentially convertible to the tristimulus values, X , Y , Z , of the color in the standard CIE system.

Tristimulus Colorimeters. Tristimulus values X , Y , Z may be obtained by direct comparison of the unknown light with an optical mixture of three primary lights in a divided photometric field. The assemblage of optical and mechanical parts required to introduce the unknown light into the test field and the mixture of the three working primaries into the comparison field is known as a tristimulus colorimeter. Since the primaries of the CIE standard coordinate system for colorimetry are imaginary, it is not possible to use these primaries as the working primaries. The tristimulus colorimeter, therefore, cannot read directly in terms of X , Y , Z , but these values may be computed as weighted sums of the readings, R , G , B , of the colorimeter by equation 4. This transformation equation simply says that the amount of the X primary required to match the RGB combination is the sum of the amount required to match the components separately; and the same is true for Y and Z .

The calibration of a tristimulus colorimeter yields the coefficients of transformation (X_r , Y_r , Z_r ; X_g , Y_g , Z_g ; X_b , Y_b , Z_b). When these coefficients cannot be determined directly by spectrophotometric measurement and computation, the measurement of lights of four known chromaticities such that no one of them can be color-matched by a mixture of any other two will give information from which they can be found.

In choosing working primaries for a tristimulus colorimeter it is natural to strive for a choice that will duplicate as many colors as possible. We have seen that it is not possible to match all colors by additive combination of three lights of fixed chromaticity, but it is an

advantage to have the color gamut as wide as possible. By choosing two of the chromaticities near to those of the extremes of the spectrum and the other one near to that of the middle (green) part of the spectrum (520 $m\mu$) we get a nearly maximum chromaticity gamut; see

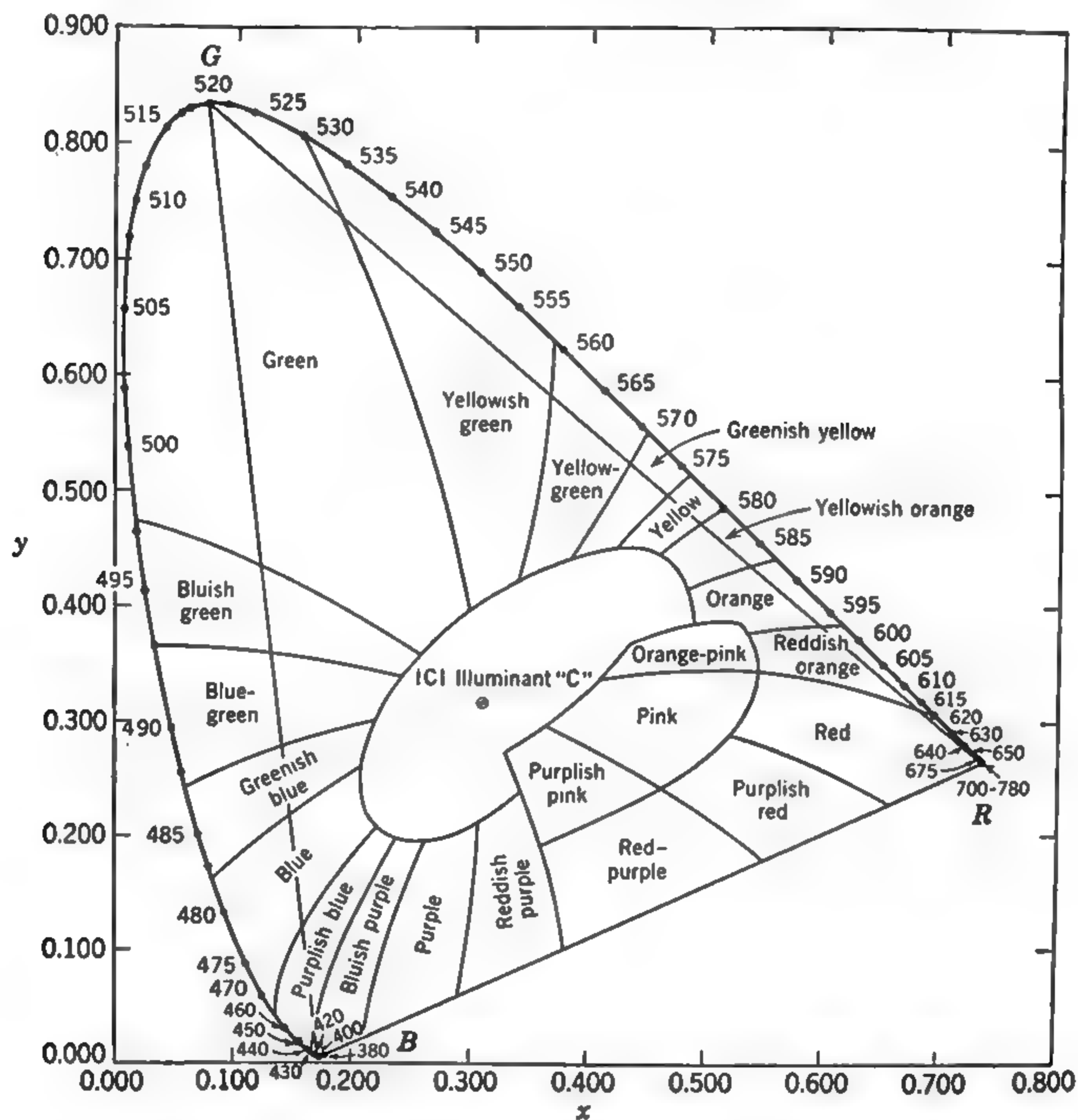


FIG. 31. Choice of real primaries to yield nearly maximum chromaticity gamut. The hue names are those proposed by Kelly (1940).

solid triangle on Fig. 31. Note that this choice of primaries fails both to cover pure yellows and to cover pure blue-greens.

A disadvantage of primaries chosen for maximum chromaticity gamut is that the two primaries (red and blue) taken near the extremes of the spectrum must necessarily have low luminosity. That is, in order to obtain a field of conveniently high luminance, radiant energy must be expended at a relatively high rate. The primaries chosen by Guild (1931) at the National Physical Laboratories in Great Britain and by Wright (1927-1928) at the Imperial College of

Science in London that have given us the standard observer for colorimetry have, therefore, not been those yielding maximum chromaticity gamut but represent a compromise between high luminance and maximum chromaticity gamut. Figure 32 shows the triangles corresponding to the primaries chosen by Wright (460, 530, and 650 $m\mu$) and to

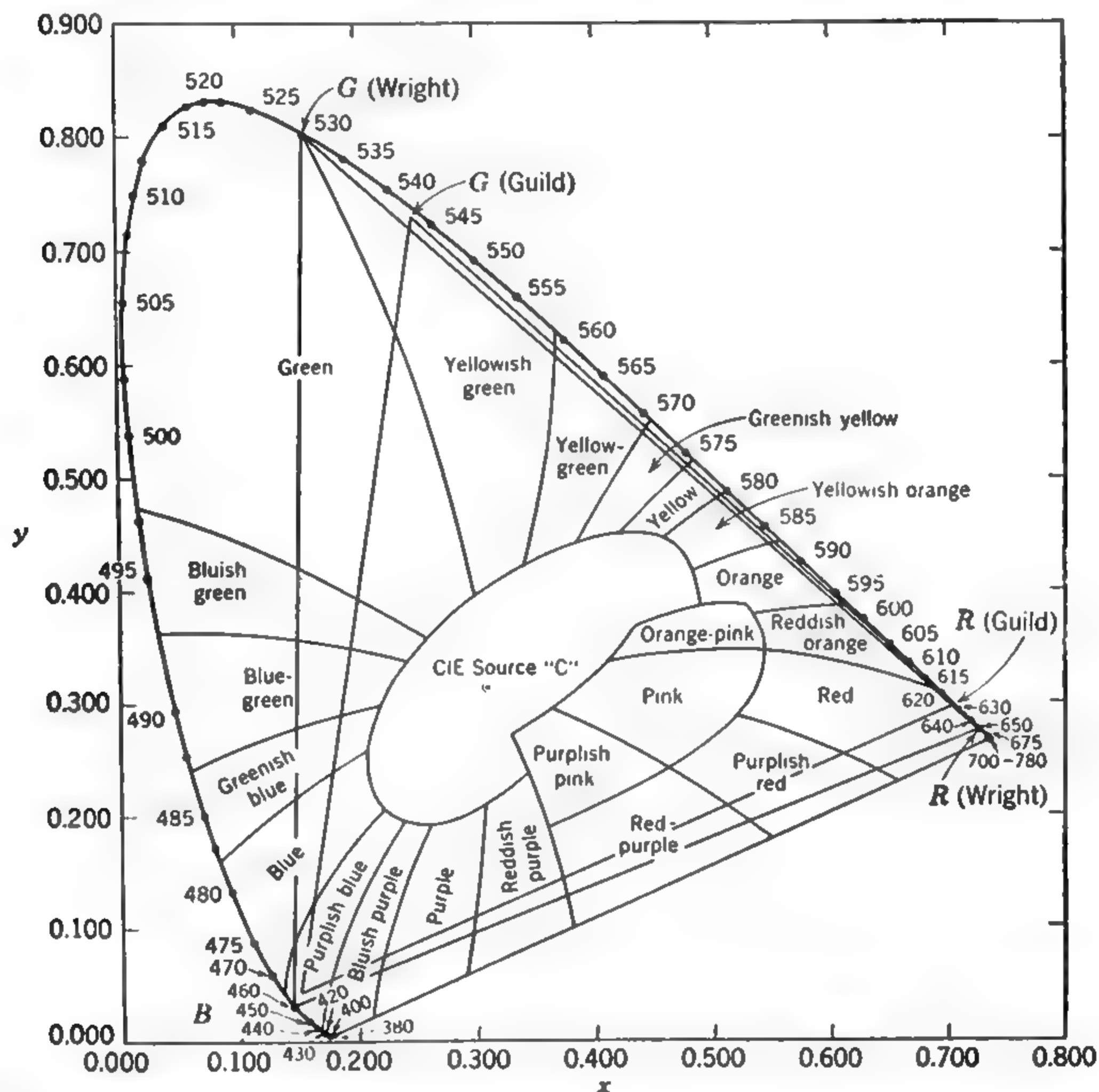


FIG. 32. Real primaries used in the Wright and the Guild tristimulus colorimeters.

the source-plus-filter primaries chosen by Guild. Note that both of these sets of primaries yield triangles covering considerably smaller portions of the area of real chromaticities than is possible with other choices (see Fig. 31). They also favor the pure yellows at the expense of pure blue-greens, the latter being less important commercially.

Attempts to use tristimulus colorimeters in industry have been very discouraging. The restricted chromaticity gamut leaves many commercially important colors outside. This forces the colorimeter to be built so as to provide the possibility of adding each primary not only

to the comparison field but also to the test field. In these cases not all the tristimulus values on the working primary system are positive; for example, G and B may be positive numbers, while R is less than zero, indicating that the blue-green to be measured had to be degraded by admixture of some of the working red primary before a match to a mixture of the green with the blue could be obtained.

The killing blow, however, is neither expense of the colorimeter nor inconvenience in reducing the results to the standard coordinate system; it is insensitivity. It seems paradoxical that a colorimeter permitting the eye to make the match should be less sensitive than the eye unaided by any instrument at all, but this is true. And this is not a matter of a few percent, it is a factor of nearly 5, or 500 percent. Binocular observation of a large field with a light surround is the rule for inspection of goods for color match in industry. Peeping with one eye through a small hole at a small, dimly lighted field with dark surroundings is what you usually get with a visual tristimulus colorimeter. The greatest handicap lies in the small angular size of field. Figure 33 shows how sharply the uncertainty of a chromaticity match increases as the angular size of the field decreases (Judd, 1930). If a tristimulus colorimeter with large chromaticity gamut is built with a favorably large field, such as one subtending 10 to 15° at the eye, and an attempt is made to measure the color of a commercial product such as the plastic cover for an electric-light switch, it will usually be found impossible to make an accurate setting for color match. The projection of the macular pigment will show up plainly so that, if the field is matched near the fixation point, it will fail to match elsewhere. Or, if it is matched in general, the center of the field will fail to match. The reason for this appearance of a projection of the macula is that the match is metameric. The mixture in the comparison field has a preponderance of energy in the long-wave, middle, and short-wave parts of the spectrum (red, green, blue) relative to the intermediate wavelengths (yellow and blue-green). Light reflected from products of commercial interest, however, have no such energy distribution in the spectrum. The result is that increasing the diameter of the field above 2° does no good. The uncertainty of the setting in terms of chromaticity amounts to about 0.005 in x or y , whereas direct comparison of two nearly identical plastics will easily serve to discriminate between plastics differing by only 0.001 in x or y . It would be very common, therefore, to find by measurements on a tristimulus colorimeter that two commercial products have the same color, but, if, as is usual, these two products show essentially nonmetameric differences, even casual direct comparison by the unaided eye would

show them not to have the same colors. This accounts for the nonuse of visual tristimulus colorimeters of large gamut for industrial applications.

Another consequence of the metameric character of color matches set up in tristimulus colorimeters is that it prevents one observer of

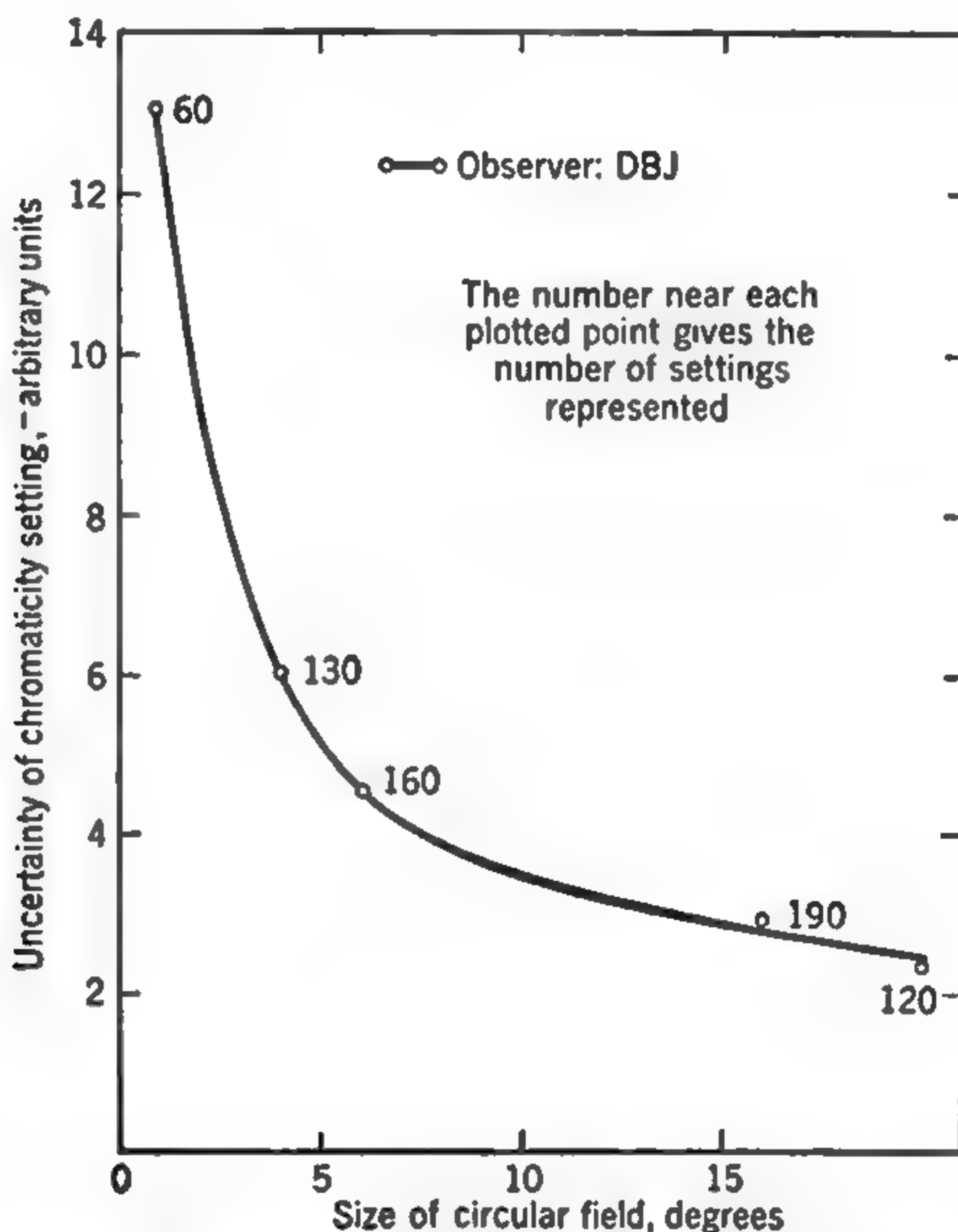


FIG. 33. Dependence of precision of chromaticity settings on angular size of field (from Judd, 1930).

normal color vision from getting the same reading as another except by chance. To mitigate the effects of these individual observer differences, it is customary to adjust the scales of the instrument separately for each observer so that for some standard (such as source C or source B) the tristimulus values will agree with those for the standard observer. Even with this correction, if a reasonable approximation to the standard values of X , Y , Z is to be assured, either the readings of a group of five or ten observers must be averaged, or a color standard yielding a spectral composition similar to that of the unknown specimen must be used.

It will be seen that tristimulus colorimeters of large gamut give but poor information regarding the unknown specimen. Their application

to product-control problems in industry is negligible. But because of the ease of calibration and the simplicity of the theory (Grassmann's laws), they are very useful visual research tools. Tristimulus colorimeters have been built and described by Allen (1924), Donaldson (1935), Guild (1925-1926), Ives (1908), Newhall (1936), Verbeek (1933, 1934), and Wright (1927-1928). The Wright instrument has spectrum primaries as we saw in Fig. 32; the other six have primaries formed by combining a light source with glass or gelatine filters. To the Guild and Wright instruments we owe our accurate information regarding the properties of the normal visual system, which have been expressed in terms of the standard observer. None of the instruments is suited to product control.

By adding three additional primary lights Donaldson (1947) has overcome the basic disadvantages of the large-gamut tristimulus colorimeter. The six primaries of this instrument have spectral distributions that cover the spectrum with some overlap. They are produced by incandescent-lamp light combined separately with each of six color filters. A second set of these six filters is mounted in a seven-hole disk at the eyepiece so that they may be put in front of it one at a time. Light from the unknown specimen fills one half of the photometric field, a combination of the six primary lights fills the other. The observer looks through one of the six filters at the eyepiece and adjusts the corresponding primary in the mixture field to an approximate brightness match. This is repeated for the remaining five filters. Then the observer looks at the photometric field without any eyepiece filter. He sees the test field and the comparison field directly. Since these have been adjusted to approximate equality for each of six parts of the spectrum separately, they will be found to be an approximate color match. Furthermore, this match will be essentially nonmetameric in character so that the field may be made as large as desired; no projection of the macula to spoil the precision of setting is visible. The final adjustment to exact color match is made by means of the red, green, and blue primaries. The instrument is calibrated in terms of the standard CIE tristimulus values; that is, the values of X , Y , and Z for one unit of each of the six primaries is known. The tristimulus values, X , Y , Z , of the unknown are found simply by adding together the contributions of each of the six primaries. This is by an extension of equation 2 (Grassmann's laws). The Donaldson colorimeter is not used for commercial products to any extent in America, but it has the properties required for successful application to product-control problems. It has a color gamut covering all but exceptional commercial products; it permits the observer to make nearly optimum

use of his ability to detect small color differences; and it is applicable to light-transmitting specimens, to light-reflecting specimens, and to self-luminous specimens.

The tristimulus colorimeters so far mentioned make up the comparison-field mixture by optical combination of light beams from different sources either so that a sum of the separate effects is obtained by continuous exposure, or so that a time-weighted average is obtained by successive stimulation at a frequency too high to produce flicker. A very simple and widely used tristimulus colorimeter based on this latter principle is obtained by taking four disks that have been cut along a radius, by interlocking them to rotate on the spindle of a motor so rapidly that neither the separate sectors nor even flicker is perceived. Such an arrangement for combining colors by rotary mixture has already been discussed under the name Maxwell disk; see Fig. 17. The four disks provide the necessary three degrees of freedom in adjustment for a match, and, if the tristimulus values of the component disks are known, the tristimulus values of the mixture can be computed as the area-weighted mean of those of the components by equation 5a.

If the disks are chosen anew for each kind of unknown color to be measured so that all are fairly similar in color to the unknown, the spectral composition of the mixture color is usually sufficiently non-metameric to require no restriction to the central 2° of the retina. Furthermore, any two observers with normal vision and some experience at making the adjustment can check each other closely. The chief drawback of this simple arrangement for product-control work in color is the time lost in adjustment of the sector-disk areas. The motor must be turned off, brought to a stop, the disks loosened and readjusted, the motor turned on and allowed to resume speed several times to obtain a final setting of reasonably good precision.

Nickerson (1931) has described a disk colorimeter that avoids the difficulties of the elementary Maxwell disk. Light reflected from the unknown specimen fills one half of a photometric field, and that from a stationary sector disk fills the other. By having the observer look at the sector disk through a rapidly rotating glass wedge, each sector is presented to view in such quick succession that no flicker is produced; and at the same time the sectors, since they are stationary, may be continuously adjusted until a color match is obtained. A further advantage is obtained by extending the rotary scanning to the unknown specimen. In this way the average color of a notably non-uniform specimen such as that made up of coarse salt crystals may be obtained. The disk colorimeter is extensively used by the U. S. De-

partment of Agriculture (Nickerson, 1946) for the color grading of food products and is well adapted to product-control colorimetry of many kinds. Frequent checks of the working standards of color are required for accurate work, lest fading or soiling of them render the previous calibration by equation 5a inapplicable.

Dominant Wavelength and Purity. In a colorimeter for determining the dominant wavelength and purity of an unknown color directly, an arrangement of optical parts must be provided so that the comparison field is filled with an additive combination of some part of the spectrum with some standard source (such as sources A, B, or C) that may be considered as having a neutral color, that is, a color having no hue. If the unknown is a light-reflecting or light-transmitting object, the source used to illuminate it may be taken as the neutral standard. Even source A (representative of incandescent-lamp light) can qualify as neutral in these circumstances, so great is the power of vision to adapt to the general color of the environment. Thus, spectrally non-selective objects appear gray both in daylight and in lamp light, and it is convenient and appropriate that the measurement should indicate this. For self-luminous objects (fluorescent lamps, television screens) the standard source taken is usually that having an equal-energy spectrum. The observer adjusts the wavelength of the spectrum component and the luminances of both the spectrum component and the neutral component until a color match with the unknown is obtained, if possible. The wavelength required for a color match is called the dominant wavelength of the unknown. The ratio of the luminance of the spectrum component to the luminance of the mixture field (spectrum plus neutral) is called the luminance purity, or colorimetric purity, of the unknown. This is the basic idea of dominant wavelength and purity. It is an attractive idea because dominant wavelength indicates pretty well what hue the color will be perceived to have; and purity, the saturation perceived. But there are two annoying obstacles that prevent a simple colorimeter being built along these lines.

In the first place, to obtain the purity of the unknown the relative luminance of the spectrum and neutral components must be measured. To the optical arrangements required to set up the match must be added some auxiliary equipment to compare these luminances. It is not sufficient to provide for the introduction of known luminances of the neutral source temporarily in the test field so that the observer may see which of them corresponds to the unknown luminances of the components. This would work very well for the neutral component, but the spectrum component is too different from the neutral compo-

nent in chromaticity to permit a reliable photometric setting by direct visual comparison. This is the awesome and old problem of photometering lights of different color, known as heterochromatic photometry. The best that can be done in the way of auxiliary equipment is to provide a way to alternate the unknown luminance with a known adjustable luminance. If the speed of alternation is set considerably below the critical frequency of flicker, it is possible to adjust the known luminance for minimum flicker. This occurs very closely at, if not exactly at, the adjustment for equality of luminance computed by the sum of the luminances of the spectral components of the light by equation 3. The settings can be made even though the two luminances being alternated belong to colors of considerably different chromaticities. We have then to add to the colorimeter an auxiliary flicker photometer.

In the second place, it is not possible to match all colors by two-part combinations of spectrum light with neutral light. We can see this from the mixture diagram of the standard system remembering that two-part mixtures are represented by points on the straight line connecting the points representing the chromaticities of the components by the center-of-gravity principle. Figure 34 shows the (x, y) -chromaticity diagram with straight lines connecting the point representing standard source C with the ends of the spectrum locus. These straight lines represent the chromaticities produced by mixtures of source C with the ends of the spectrum. If source C is taken as the neutral standard, these lines correspond to constant dominant wavelength, one at $380\text{ m}\mu$, the other at $700\text{ m}\mu$. The area on the diagram inclosed by these lines combined with the spectrum locus corresponds to colors that can be duplicated by mixtures of source C with some part of the spectrum, and such colors are called spectral colors. The remainder of the area, that inclosed by these two lines of constant dominant wavelength and the straight line connecting the extremes of the spectrum locus, corresponds to real colors that cannot be so duplicated, and these colors are called nonspectral colors, or, sometimes, purples.

To measure nonspectral colors in terms of the spectrum it is necessary to provide an optical arrangement whereby the spectrum component may be added to the unknown color instead of to the neutral standard. The observer then adjusts the wavelength and amount of the spectrum component until this component combined with the unknown color matches the neutral standard. The required wavelength is called the complementary wavelength of the unknown color. Luminance purity for nonspectral colors has been defined in two ways (Judd, 1926): one as the ratio of the luminance of the spectrum com-

ponent, counted as negative since it is added to the unknown, to the luminance of the unknown; the other as the ratio of the luminance of the spectrum component to the luminance of the mixture, all luminances taken as positive. Neither of these ways of expressing the purity of nonspectral colors is very convenient, and neither has been used to any considerable extent.

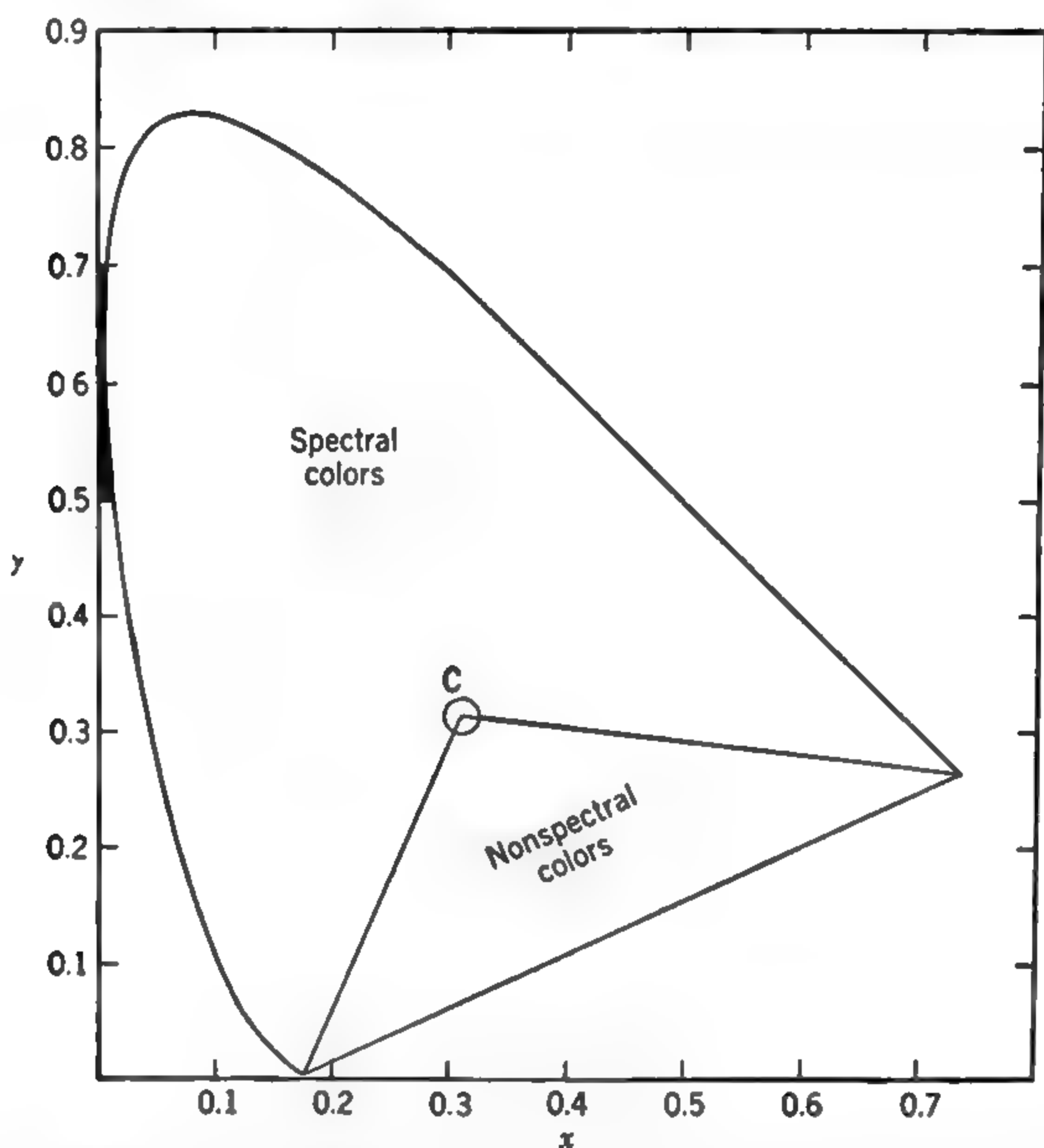


FIG. 34. Division of real chromaticities into spectral and nonspectral parts.

Equipment to measure dominant and complementary wavelength and purity directly is, therefore, complicated and inconvenient. Since the matches set up are metameric in character, this equipment, like the tristimulus colorimeter, must have an observing field of not more than 2° subtense at the eye. It is thus beset with the handicap of low precision of setting, and, because of the metamerism (disparity in spectral composition), two observers ordinarily obtain settings disagreeing by even more than the large uncertainty in the setting. Furthermore, the method has a handicap not possessed by the tristimulus colorimeter. Oftentimes the actual setting up of the match by two observers will agree closely, but their photometry of the relative lumi-

nances of the components by flicker photometry will disagree. The sum and substance is that colorimeters to measure dominant wavelength and purity directly are even less adapted to practical control than tristimulus colorimeters. Priest (1924) and Nutting (1913) years ago used them for research purposes, and more recently the Institute of Paper Chemistry (1941, 1945) has used one in a study of color blindness.

The only useful result from the very considerable attention given to such colorimeters is the idea of a chromaticity specification more easily understood than chromaticity coordinates x and y . Dominant wavelength indicates what part of the spectrum has to be mixed with the neutral standard, and purity indicates the degree of approach of the unknown color to the spectrum color so designated. Those who are familiar with the colors of the spectrum can easily visualize what is meant by dominant wavelength of 600 $m\mu$ and purity of 50 percent. They know that the spectrum at 600 $m\mu$ usually looks orange in color; so they visualize the color indicated as one of orange hue of about half the saturation of spectrum orange. The fact that such a color is usually perceived as more pinkish than this is a drawback, but not enough to vitiate the usefulness of the designation. It is usually said that colors of constant dominant wavelength are perceived to have nearly, but not exactly, constant hues. It has become fairly common practice to convert designations of chromaticity found in terms of the coordinates x and y into terms of dominant wavelength and purity. This may be done graphically as shown in Fig. 35. The problem illustrated in Fig. 35 is the determination of the dominant wavelengths of sources A and B for source C as the neutral standard. The method is to draw a straight line through the point representing the neutral standard (source C) and the point representing the color to be evaluated, and then to extend this straight line until it intersects the spectrum locus. The wavelength of the intersection is the required dominant wavelength of the color to be specified. For source B the intersection falls at 578.7 $m\mu$; for source A, at 583.3 $m\mu$.

The formula for computing luminance (or colorimetric) purity, p_c , of an unknown color of chromaticity coordinates x, y , is:

$$\begin{aligned} p_c &= (y_b/y)(x - x_w)/(x_b - x_w) \} \\ &= (y_b/y)(y - y_w)/(y_b - y_w) \} \end{aligned} \quad (10)$$

where x_w, y_w are the chromaticity coordinates of the neutral standard, and x_b, y_b , those of the spectrum color. The expression for p_c to be used in calculation is whichever of the two forms given can be determined

with the least rejection error. Since the idea of purity is used only to indicate how closely the unknown color approaches that of the spectrum component, however, it has become customary to omit the ratio y_b/y

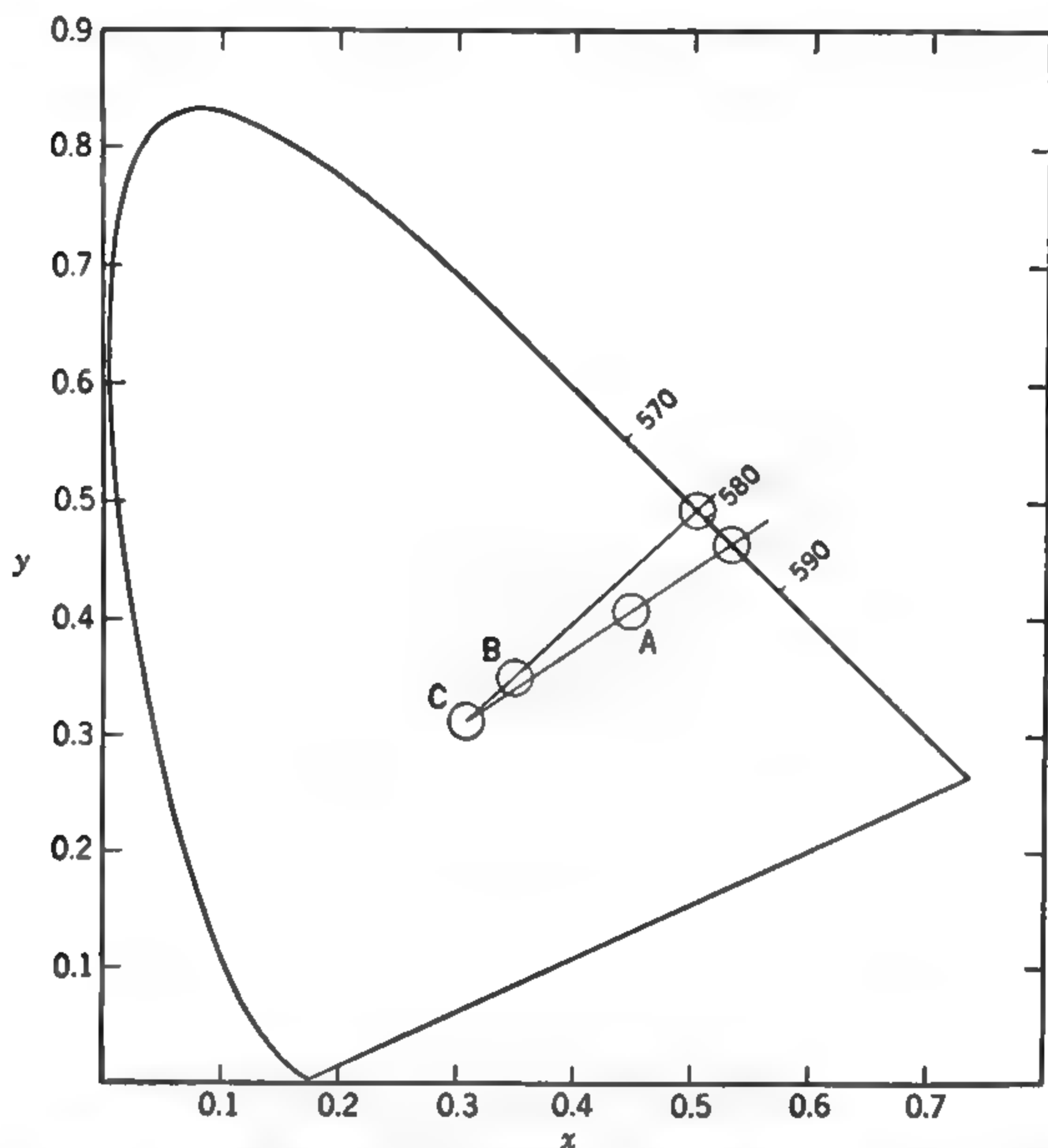


FIG. 35. Graphical determination of dominant wavelength and purity of standard sources A and B relative to source C on the (x, y) -chromaticity diagram.

in calculating purity. The resulting quantity is called excitation purity p_e , and as inspection of the formula will show:

$$\left. \begin{aligned} p_e &= (x - x_w)/(x_b - x_w) \\ &= (y - y_w)/(y_b - y_w) \end{aligned} \right\} \quad (11)$$

is merely a ratio of distances on the standard chromaticity diagram. For example, from Fig. 35 it may be seen immediately that the excitation purity of source B relative to source C as the neutral standard is about 20 percent, and that of source A is about 60 percent. These purities correspond to the slightly yellowish color of source B for an observer adapted to daylight, and to the considerably more saturated yellowish color of source A under the same conditions.

The *Handbook of Colorimetry* (Hardy, 1936) shows large-scale sections of the (x, y) -chromaticity diagram on which the loci of dominant wavelength and excitation purity have been drawn at frequent intervals. These charts serve to convert chromaticity coordinates x, y to dominant wavelength and purity with a precision adequate for most commercial purposes, and many color specifications are written in these terms even though the intent of the contract is that conformity to the color clause shall be determined by spectrophotometry and subsequent evaluation of the luminous reflectance and chromaticity coordinates x, y . The same color requirements may often be stated more simply in terms of x and y directly, but the dominant wavelength and purity form is preferred by many purchasers because they can more easily understand what the contract says, and so be more sure that the requirement stated applies exactly to their needs.

Subtractive Colorimeters. If the visual colorimeter provides a comparison field whose color is controlled by introducing three filters into a single beam illuminating the field, the name given to it is a subtractive colorimeter. Each of the three filters subtracts a certain fraction of each part of the spectrum of the incident light.

Subtractive colorimeters can be used to measure the colors of self-luminous areas, of light-reflecting specimens, and of light-transmitting specimens, but their application to the last is by far the most direct and convenient. Since the comparison field is also controlled by light-transmitting filters, the same light source can easily be used for both in this application, and small variations in the light source can be tolerated since they leave the colorimeter settings unchanged.

Calibration of subtractive colorimeters so as to obtain from them tristimulus value Y and chromaticity coordinates x, y is a major difficulty. It has to be based on equations 6 and 8 and requires lengthy computations.

Jones Subtractive Colorimeter. In the Jones subtractive colorimeter (Jones, 1920) the color of the comparison field is controlled by allowing incandescent-lamp light or artificial daylight to pass through three wedge filters in succession. The color match is obtained simply by adjusting the positions of the gelatine wedges dyed with cyan, magenta, and yellow dyes, respectively. These wedges absorb principally red, green, and blue light, respectively; so this colorimeter is equivalent to a tristimulus colorimeter in which the primaries vary somewhat in chromaticity from the thin to the thick part of the wedge. A neutral wedge is also provided so that, if desired, only two of the chromatic wedges need to be used to obtain the match. This choice of wedges is more convenient for general use because the operator can understand

more easily what adjustment has to be made to correct a small residual color difference. If he sees a hue difference, the ratio of the amounts of the two chromatic wedges has to be changed. If he sees the color of the comparison field not sufficiently saturated, he must increase the amounts of both chromatic components and reduce the amount of neutral wedge. If he sees the comparison field too dark, he must reduce the amount of neutral wedge alone. The spectral composition of the light transmitted by the wedges is fairly similar to that leaving the commercial product to be tested. On this account the matches set up are usually only moderately metameric, and a large field of view is often useful.

The unique merit of the wedge subtractive colorimeter, however, is in the measurement and control of the processing of photographs in color (Evans, 1948). The chromatic wedges are then dyed with the same dyes used in the colored photograph. When a match is obtained, both the test field and the comparison field are colored by the same amounts of the same dyes. The spectral compositions of the two fields are thus automatically made identical. This assures that the settings of any two observers having normal trichromatic vision will not significantly disagree, and it also permits use of a large observing field with the resultant increased precision. Furthermore, the answer comes out in terms that tell the operator immediately what he wants to know, that is, the amounts of cyan, magenta, and yellow dye in the test area of the photographic transparency itself; no calibration or reduction to fundamental terms needs ordinarily to be made. The only drawback of the gelatine-wedge subtractive colorimeter for this use arises from failure of the wedge colors to be perfectly permanent.

To summarize, the gelatine-wedge subtractive colorimeter provides color matches for most commercial products, and these matches are only moderately metameric. The instrument is therefore capable of high precision, and usually the settings on it are essentially the same, regardless of what observer makes them. The impermanence of the wedges, however, discourages calibration in the CIE system. There is good prospect that the wedges will have changed before the long computations have been finished!

Lovibond Tintometer. The Lovibond Tintometer, in one variation or another (Lovibond, 1887, 1890; Knapp, 1910; Priest, 1920–1921; Lowry and McHatton, 1921, 1922; Bailey, 1925; Schofield, 1939) is widely used for color measurement of the articles of commerce that transmit light, such as lubricating oil, paint vehicles, vegetable oils, sugar solutions, and beer; and it is used for some light-reflecting materials, such as oleomargarine. The color of the comparison field is

controlled by introduction of Lovibond red, yellow, and blue glasses. The coloring materials of these glasses are gold, chromium, and cobalt, respectively. These coloring materials are confined to a thin layer of glass flashed onto an essentially clear nonlight-absorbing glass support. The unit of the scale defined by each set is arbitrary, but the three units are related by being adjusted so that, for observation by daylight, subtractive combination of one unit of each of the red, the yellow, and the blue scales results in a filter perceived as nearly neutral. Each scale is exemplified by many glasses, each glass being marked with the number of unit glasses to which it is equivalent. If two or more glasses of the same coloring material are used in combination, the number of unit glasses to which the combination is equivalent is found simply by adding the numbers of units for each individual glass of the combination. The Lovibond glasses thus are equivalent to three wedges except that the color must be varied in small discrete steps rather than continuously. In effect, the Lovibond numerals express in arbitrary units the thickness of three layers of coloring material required to produce the color by subtractive combination.

The Lovibond Tintometer is an instrument designed to use the precision built into the Lovibond glasses to the best possible advantage. It is so arranged that the unknown specimen and a white surface are viewed side by side, suitably illuminated. These two fields of view are seen through an eyepiece, and Lovibond glasses of one, two, or three scales can be introduced into the field of view by a simple system of sliding racks. The unknown color may be matched by a glass from one single rack, or by a combination of two or three glasses. If all three kinds of Lovibond glass (red, yellow, and blue) have to be introduced, this means that the color has some grayness in it, and the lowest of the three amounts is a measure of this grayness, the chromatic character of the unknown being indicated by the additional amounts of the other two required for the match. The instrument is designed to deal with both liquid and solid specimens. Although the matches set up are only moderately metameric as a rule, the size of field is 2° or less. This conforms to the observing conditions of the CIE standard observer, and the calibrations of the Lovibond glasses are, in general, carried out by the maker with such precision that any errors remaining cannot be detected under these observing conditions.

A spectrophotometric analysis of the Lovibond glasses has been made by Gibson, Harris, and Priest (1927) at the National Bureau of Standards. Figure 36 shows the spectral transmittances of Lovibond red, yellow, and blue glasses of 10 units each, and it also shows the spectral transmittances of the approximately neutral filter produced by

combining these glasses. Note that all three kinds of Lovibond glasses transmit freely in the long-wave extreme of the spectrum (700 $m\mu$). This restricts the gamut of green colors producible by combination of these yellow and blue glasses to rather low purity, and it keeps the neutral combinations from being even approximately nonselective in spectral transmittance. Note in Fig. 36 the variation by a factor of more than 40 in spectral transmittance of the combination (10R + 10Y + 10B) nominally neutral. This high transmittance of

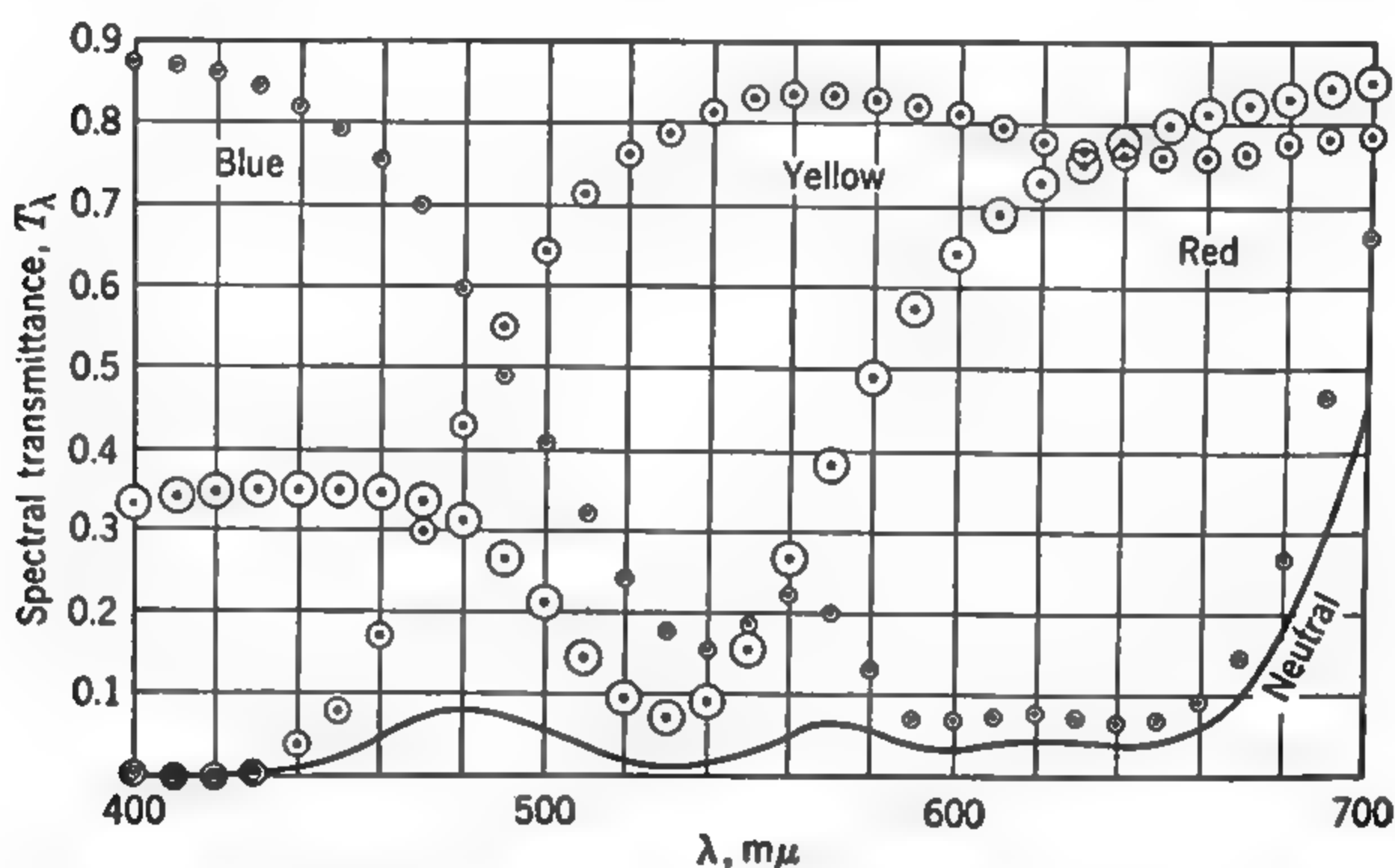


FIG. 36. Spectral transmittances of Lovibond glasses 10 Red, 10 Yellow, and 10 Blue, and of the combination of these three, nominally neutral.

all three kinds of Lovibond glass (red, yellow, blue) in the long-wave end of the spectrum also causes three-part combinations of Lovibond glasses of equal amount to drift in chromaticity away from neutral toward red as the number of Lovibond units is increased. This makes it unfeasible in general to derive chromaticity coordinates, x , y , of a color from its specification in terms of amounts on all three of the Lovibond scales, red, yellow, and blue.

Another method of using the Lovibond glasses has been developed to overcome this drawback. It is known as the Lovibond-Schofield system (Schofield, 1939). In this system an auxiliary device is provided for controlling the luminance of the comparison field independently. The luminance can be reduced by turning a knob, and the amount of reduction is read from a scale on the instrument. This device makes it unnecessary to use all three kinds of Lovibond glasses (red, yellow, blue) at a time. With this modified system, as with the original Lovibond system, it is an easy matter for anyone with normal color vision

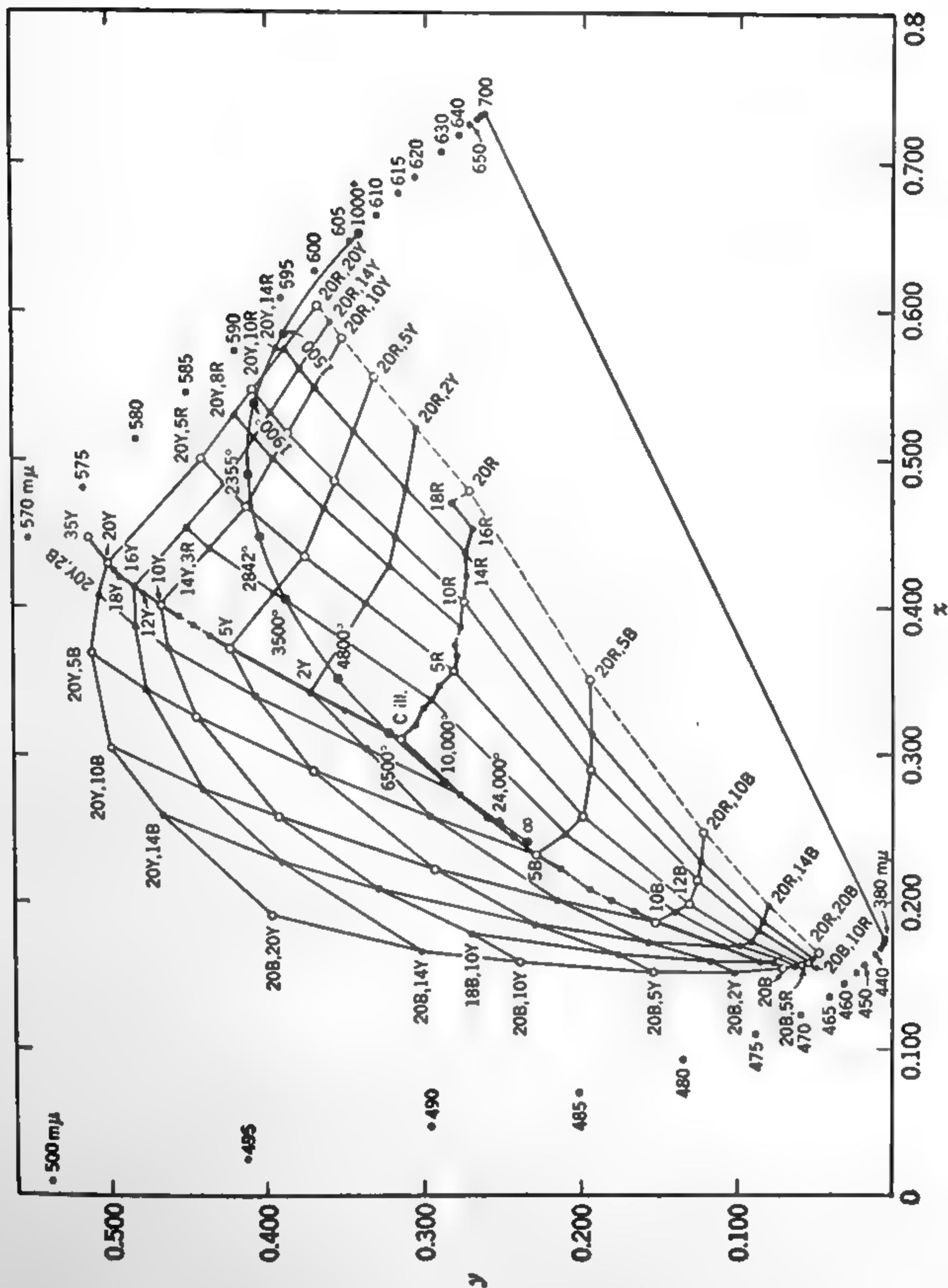


Fig. 37. Chromaticities of two-part combinations of Lovibond glasses illuminated by source C (after Haupt and Douglas, 1947).

to understand the meaning of the numbers read from the instrument. Once written down, they serve as a permanent record of the color. Conversion graphs have been prepared from which, given the instrumental readings in the Lovibond-Schofield system, the CIE color specification (Y, x, y) can be obtained quickly and easily to a degree of accuracy that is sufficient for most purposes. The illuminant is source A, but provision is made in the eyepiece for liquid filters to convert either to source B or to source C so that the colors of nonfluorescent specimens may be determined for these sources in addition to source A, if desired.

Figure 37 taken from the work of Haupt and Douglas (1947) shows the chromaticity of two-part combinations (up to 20 units) of the central areas of Lovibond red, yellow, and blue glasses in possession of the National Bureau of Standards. The illuminant is standard source C (representative of average daylight), and the plot used is the (x, y) -chromaticity diagram of the standard CIE system. Figure 38 taken from *NBS Circular 478* (Judd, 1950) gives similar information for standard source A (representative of the gas-filled incandescent lamp). The relative smallness of the color gamut of the Lovibond glasses shown in Figs. 37 and 38 arises from the fact that no combinations involving more than 20 Lovibond units of each kind are shown. By taking glasses equivalent to higher multiples of the Lovibond unit the coverage in the purplish red to greenish yellow hue range can be made virtually complete, and that in the blue to reddish purple range considerably extended, but very little extension in the green direction results by taking more Lovibond units of yellow and blue, for the reason already discussed in connection with Fig. 36. The wide application of the Lovibond glasses to commercial color grading indicates, however, that this gamut restriction is not very important.

The large-scale graphs that may be purchased from Tintometer, Ltd., give information for standard sources B and C similar to that in Figs. 37 and 38. They refer, moreover, not to individual sets of Lovibond glasses, but rather to carefully derived ideal master standards of the system. They provide, therefore, a more authentic basis for converting Lovibond readings to luminous transmittance and chromaticity coordinates x, y on the standard CIE system for any set of Lovibond glasses taken at random and should be used unless a conversion chart is available that is based upon the actual set of glasses used to obtain the readings.

The question of the reliability of the Lovibond numeral has arisen frequently among American users of these glasses. A supplier of cot-

tonseed oil in the south would grade a tank car of oil by comparison with his set of Lovibond glasses. If the color of a $5\frac{1}{4}$ -inch layer of the bleached oil were no darker than 35-yellow, 7.6-red, it was called prime summer yellow and would bring the highest price. Darker oils would have to be sold at lower prices. Even if the color measurement showed 35-yellow, 7.7-red, the difference in dollars corresponding to

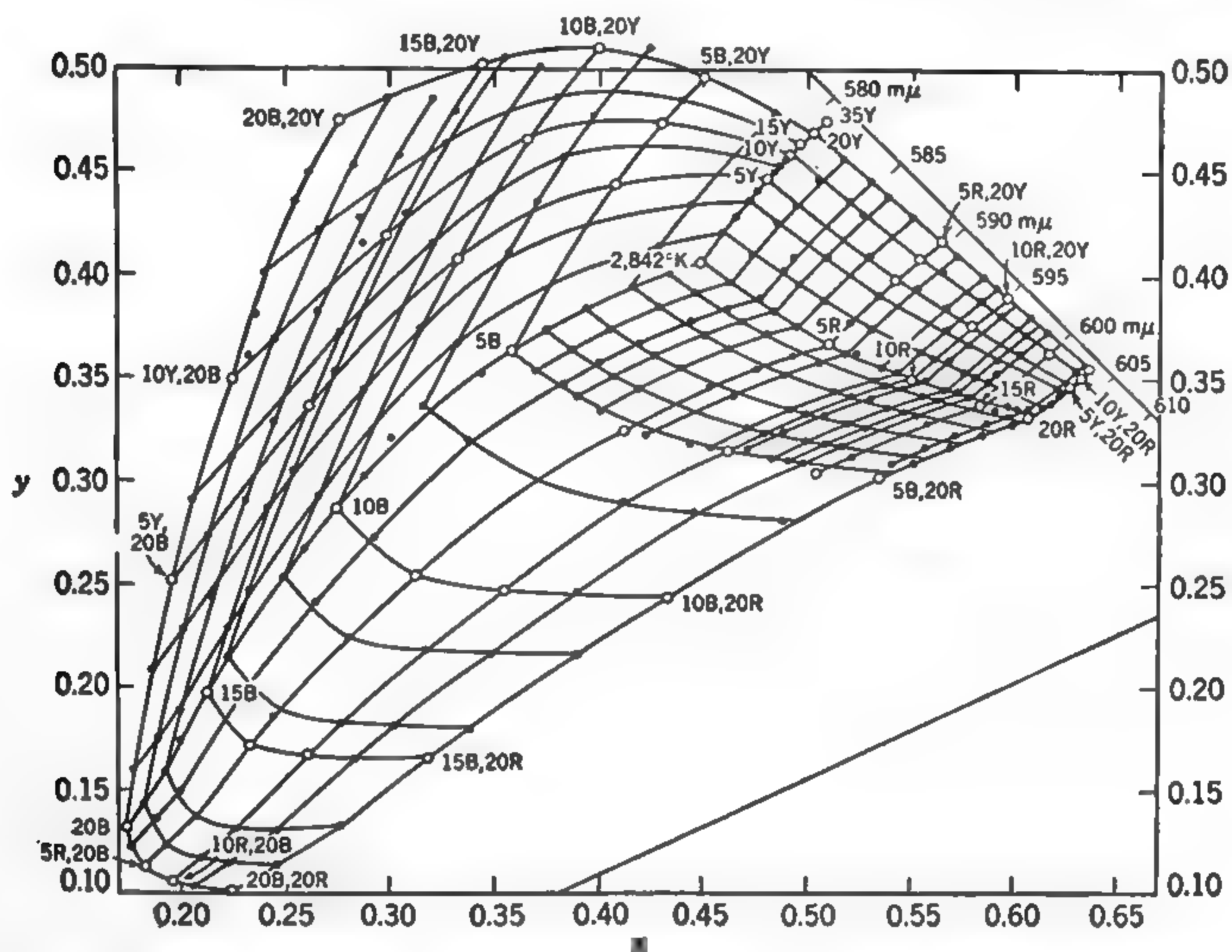


FIG. 38. Chromaticities of two-part combinations of Lovibond glasses illuminated by source A (after Judd, 1950).

this difference of only 0.1 Lovibond red would be appreciable when one is selling cottonseed oil in tank-car lots. The northern processor has had a considerable monetary motive to find Lovibond glasses that would permit him to assign a color notation to the oil higher in Lovibond red by only a few tenths of a unit. In the past it has not been difficult to find such glasses. Comparisons of Lovibond red glasses in actual use in the purchase and sale of vegetable oils have revealed important discrepancies (Judd and Walker, 1928; Walker, 1934; Haupt, 1938). A scale of the red glasses used in combination with the 35-yellow glass has been constructed by Priest and Gibson (Gibson and Haupt, 1934) at the National Bureau of Standards, and thousands of Lovibond red glasses have been recalibrated on this scale

(Haupt, 1938), some at the National Bureau of Standards and some at the Electrical Testing Laboratories.

In view of the rather bitter controversies arising from the use of Lovibond red glasses in America, it is of interest to look further into the sources of the discrepancies that have been found between the actual colors of such glasses and the colors indicated by the Lovibond numerals engraved on the glasses by the makers. Prior to 1940 separate Lovibond glasses were available in America at low cost for use in Lovibond Tintometers and as replacements for glasses broken incidental to such use. These were in the form of rectangles 2 by $\frac{5}{8}$ inches. Some American instrument manufacturers followed the practice of using these Lovibond glasses in their own instruments so that light passed through the centers of the glasses. Others cut them up into 4 or 5 pieces and used these pieces in separate instruments. All depended upon the reliability of the Lovibond color scales to give meaning to the readings obtained, and nearly all used observing fields considerably larger than 2° to raise the precision of the determination to accord with the great attention paid to small color differences.

What was the view of Tintometer, Ltd., one of the foremost suppliers of working standards of color in the world, on all of this? They viewed it with angry disapproval. Their business reputation depends vitally on the integrity of the Lovibond color scales. They had maintained the uniformity of these scales for many years very creditably in the face of great difficulties. They knew better than anyone else that the thickness of the flashed layer of their glasses could not be controlled so as to be adequately uniform over the whole face of the 2 by $\frac{5}{8}$ inch glass; so they calibrated only the portion of the glass effective in the Tintometer. This is a small fraction of *the lower third* of the glass. The color grading has been carried out throughout the years with such care that the makers believe the number of glasses engraved with numerals detectably in error to be negligible, *provided that the glasses are used in the Tintometer* as they are intended to be.

The use of the Lovibond glasses in large-field colorimeters has dealt Tintometer, Ltd., a double blow.

In the first place American instrument makers have used the Lovibond color scales to compete with the Tintometer itself. Tintometer, Ltd., regards the Lovibond color scales as their own property applicable only to their own instruments. They feel that use of Lovibond glasses in other instruments is quite improper; in Britain it simply would not be done without specific authorization by the manufacturer. Perhaps monopoly by gentleman's agreement is the British way.

The American view is quite different. Loose glasses once purchased become the sole property of the buyer to use as he pleases; no specific authorization for any use is needed. Tintometer, Ltd., should be held responsible, American manufacturers think, for the entire area of the glass bearing their engraved marks, to the precision of 0.1 unit indicated by those marks. Purchase of parts from the best sources of supply and assemblage of them into units adapted to customer needs are essential methods of competitive American industry and have much to do with its efficiency. American instrument makers are encouraged to build better colorimeters by our high protective tariff on foreign-built instruments. The Lovibond color scales are public property, we think; anyone can use them. This view is confirmed by the publication of fundamental information about them by the National Bureau of Standards (Gibson, Harris, and Priest, 1927).

In the second place, the use of Lovibond glasses in large-field colorimeters unauthorized by the makers has resulted in a decline in confidence of the American user in the calibration of Lovibond red glasses by the maker. The American user finds that the color of a Lovibond glass fails by a detectable amount to conform to that indicated by the maker's numeral engraved on the glass. He may complain to the American instrument maker but would be told, of course, that genuine Lovibond glasses have been used. Perhaps neither he nor the instrument maker would know that an uncalibrated portion of the Lovibond glass was actually effective in the instrument. Note that Figs. 36, 37, and 38 refer to uncalibrated portions of one individual set of glasses. A complaint directed to Tintometer, Ltd., must bring the reply that this use of the Lovibond glasses is unauthorized and Tintometer, Ltd., cannot assume any responsibility. The American user is frustrated and finds he must then pay the National Bureau of Standards to recalibrate the central portion of his Lovibond red glasses. He notes the difference between the maker's numeral and the NBS numeral and remembers this as the size of the mistake made by Tintometer, Ltd., in the original calibration.

Exactly how much of such discrepancies corresponds to errors in the original calibration and how much to failure to use the calibrated area is not known. However, the officers of Tintometer, Ltd., cannot be expected to feel happy about being undeservedly held responsible for all the errors resulting from the use of uncalibrated portions of their glasses in instruments built by their competitors.

In the face of this double blow, Tintometer, Ltd., discontinued the practice of selling whole sets of loose Lovibond glasses during World War II. They had trouble making even enough glasses for their own

instruments. The single glasses sold now are in the form either of $\frac{9}{16}$ -inch circles or of rectangles of $\frac{3}{8}$ by $\frac{3}{4}$ inches (about one-fifth of the previous size) so that there can be no possibility of an unwary user viewing an uncalibrated portion. The production of Lovibond glasses has been considerably increased since World War II, but American use of them has declined because whole sets of glasses were for several years obtainable for use in American instruments only by purchasing an expensive Tintometer, paying the considerable import duties, and then cannibalizing the instrument to obtain the glasses.

The question whether glass working standards of color organized on the subtractive principle like the Lovibond glasses can be provided with enough precision to meet the most critical needs of industry at a price low enough to encourage employment of them in large-field colorimeters has yet to be answered. Somewhat greater precision in calibration is required than is now claimed for Lovibond glasses. Perhaps Tintometer, Ltd., will still solve this problem. The American trend, however, accelerated by nonavailability of Lovibond glasses, is toward other methods.

To summarize, the Lovibond glasses provide moderately metameric color matches for many commercial products. Because the metamerism is moderate, the problem of one observer's failing to check another is unimportant. So far as is known, the Lovibond glasses, unless the flashed layer becomes scratched by careless use, are permanent in color. The glasses are convenient to use, either in the Tintometer or in other applications, and the results in Lovibond notation are easy to understand. Furthermore, the results can be converted to standard CIE terms, if it is desired to compare them with those obtained by other methods. However, the Lovibond numerals may be found to be significantly in error, if the glasses are used in large-field colorimeters. Only if Lovibond glasses are used in colorimeters made by Tintometer, Ltd., will the makers guarantee the calibrations. Complete sets of large Lovibond glasses can no longer be bought from Tintometer, Ltd.

REPRODUCTION OF PICTURES IN COLOR

Pictures in color add greatly to the pleasure and richness of our daily lives. Colored prints of the great masterpieces of painting recall for us the beauties of the art galleries that we have seen and sharpen our appetites for those we have not yet visited. Colored photographs of scenes of family life and vacation trips serve as vivid reminders of the pleasant parts of our lives. Colored illustrations add to our enjoyment of magazine articles, and the increasing use of color in advertise-

ments in the face of the rather high cost of producing a picture in color is excellent proof of the impact that such pictures can give to an advertisement. Motion pictures in color have enriched our entertainment for many years, and distant scenes in and out of the studio can now be brought to us in color by television.

The printing, lithographic, photographic, motion-picture and television industries are greatly concerned with the measurement of color, and the very existence of these industries is proof that many and diverse color problems have been well solved. But not many outside those industries have stopped to realize that the automatic reproduction of a picture in color is, itself, color measurement on a grand scale. To reproduce reasonably well a picture in color two processes are required. First, there must be made a record of the color of each of the many picture elements in the scene. This amounts to about 100,000 color measurements. As we have already seen, each color requires three numbers for its specification; so the record must give 100,000 triads, each triad being the tristimulus values of a particular picture element. This may take the form of three separation negatives obtained by photographing the scene three times, first through a red, then through a green, and finally through a blue filter, each negative recording an approximation to one of the tristimulus values for each of about 100,000 elements of the picture. Secondly, these 100,000 records of the tristimulus values are used to control the colors of the finished picture. Automatic color reproduction is thus automatic tristimulus colorimetry repeated about 100,000 times for each picture; then these 100,000 color spots are set up in what amounts to 100,000 correspondingly placed tristimulus-colorimeter comparison fields to form the reproduction of the scene.

In color photography these 100,000 triads of tristimulus values could be found in actual numbers by reading the transmittance of each spot on the separation negatives, or their equivalent, with a microdensitometer. In color television, these tristimulus values are more easily available; they are the signals broadcast by the transmitter, line by line, and field by field, to control the luminous intensity of the flying spot on the cathode-ray tube out of which is built at the rate of 144 a second the black-and-white images on the television receiver. These images give in sequence the tristimulus values for the red, then the blue, and finally the green working primary.

Of course, it would be quite useless to try to put down in numerical form these 100,000 triads of tristimulus values either from the photographic-film record or from the radio signal. The record is used directly to build up the correct color at the proper spot of the picture.

In one form of receiver for color television each picture element is built up by black-and-white television, and the field is rendered in its correct color by successive presentation of the three primary colors by a sector disk rotating in front of the television screen and synchronized with the camera. In most forms of picture reproduction other than television the correct color of each picture element is formed by subtractive combination of dyes as in the Jones subtractive colorimeter, the cyan dye serving to control the amount of red light reflected or transmitted from the picture, the magenta dye controlling the amount of green light, and the yellow controlling the amount of blue light.

An important question in reproduction of pictures in color is color fidelity—how faithfully the colors of the original scene are reproduced. This is not the whole of the problem of producing pictures that the public will like. We know too little about what makes us see objects and people from the mosaic of colored patches presented to the eye from real scenes to state with confidence that a completely faithful reproduction (not yet achieved, by the way) would always look good. In fact, there are some who take the position that perfect color fidelity usually leads to poor pictures and should be avoided on purpose. They say that intentional systematic deviations from fidelity can make the picture better than the original itself. This is adding art to science. But even if you intend to try to improve on the original scene, it is a great help to have a faithful reproduction to start with. You could not get very far with intentional improvements if the basic color fidelity of the picture was so poor that it would not yield any reds, for example, in the picture, or so poor that greens in the original scene were rendered as reds in the picture. So, reasonably faithful reproduction of colors must be built into any reproduction system, even if the final aim is to improve artistically upon the original scene by intentional deviations from color fidelity.

Once the idea is grasped that reproduction of pictures in color is simply tristimulus colorimetry repeated about 100,000 times for each picture, it becomes evident that the basic theory of color fidelity is simply the theory of tristimulus colorimetry, the outlines of which have already been laid down in the preceding pages. This basic theory is one of the tools that serve to solve the problems of color reproduction whether the medium be printing ink, the transparent dyed layers of Kodachrome slides, the motion-picture film in Technicolor, or the flying spot of light on the screen of a television receiver.

It is beyond the scope of this treatment to show in detail how the basic theory applies to the hundreds of color-reproduction systems already developed, particularly those in which the end result is a sub-

tractive combination of pigments or dyes and in which account must be taken of the fact that the red, green, and blue primaries controlled by these colorants varies somewhat with their amounts. It seems appropriate, however, to point out how colorimetric theory applies to television in color where the analogy is exact. We can at least see very easily why television in color requires three separate independent signals for each picture element, and why the primary colors modulated by these signals are always chosen to be a red, a green, and a blue. And finally we can make clear the basic difficulties of producing a truly faithful rendition of a scene in color from mixtures of red, green, and blue light.

Why television in color requires three separate independent signals is not far to seek; it resides in the human eye itself. Each human eye (Fig. 1) casts an image of the visual field onto the mosaic of light-sensitive elements known as the retina (Fig. 3). In the eyes of some people all these elements are alike in their spectral response (Fig. 2). These are totally color-blind people. They can discriminate light from dark, but that is all (see Table 3). Most totally color-blind people have only retinal rods (see Fig. 2, rods) and are comfortable only in dim light. Others have cones of identical spectral sensitivities (see Fig. 2, cones). These people do not need color television, because they could not tell it from black-and-white television anyway.

Then there are people whose eyes have two kinds of cones instead of only one kind of receptors. Figure 29*d* shows the kinds of spectral response with which their cones are equipped. Take any pair of curves you like. We have people who respond in accord with any of the three possible combinations. These are partially color-blind people. They would be satisfied with two-color television. But over 95 percent of the population have eyes equipped with three kinds of cones, all three of the kinds shown in Fig. 29*d*; or, at least, they respond as if they had these three kinds of cones. To make all scenes look natural to people of normal color vision requires three-color television.

Now as to why the primary colors in television are always chosen to be red, green, and blue. This can be seen from Fig. 28, the (x, y) -mixture diagram of the standard CIE system. It is obvious from the curved shape of the spectrum locus that there are no three colors whose mixtures will produce all other colors. It is equally obvious that the widest gamut is produced by choosing spectrum red, spectrum green, and spectrum violet as the primaries.

In color television the primary colors are produced by placing color filters of glass, plastic, or gelatine in front of a cathode-ray tube, or by using phosphors whose spectral emittance is already highly selective

with respect to wavelength. Although good approximations to spectrum red, spectrum green, and spectrum violet can be so produced, the filters required to do it must necessarily absorb a large fraction of the radiant flux emitted by the tube. Similarly, phosphors producing spectrum red and spectrum violet by spectrally selective emission must necessarily have very low luminosity or luminous efficiency. That is, many watts of radiant flux are required to produce a lumen either of spectrum red or of spectrum violet light. This waste of radiant flux is intolerable; so the actual choice of primary colors results from a compromise between the desire to include as wide a color gamut as possible and the conflicting desire to have a bright picture. Instead of extreme spectrum red, the usual compromise is a somewhat orange-red; instead of extreme spectrum violet, a somewhat desaturated blue; and instead of spectrum green, a somewhat desaturated yellowish green.

Figure 39 shows a reasonable compromise. The red primary is at $x = 0.675$, $y = 0.325$; the green at $x = 0.22$, $y = 0.68$; and the blue at $x = 0.13$, $y = 0.08$. The dashed lines connecting these points form the edges of the triangle representing the gamut of the system. The hue names separated by curved solid boundaries are those suggested by Kelly (1943) and are to be discussed in a later chapter. They serve to show approximately what colors are included in the gamut. It is seen that by this compromise gamut the hope of rendering accurately the saturated bluish greens and saturated purples in any scene is given up. We may say that red, green, and blue are chosen as the primary colors in color television because they represent the best compromise between the conflicting desires to have a maximally colorful picture and to have a maximally bright picture.

We have thus encountered one of the basic limitations of three-light rendition. This difficulty is not peculiar to color television. Similar limitations apply to color photography and to three-color process printing. They arise from the kind of receivers of radiant energy that we have in our eyes.

Now let us come to fidelity of reproduction of colors that do fall within the gamut. From the chromaticities of the primaries it is possible to compute what spectral sensitivity is required for the camera generating the signals for control of each primary (Hardy and Wurzburg, 1937). We have merely to compute the tristimulus values of the spectrum relative to the working primaries of the system. Let us work out the spectral sensitivities of the camera required for the working primaries shown in Fig. 39:

Red: $x = 0.675, y = 0.325, z = 0.000.$

Green: $x = 0.220, y = 0.680, z = 0.100.$

Blue: $x = 0.130, y = 0.080, z = 0.790.$

Let us set the units of the working primaries so that by one unit of the red primary we mean the mixture of the CIE primaries designated as

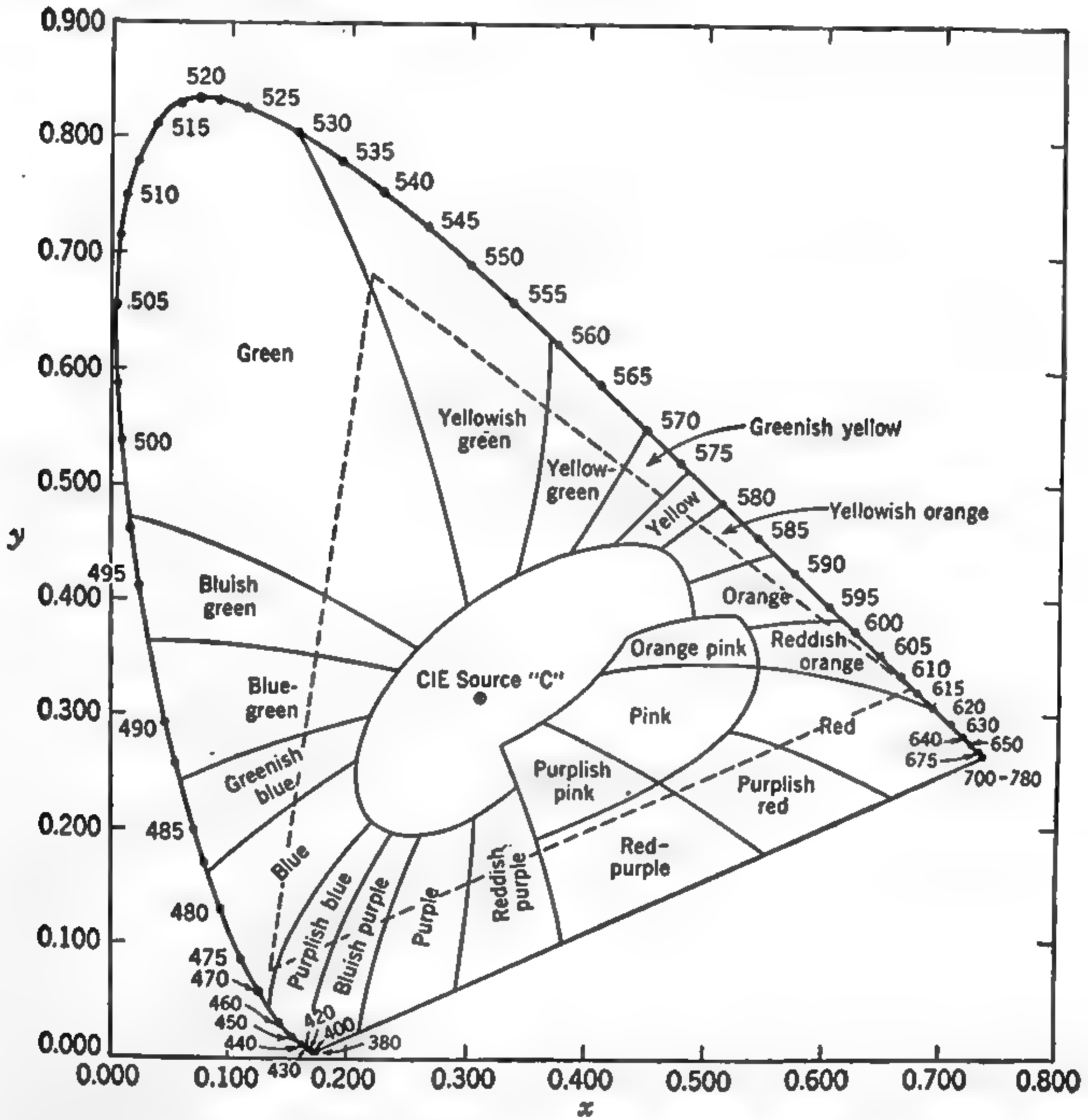


FIG. 39. Working primaries suitable for color television shown on the (x, y) -chromaticity diagram. The hue names are those proposed by Kelly (1940).

$X = 0.675, Y = 0.325,$ and $Z = 0.000,$ and a similar choice for the green and blue working primaries so that we may write:

- One unit of red : $X = 0.675, Y = 0.325, Z = 0.000.$
- One unit of green: $X = 0.220, Y = 0.680, Z = 0.100.$
- One unit of blue: $X = 0.130, Y = 0.080, Z = 0.790.$

If we have a color specified by the tristimulus values, R , G , B , in the working system, we can compute by Grassmann's laws the tristimulus values, X , Y , Z , of that same color in the standard system. The argument runs like this. The tristimulus value R means R units of the red primary, and, since we know what each unit corresponds to, we can write the contribution from the red primary as $X = 0.675R$, $Y = 0.325R$, $Z = 0.000R$. Similarly the contributions from the green and blue primaries are:

$$X = 0.220G \quad Y = 0.680G \quad Z = 0.100G$$

$$X = 0.130B \quad Y = 0.080B \quad Z = 0.790B$$

The tristimulus values, X , Y , Z , of the color, R , G , B , are found by adding together the contributions corresponding to the red, green, and blue primaries taken separately, thus:

$$\left. \begin{aligned} X &= 0.675R + 0.220G + 0.130B \\ Y &= 0.325R + 0.680G + 0.080B \\ Z &= 0.000R + 0.100G + 0.790B \end{aligned} \right\} \quad (12)$$

This set of transformation equations could have been written down immediately from equation 4; but the derivation of them from Grassmann's law, just given, shows how equation 4 itself was derived.

To find the tristimulus values of the spectrum relative to the working primaries (R , G , B), the reverse transformation equations must be found. This may be done by solving explicitly for R , G , and B , as in equation 4d, and substitution in this equation yields:

$$\begin{aligned} DR &= (0.680 \times 0.790 - 0.080 \times 0.100)X + \\ &\quad (0.130 \times 0.100 - 0.220 \times 0.790)Y + \\ &\quad (0.220 \times 0.080 - 0.130 \times 0.680)Z \end{aligned}$$

$$\begin{aligned} DG &= (0.080 \times 0.000 - 0.325 \times 0.790)X + \\ &\quad (0.675 \times 0.790 - 0.130 \times 0.000)Y + \\ &\quad (0.130 \times 0.325 - 0.675 \times 0.080)Z \end{aligned}$$

$$\begin{aligned} DB &= (0.325 \times 0.100 - 0.680 \times 0.000)X + \\ &\quad (0.220 \times 0.000 - 0.675 \times 0.100)Y + \\ &\quad (0.675 \times 0.680 - 0.220 \times 0.325)Z \end{aligned}$$

$$\begin{aligned}
 D &= 0.675 \times 0.680 \times 0.790 + 0.130 \times 0.325 \times 0.100 + 0.220 \\
 &\quad \times 0.080 \times 0.000 - 0.130 \times 0.680 \times 0.000 - 0.675 \\
 &\quad \times 0.080 \times 0.100 - 0.220 \times 0.375 \times 0.790 \\
 &= 0.3050
 \end{aligned}$$

whence:

$$\left. \begin{aligned}
 R &= 1.735X - 0.527Y - 0.232Z \\
 G &= -0.842X + 1.749Y - 0.038Z \\
 B &= 0.106X - 0.221Y + 1.271Z
 \end{aligned} \right\} \quad (13)$$

The accuracy of the computation may be (and has been) checked by solving explicitly by equation 4d for X, Y, Z , from equation 13 to see whether equation 12 is found again. Each column of coefficients in equation 13 gives the tristimulus values, R, G, B , of one of the primaries of the standard CIE system. The first column (1.735, -0.842, 0.106) gives the tristimulus values, R, G, B , for one unit of the X primary ($X = 1, Y = Z = 0$). From the fact that one of these values (-0.842) is less than zero, we know that the X primary of the CIE system is outside the gamut of the (R, G, B) system of working primaries; but, of course, we knew this already just by looking at Fig. 39, and we knew it even without looking because all the primaries of the CIE system are imaginary and cannot possibly be inside the gamut of any color system based upon real primaries. Note that each of the three columns has at least one negative coefficient.

The application of equation 13 is very general. If we have any color specified by its tristimulus values, X, Y, Z , to reproduce on the television screen by the (R, G, B) system of working primaries, equation 13 yields the signal strengths (R, G, B) that must be applied to a linear receiver to do it. For example, if it be desired to show on the television receiver the color of a light having an equal-energy spectrum ($X = Y = Z = 1$), substitution in equation 13 shows that the required signal strengths are $R = 0.976, G = 0.869, B = 1.156$. And, in general, equation 13 shows the signal strengths as three positive numbers required for any color within the gamut of the working system. We may even ask of equation 13 what are the signal strengths required to render each of the spectrum colors on the receiver. Of course, we know from Fig. 39, that for each spectrum color except one (612 m μ , the red working primary) at least one of the values of R, G, B will be less than zero. Figure 40 shows the tristimulus values, $\bar{r}_\lambda, \bar{g}_\lambda, \bar{b}_\lambda$, for all colors of the equal-energy spectrum as functions of wavelength obtained by

substituting from Table 6 the tristimulus values, \bar{x}_λ , \bar{y}_λ , \bar{z}_λ , of the spectrum colors of the standard CIE system for X , Y , and Z , respectively, in equation 13.

In Fig. 40 the positive lobes are shown as solid lines; the negative, as dotted. Note that, as expected, these negative lobes do indeed extend over all spectrum colors except the one ($612\text{ m}\mu$) corresponding to the red primary; that is, Fig. 40 shows, as expected, that spectrum

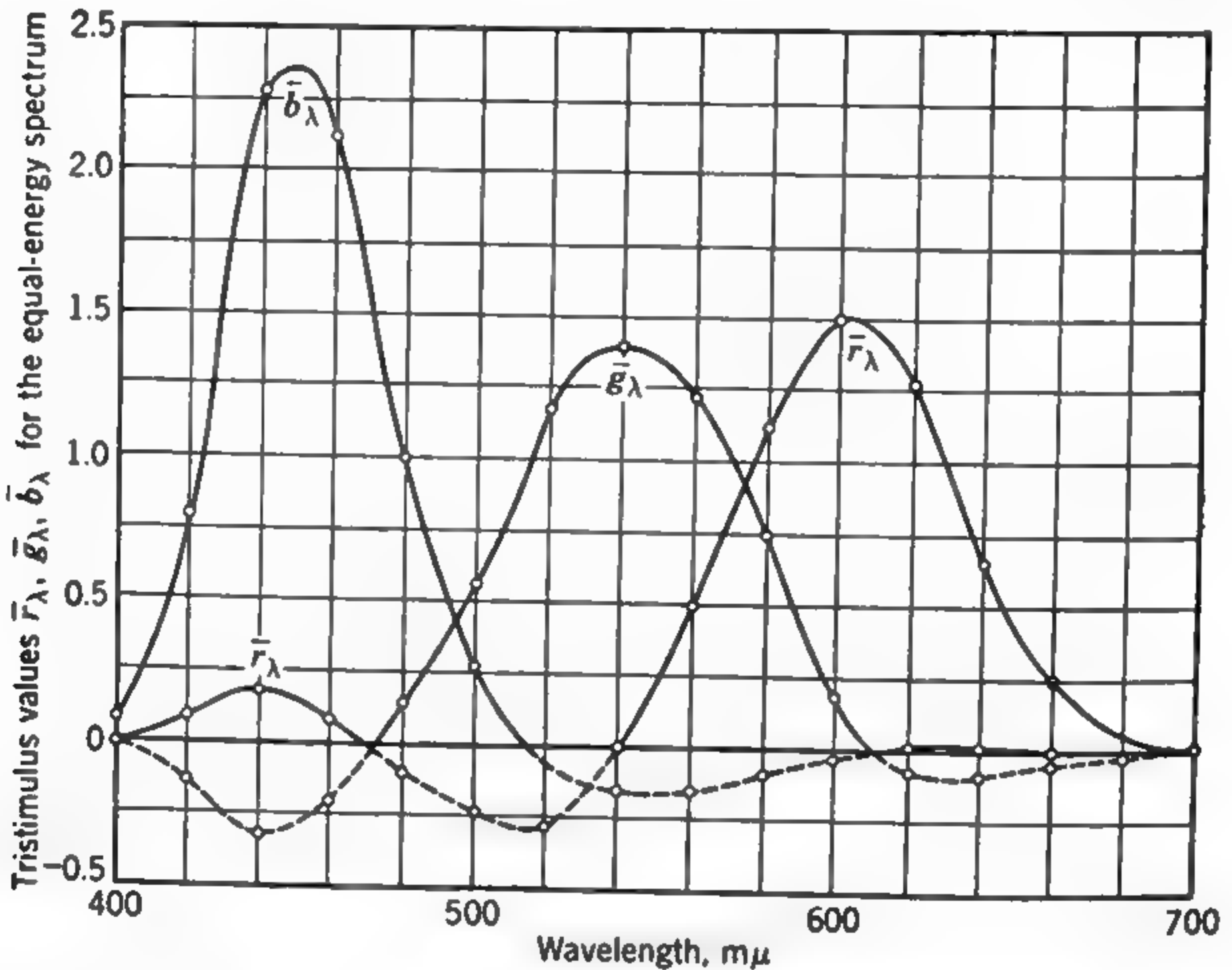


FIG. 40. Spectral sensitivities of the television camera components (R , G , B) required to control the working primaries of Fig. 39 so as to produce perfect color fidelity. Note the portions (dotted) of the curves indicating negative sensitivity.

colors in general cannot be reproduced on a television tube having the working primaries, R , G , B , no matter what signal strengths are fed to them. Consider, however, the signal strengths required to reproduce the color produced by adding together unit amounts of the two spectrum colors of wavelengths 500 and 600 $\text{m}\mu$, respectively. By Grassmann's law (equation 2) we may compute the tristimulus values, X , Y , Z , of this color from those of the components (Table 6), thus:

Wavelength, $\text{m}\mu$	X	Y	Z
500	0.0049	0.3230	0.2720
600	1.0622	.6310	.0008
Sum	1.0671	0.9540	0.2728

Insertion of the tristimulus values (1.0671, 0.9540, 0.2728) of this mixture color into equation 13 reveals that this mixture color can be reproduced on the television tube, because the required signal strengths, R, G, B , are:

$$R = 1.285 \quad G = 0.760 \quad B = 0.249$$

all of which are positive. But now note that these signal strengths may be computed directly from the ordinates of Fig. 40 by the same method:

Wavelength, mμ	\bar{r}	G	B
500	-0.225	0.551	0.275
600	1.510	.209	-.026
	<hr/>	<hr/>	<hr/>
Sum	1.285	0.760	0.249

It is evident that, if we had three cameras whose spectral sensitivities conformed, respectively, to the three curves of tristimulus values of the spectrum given in Fig. 40, and whose total response was equal to the sum, wavelength by wavelength, of the responses corresponding to the positive lobe, minus the sum, wavelength by wavelength, of the responses corresponding to the negative lobe, the three cameras, taken together, would constitute an automatic computer of the signal strengths required to render on the television tube an exact duplicate of any scene toward which the camera was pointed.

This result is theoretically rigorous and satisfying, but from the practical standpoint it is somewhat disappointing. It forces us to think of building a camera with a sensitivity less than zero for some parts of the spectrum; that is, for example, the camera controlling the amount of the red working primary must not only be sensitive to the red parts of the scene in accord with the positive lobes of the curve of \bar{r}_λ in Fig. 40 but also supply a red signal from the blue-green parts that is less than zero in accord with the negative lobe; that is, it must do this if the colors of the television screen are to duplicate accurately, element by element, those of the scene. This seeming impossibility can be achieved, however. What is required is a two-camera control for each primary, making six cameras in all. One of the two cameras has to be given a spectral sensitivity corresponding to the positive lobes of the sensitivity curve (say in the red and violet portions of the spectrum); the other, to the negative lobe (blue-green). Then by electronic means the difference in output of the two cameras would have to be used to control the amount of the (red) primary in the television receiver.

Analogous requirements apply approximately to color photography. Instead of three separation negatives, six are required for perfect reproduction. The three made in accord with the negative lobes would have to be printed as positives, bound with the main negatives in register, and so supply the corrections in reverse. In spite of the fact that the cyan, magenta, and yellow dyes used in color photography control amounts of red, green, and blue light having chromaticities varying with the amounts, one or two correction positives (known as masks) are sometimes made, and this practice is known as masking (MacAdam, 1938, 1940; Spencer, 1935; Murray, 1934, 1937). The spectral sensitivities by which the masks are made, however, usually correspond to the positive lobes of the other two primaries; that is, the masks are made from the three main separation negatives themselves, rather than from special emulsions duplicating the negative lobes; so the process gives only an approximate correction, though, to be sure, it often results in a very worth-while improvement.

Whether six-camera controls for televising in color are worth while is doubtful. The predominating practice has been to adjust the camera sensitivities approximately according to the positive lobes. The chief result of neglecting the negative lobes is that colors near the boundaries of the gamut are rendered somewhat less saturated than the colors of the original. An approximate correction is possible without six-camera controls by using a negative response from a combination of the other two controls as in color photography, but a number of considerations discourage this practice. First, a similar result can be obtained simply by setting the contrast of the receiver abnormally high. Second, telecasting in color is beset by other impediments to color fidelity (such as cross-talk) more important than neglect of the negative lobes. Third, with cross-talk satisfactorily reduced, remarkably good pictures result from controls based on the positive lobes alone; so perhaps rigorous, element-by-element color fidelity is an unnecessary refinement in color television. Nevertheless, the foregoing theory has assisted in selection of the spectral sensitivities of the camera controls and has acted as a guide in the development of color television.

COLORIMETRY BY DIFFERENCE

In the fundamental colorimetry of lights and objects by means of the spectroradiometer or the spectrophotometer, a single standard is used for each class of specimen. Opaque surfaces are referred either to the ideal perfect diffuser or to a sufficiently thick layer of magnesium oxide deposited from the smoke of magnesium turnings or ribbon burning in air (*NBS Letter Circular* LC-547, TAPPI T 633

m-47, ASTM D 307-44 and D 986-48 T). Transparent objects, such as gelatine films, crystal, or glass plates, are referred to the same thickness of air; transparent solutions, either to the same thickness of distilled water or solvent, or, for concentrated solutions, to a thickness of solvent equivalent to that in the solution. Self-luminous objects, such as fluorescent lamps, cathode-ray tubes, television screens, and incandescent lamps, are measured relative to one of the standard sources, usually source A, or they are measured relative to source A viewed through a filter of known spectral transmittance. The analysis of the spectra of lights and objects with reference to these standards can go forward, regardless of how similar or how different the light or object is to the standard. The method is fundamental and quite general.

If it is necessary to determine the color of a light or object in terms of a standard having spectral characteristics markedly different from the unknown, spectral analysis followed by computation by means of the standard observer is the shortest way to obtain an accurate, unambiguous answer. Thus we have already seen that many visual colorimeters (tristimulus, dominant wavelength and purity, subtractive) have severe limitations. But, if the unknown and the standard are nearly alike in spectral character, that is, if the unknown and the standard form an essentially nonmetameric pair, then it is possible to obtain an accurate, unambiguous result by direct comparison, either visual or photoelectric. Furthermore, the answer can be got more quickly and at less cost than by means of the spectroradiometer or spectrophotometer.

For example, the colors of nearly clear glass plates can be obtained quickly and precisely by direct comparison, because the spectral flux distribution of the emergent beam is very similar to that of the incident beam transmitted through so much air with which it is compared. Likewise, the comparison of a near-white specimen with the magnesium oxide reflectance standard is a simple colorimetric problem, as is also the determination of the colors of incandescent lamps nearly equivalent to standard source A.

Highly selective absorbers and emitters, like the rare-earth glasses and gaseous discharge tubes, present difficult colorimetric problems. In general, the greater the deviation in spectral composition between the unknown specimen and the standard, the greater is the uncertainty of the result obtained by a visual or a photoelectric colorimeter.

Modern photoelectric spectrophotometers have greatly extended the application of both visual and photoelectric colorimetry. They have supplied a rapid, practical way to calibrate working standards of color. If a fairly large group of specimens is at hand to be measured, say 20

or more, all of similar spectral composition, the most satisfactory way to measure them in the present state of colorimetric science is to evaluate one or two of them carefully by means of the spectrophotometer to serve as working standards, then obtain the color specifications of the rest by visual or photoelectric determination of the difference between specimen and standard. Note how many factory color-control problems fall in this category. Variations in manufactured goods come about from small changes in the proportions of a small number of colorants, or from variations in the manufacturing process such as temperature, humidity, acidity, alkalinity, or from minor variations in the colors of raw materials. It is rare that any such influences introduce importantly metameric differences between successively produced goods.

Another important application of colorimetry by difference from a spectrally similar standard is the solution of such problems as the color changes produced by various deteriorative agents (weathering, heat, salt spray, acid, alkali, abrasion, and so on), or those produced by admixture of a colorant, or toner. In these problems, the color standard is the original unexposed or untuned specimen; and the chief concern is to evaluate the character and amount of the color differences, the absolute evaluation of the colors being of secondary interest, or required only approximately. For these applications colorimetry by difference would be used in preference to indirect colorimetry by means of the spectrophotometer because it is quicker and cheaper.

Another application is to fluorescent materials. The color measurement of fluorescent reflecting materials must now be accomplished by irradiating the sample with the source of interest (such as daylight) and obtaining the color specification by difference from a nonfluorescent standard previously calibrated by a spectrophotometer; there is no other way to do it. No spectrophotometer is currently available that will make accurate measurements on a fluorescent sample. This application, less general than the others, may yet loom large in the next few years. Fluorescent dyed fabrics are used for signaling by the armed forces; fluorescent painted fabric is used for life-raft sails; and fluorescent dyes are used to color the sea itself in air-sea rescue work. More than 60 percent of the textile swatches serving as color standards in the Standard Color Card of America (TCCA, 1940) were found to be detectably fluorescent (Reimann, Judd, and Keegan, 1946), and more than 25 percent are so strongly fluorescent as to render the commercially available spectrophotometers inapplicable. Furthermore, an important development in soaps and detergents is the so-called "optical bleach," a dye that fluoresces blue from ultraviolet energy. Opti-

cally bleached fabrics are thus fluorescent, and the assessment of the merit of the soaps containing these dyes requires the measurement of the colors of these fabrics by difference from a nonfluorescent color standard.

Finally, there is the application of colorimetry by difference from a standard to colors of nonturbid solutions. In these applications generally the color changes of interest arise from alteration of the amount of one of the constituents in the solution. If the standard is a solution of known concentration of the constituents of interest, spectral similarity between standard and unknown is assured, and colorimetry by difference may be used to evaluate the concentration of the constituent in the unknown solution. The color differences are usually measured directly by altering the thickness of the unknown or the standard solution until a color match is produced, and the answer is found directly as the concentration of the constituent in the unknown. This method is used as the final operation in thousands of quantitative chemical tests (Mellon, 1950), such as the analysis of alloys (Hague, 1944; Snell and Snell, 1948). So manifold are the uses of this kind of chemical colorimetry and so widespread is the use of color comparators (both visual and photoelectric) to carry it out that to many people chemical colorimetry is the whole of colorimetry and the simple color comparator used is dignified by the name colorimeter. Chemical colorimetry uses the color of the solution as an indication of the concentration of one of the substances in solution. The daylight color gamut produced by such variation depends upon the spectral transmittance of the unknown substance in solution and may follow a complicated course in the color solid. However, the variation is necessarily one dimensional and may be followed with equal validity by viewing sample and standard through a color filter, and this is often done whenever a filter can be found that increases the perceptibility of the color differences. If the filter chosen is very selective with respect to wavelength, it may isolate a restricted region of the spectrum. Such a filter simplifies the color gamut by making it a light-to-dark variation at nearly constant chromaticity, and the operation becomes photometric rather than colorimetric in nature as in the colorimetry of sugar solutions (Ambler and Byall, 1931; Brewster, 1936; Zerban and Sattler, 1936, 1951; Keane and Bryce, 1937). Absorptimetry, the name used by Mellon (1950) for this method, is quite apt because what is being measured is the absorptive capacity of a system for radiant energy. As in other applications of colorimetry by difference, the spectrophotometer is indispensable for the preliminary research to set up the standards used in such routine determinations.

Among the most time-saving and valuable of the tools and technics of color measurement in commerce are the devices for carrying out colorimetry by difference from a standard spectrally similar to the unknown.

Color Comparators. As the name implies, the essential property of a color comparator is to supply two near-by, but not necessarily juxtaposed, fields to the view of the operator, one illuminated by light from the standard, the other by light from the unknown. Two identical teacups in the breakfast room make a color comparator. By its use can be told at a glance whether two cups of tea are equally strong, or whether two cups of coffee poured from the same pot have equal amounts of cream added. Scarcely more complicated is the color comparator formed by holding two test tubes side by side up to a window as is sometimes done in titration to determine alkalinity or acidity of a solution by addition of a dye used as an indicator. If the chemist prepares a series of standard solutions in test tubes, he may use this simple color comparator to determine approximately the concentration of solutions of the same constituent within the range of the standards. Some comparators substitute cells of rectangular cross section for the test tubes, and line them up in racks, straight-line or circular, for convenient handling and viewing. Others have built-in illuminators and an optical device to bring the fields to be compared into juxtaposition. For dilute solutions Nessler tubes with flat ends are used. The light is introduced at the bottom and is viewed after suffering absorption from passing up through the entire height of the solution in the tube. Color changes can be produced by adding more of the solution, either standard or unknown. The Duboscq-type comparator has an optical glass plunger that can be pushed more or less deeply into the unknown solution, thus adjusting the effective height of the column reversibly and very conveniently. Other comparators are supplied with sets of permanent glass standards adapted to a particular chemical test, such as the Hellige color comparators for pH control of electrotyping and electroplating solutions, wash water in laundries, and water in swimming pools and filtration plants, for color of whiskey, beer, varnish, and water, and for medical tests (such as liver function, blood, and urinalyses). For details of such comparators reference may be had to Mellon's excellent treatment (1950).

Color comparators are simple and practical. A manufacturer would be foolish to use a spectrophotometer for a job requiring only color comparison. It often pays, however, to use a photoelectric transmissometer with spectral filters for this job, such as that designed by Clifford and Brice (1940). These are called abridged spectropho-

tometers or filter photometers. For details see Müller's analysis of filter photometers (Müller, 1950). The Macbeth-Ansco Color Densitometer introduced in 1951 is a filter photometer applicable to small specimens because of its optical system and applicable to very dark specimens because of the high sensitivity of the photomultiplier tube used as the light detector.

Martens Photometer. One of the most useful visual devices for determining relative luminance of surfaces of nearly identical spectral selectivity is the Martens photometer (Martens, 1900). Figure 41 shows the Martens photometer combined with a diffuse illuminator to form the Priest-Lange reflectometer (Priest, 1935). This reflectometer is intended for the measurement of luminous reflectance of opaque specimens relative to reflecting standards of similar spectral selectivity of reflectance. It is also adaptable to the measurement of luminous transmittance of transparent plates relative to transmitting standards similar in spectral selectivity of transmittance. Finally, the Martens photometer removed from the mounting shown in Fig. 41 may be used to determine the luminance of an unknown self-luminous surface, provided that a standard of known luminance is available that differs from the unknown only in an essentially nonmetameric way.

The superior usefulness of the Martens photometer arises from the convenience of the adjustment for equality of brightness between the two halves of the photometer field and from the fact that in a well-made instrument the dividing line between the half-fields is exceptionally narrow so that it is often invisible when a brightness match has been set. The addition of the simple lens in front of the photometer permits the object viewed to be thrown into focus and thus extends the use of the device to the reflectometry of small specimens such as gems, picture elements, postage stamps, and natural and artificial teeth.

The Martens photometer is customarily used in one of two ways. In the interchange method the specimen is compared directly with the standard and the angle θ_1 , required for a brightness match, is read from the instrument scale set so that the zero of the scale corresponds to maximum obscuration of the comparison side of the field. Then specimen and standard are interchanged, and the angle of brightness match, θ_2 , is read. The luminance ratio, L/L_s , is found as:

$$L/L_s = (\tan \theta_1)(\cot \theta_2) \quad (14)$$

In the substitution method the standard is first compared with a comparison surface and the angle, θ_1 , for brightness match found. See in Fig. 41, for example, wedge *W* beneath the photometer which directs the line of sight to an area of the white-lined sphere which here forms

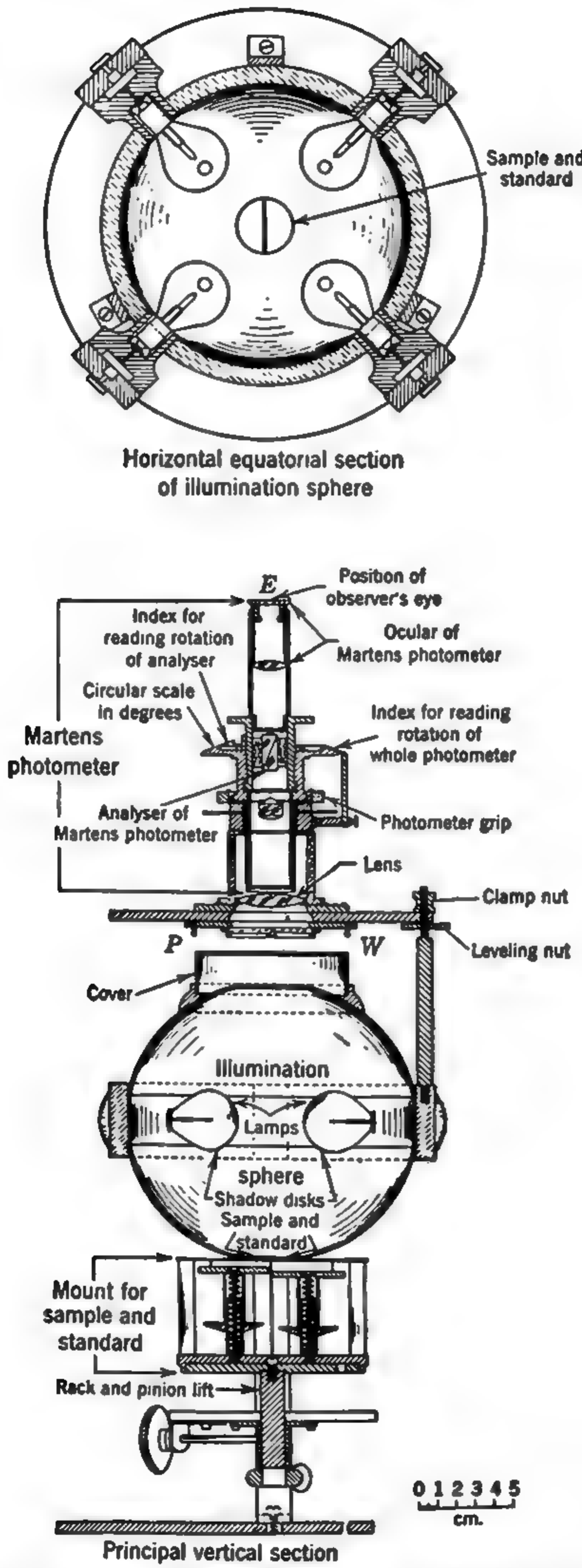


FIG. 41. Vertical cross section of the Priest-Lange reflectometer showing Martens photometer, diffuse illuminator, and specimen holder (after Priest, 1935). Also (above) horizontal cross section of the illuminator.

the comparison surface. Then the specimen is substituted for the standard, and angle θ_2 found. The luminance ratio is computed as:

$$L/L_s = (\tan^2 \theta_2)/(\tan^2 \theta_1) \quad (15)$$

If specimen and standard are of considerably different chromaticities, the setting for equality-of-brightness is not adequately reproducible. If the specimen and standard form a strongly metameric pair (that is, have considerably different spectral compositions but nearly identical chromaticities), a projection of the macular pigment of the observer may appear on the photometric field of the instrument (usually 6° in diameter). Such a projected pattern may make it impossible to obtain a precise and reproducible setting for brightness equality. Reproducibility of setting can be improved for such metameric pairs by stopping down the field to about 2° , thus keeping the whole field within a reasonably uniform part of the macular projection, but the smaller photometric field brings with it a reduced precision of setting. These limitations apply to all visual photometers. They render difficult and uncertain the visual photometry of mercury-arc lamps, and, to a lesser degree, fluorescent lamps, by comparison either with incandescent-lamp standards of luminous intensity, or with such standards combined with "color-matching" filters of known spectral transmittance.

The Martens photometer finds hundreds of practical applications. For example, it has been used at the National Bureau of Standards in the outdoor colorimetry of fluorescent fabrics otherwise too bright to be compared to nonfluorescent color standards, in the routine checking of Lovibond red glasses and color standards for railroad signals, in the measurement of manila-rope fiber (Becker and Appel, 1933; Becker, 1933; Reimann and Mease, 1940), and it has served as an integral part of a colorimeter designed to test the fidelity of color television by comparison of both a test color and the rendition of it by television with combinations of the Lovibond red, yellow, and blue glasses (Judd, Plaza, and Balcom, 1950).

Chromaticity-Difference Colorimeter. The determination of chromaticity coordinates x, y by comparison of the unknown specimen with a working standard of similar spectral reflectance can be carried out visually with high precision by means of a differential colorimeter described by Judd (1939, *a*). The adjustment of the chromaticity of the comparison field to match the standard field is by two double wedges, one of greenish and the other of yellowish glass. Since the light from the comparison field must pass through both the yellow and the green wedges, it is sometimes called a subtractive colorimeter; see Fig. 42,

which gives a schematic diagram. The standard and comparison fields are brought into juxtaposition by means of a Lummer-Brodhun cube having a double trapezoid pattern subtending 9 by 13° at the observer's eye. The adjustment to near-equality of brightness to facilitate detection of chromaticity differences is by movement of the projection lamp that illuminates both standard and comparison surfaces. The

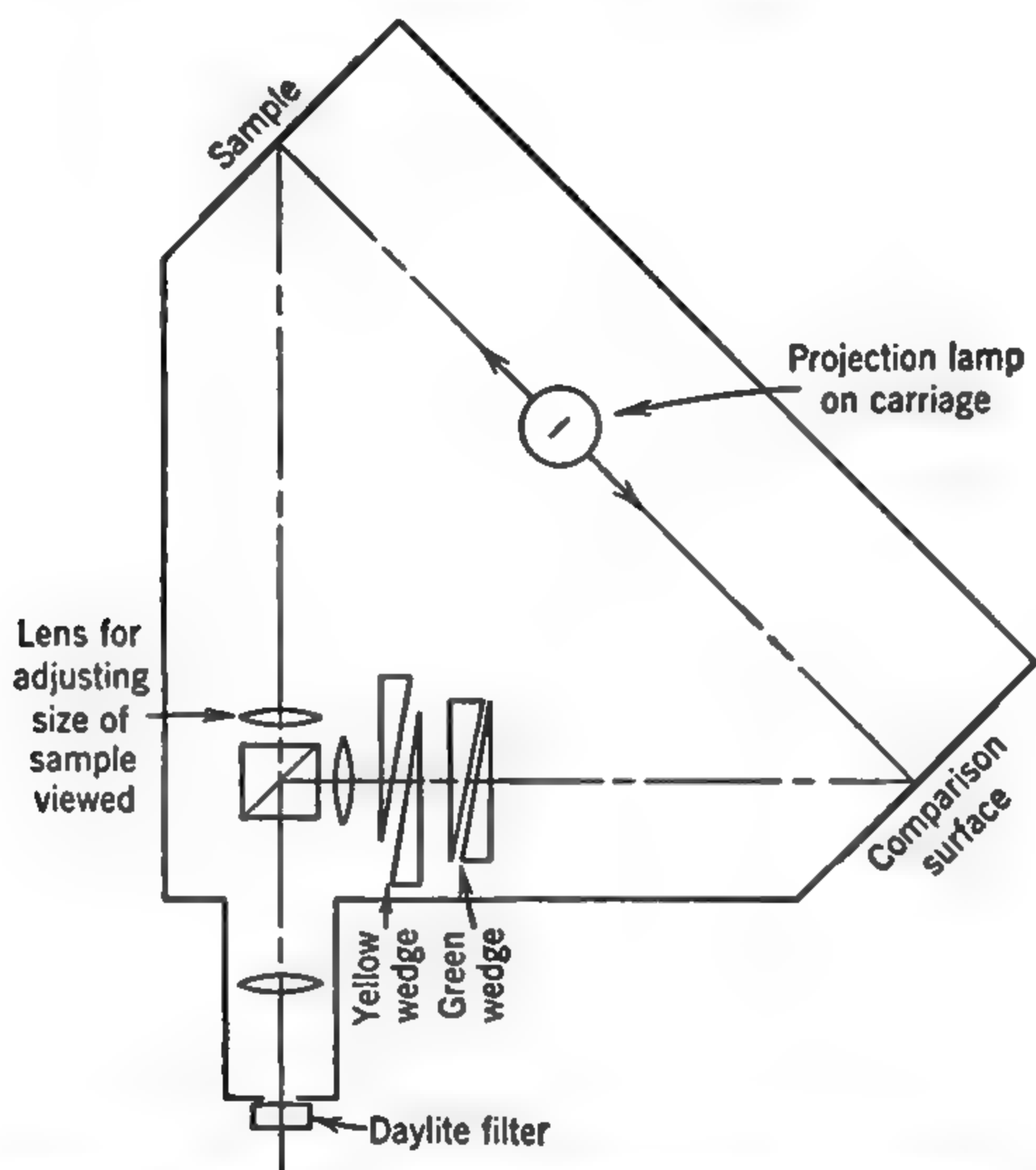


FIG. 42. Schematic diagram of chromaticity-difference colorimeter (after Judd, 1939, *a*).

comparison is made either by incandescent lamp or by artificial daylight obtained by insertion of Corning Aklo and Corning Daylite glass found to yield for comparing nearly nonmetameric pairs a satisfactory heat-free duplication of standard source C.

A substitution method is usually employed with this colorimeter, a match first being set up between the standard and comparison surfaces by adjustment of the wedges. Then the unknown specimen is substituted for the standard, and the wedges readjusted to restore the match. Auxiliary yellow and green filters may be inserted either in the beam reflected from the specimen or in the comparison beam, if the specimen differs sufficiently from the standard to require it. The differences in wedge settings can be calibrated in terms of the difference

in the chromaticity coordinates, x , y , from the known spectral transmittances of the wedges by computation as in Table 8 or Table 13, provided that the spectral directional reflectances of the comparison surface are known approximately.

The large angular size of the field (9 by 13°) takes full advantage of whatever ability the observer may have to discriminate small chromaticity differences. Note from Fig. 33 that fields larger than this promise little if any improvement in precision of setting. The instrument also provides for a high field luminance. With a 500-watt projection lamp only 7 inches away from both standard and comparison surface, the field luminance is about one-third that (10 lamberts) produced by the noon winter sun illuminating a snow-covered field. Some observers find a field as bright as this painful, but the added precision permits fewer settings to be made, and it has often seemed preferable to risk a headache to get the job done quickly. If specimen and comparison surfaces are similar in spectral composition, the settings for match between specimens of light color may be repeated easily within 0.001 in chromaticity coordinates x , y . If the surfaces are not spectrally similar, or if they are viewed through many yellow and green filters, a faint projection of the macular pigment may appear in the photometric field. Such an appearance lowers the precision of setting somewhat and is also correlated with failure of the result to agree with indirect colorimetry by way of the spectrophotometer (Reimann, Judd, and Keegan, 1946) and the standard observer. This failure may often be by as much as 0.003 in x or y .

The chromaticity-difference colorimeter has the disadvantage of requiring a skilled operator. The calibrations too are time consuming, and constant vigilance has to be maintained lest calibrations made for one comparison surface fail to apply to another because of metameric difference between them. Furthermore, the instrument is not applicable to some spectrally selective specimens because nearly homogeneous radiant energy is too little changed in spectral composition by passage through the wedges.

The colorimeter has been used to check enamel color standards set up by the National Hosiery Manufacturers Association as an aid in the color grading of raw silk to minimize unlevel or "two-tone" dyeing of hosiery, and, in general, for inspection of working standards, transparent and opaque, for conformity to a master standard. These include the standards for Kitchen and Bathroom Accessory Colors set up by the National Retail Drygoods Association (NBS Commercial Standards CS62-38 and CS63-38), Lovibond red glasses, and the color

standards used in the inspection of glass and plastic ware for airplane running and signal lights. It has also been used to set up a color standard for ruby mica (Judd, 1945). But the chief use is for general colorimetry by difference, both for fluorescent and nonfluorescent specimens (Reimann, Judd, and Keegan, 1946).

Photoelectric Tristimulus Colorimeters. If three photocells could be adjusted, as by glass filters, so that their responses were proportional

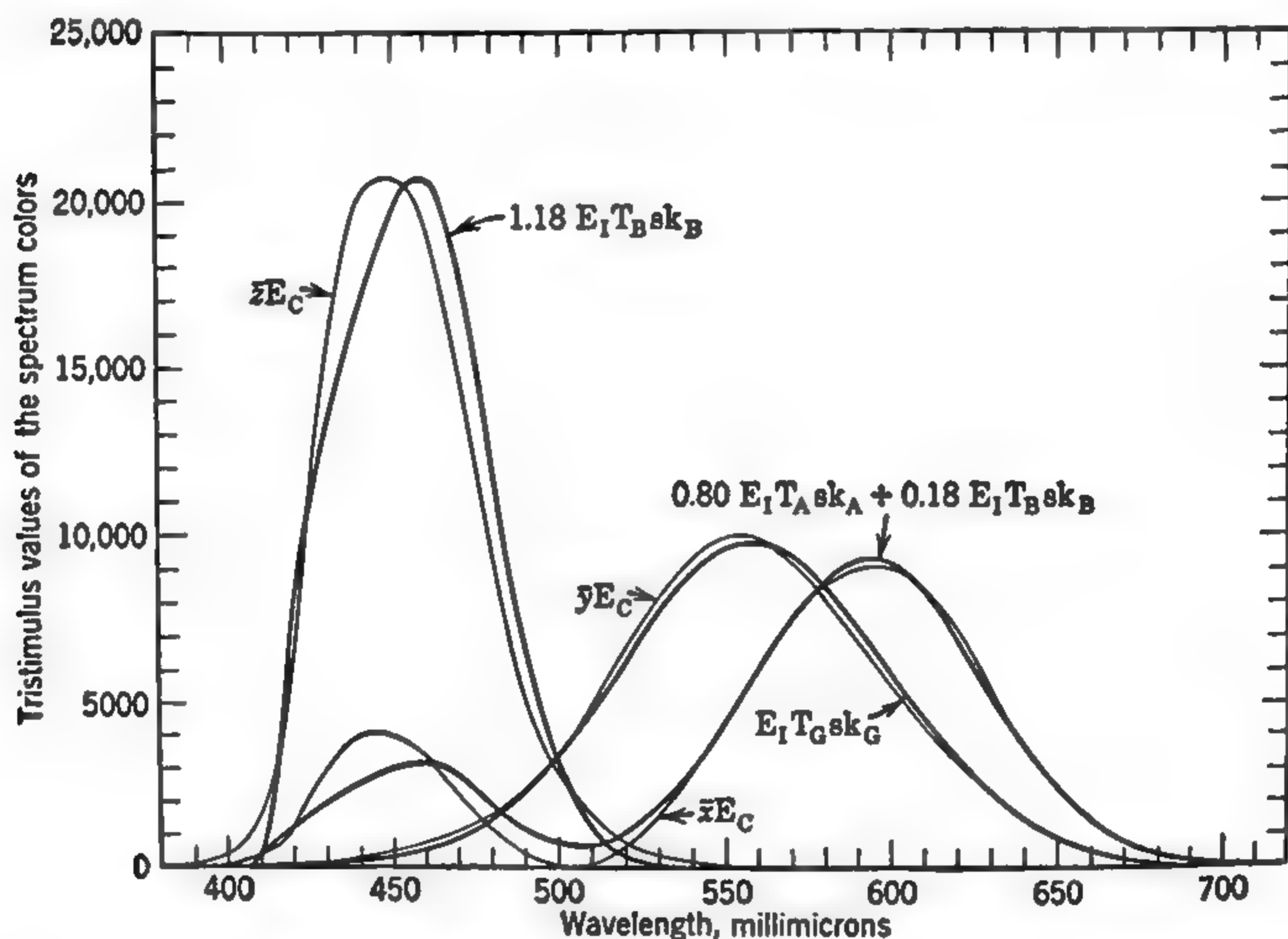


FIG. 43. Curves showing the degree to which the Hunter tristimulus filters combined with incandescent lamp and barrier-layer cell duplicate the CIE tristimulus values of the spectrum of standard source C (after Hunter, 1942).

throughout the visible spectrum to some linear combination (as in equation 4d or equation 13) of the standard CIE distribution curves (see Fig. 14), then they could be used to test directly whether any two light beams have the same color (Gibson, 1936; Guild, 1934; Hunter, 1942). The response of the first photocell might give the X -tristimulus value of the light beam falling on it, the response of the second would give the Y -tristimulus value, and the response of the third, the Z . If a second light falling on these three photocells gave the same values of X , Y , Z , that would be proof that the two lights had the same color according to the standard observer. In this application the photocells are devices for computing automatically the tristimulus values, X , Y , Z , according to equation 8.

Several good attempts to build a photoelectric tristimulus colorimeter suitable for general use have been made (Barnes, 1939; Dresler and Frühling, 1938; Guild, 1934; Hunter, 1940; Perry, 1938; Winch and Palmer, 1937) but with incomplete success (Van den Akker,

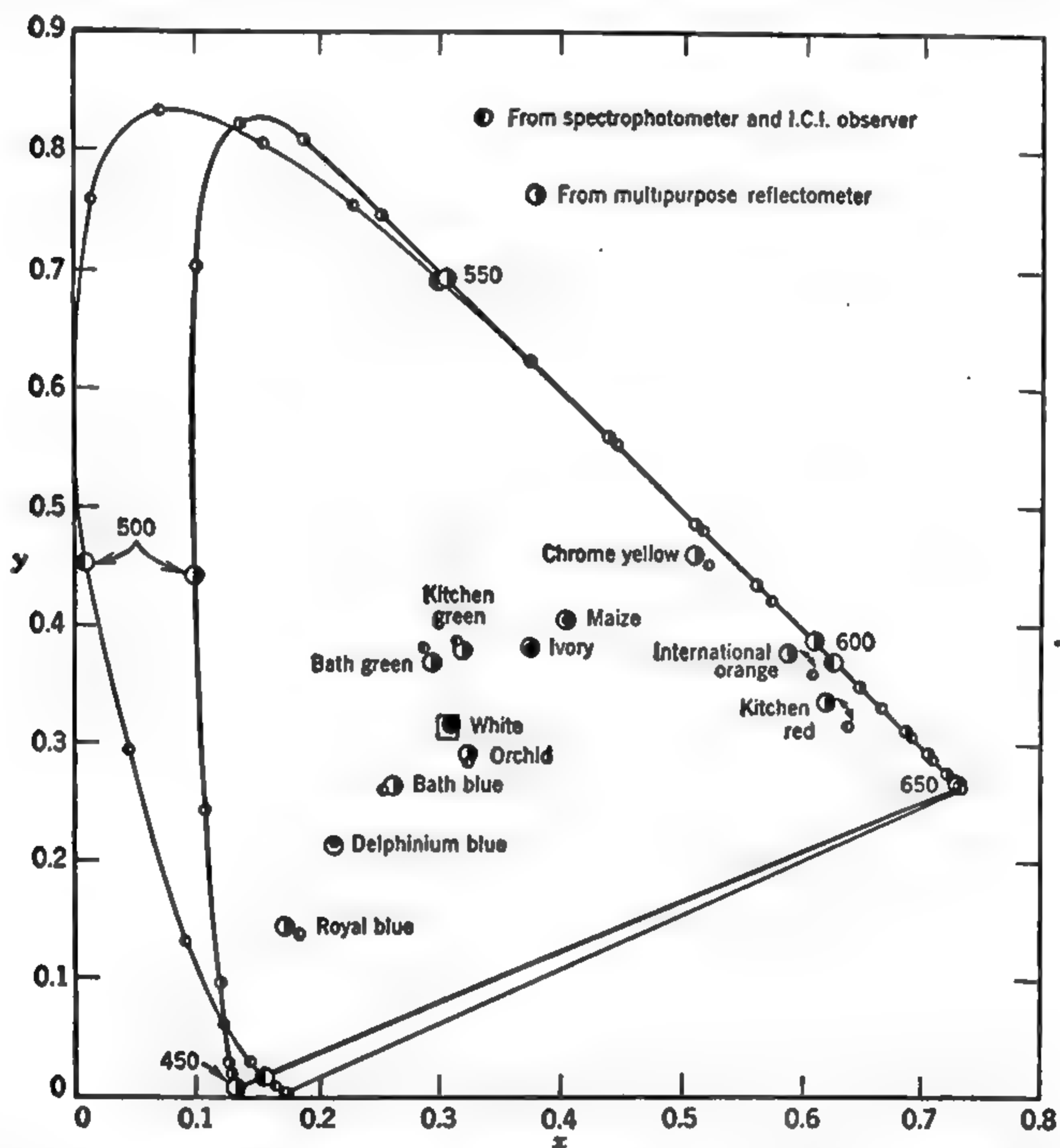


FIG. 44. Chromaticity discrepancies expected from the use of the Hunter tristimulus filters (after Hunter, 1942). All measurements are referred to magnesium oxide as the standard white.

1937). Figure 43 shows the degree to which filters designed by Hunter (1942) to duplicate the CIE standard observer and simultaneously to adjust a projection lamp to CIE standard source C have succeeded. If A , G , and B represent the settings obtained for a specimen relative to those for a standard magnesium oxide surface with the amber, green, and blue filters, respectively, approximate tristimulus values X , Y , Z may be found as:

$$\left. \begin{aligned} X &= 0.80A + 0.18B \\ Y &= 1.00G \\ Z &= 1.18B \end{aligned} \right\} \quad (16)$$

The approximation to the X value has to be found by combining the readings of two filter-photocell combinations because it has so far

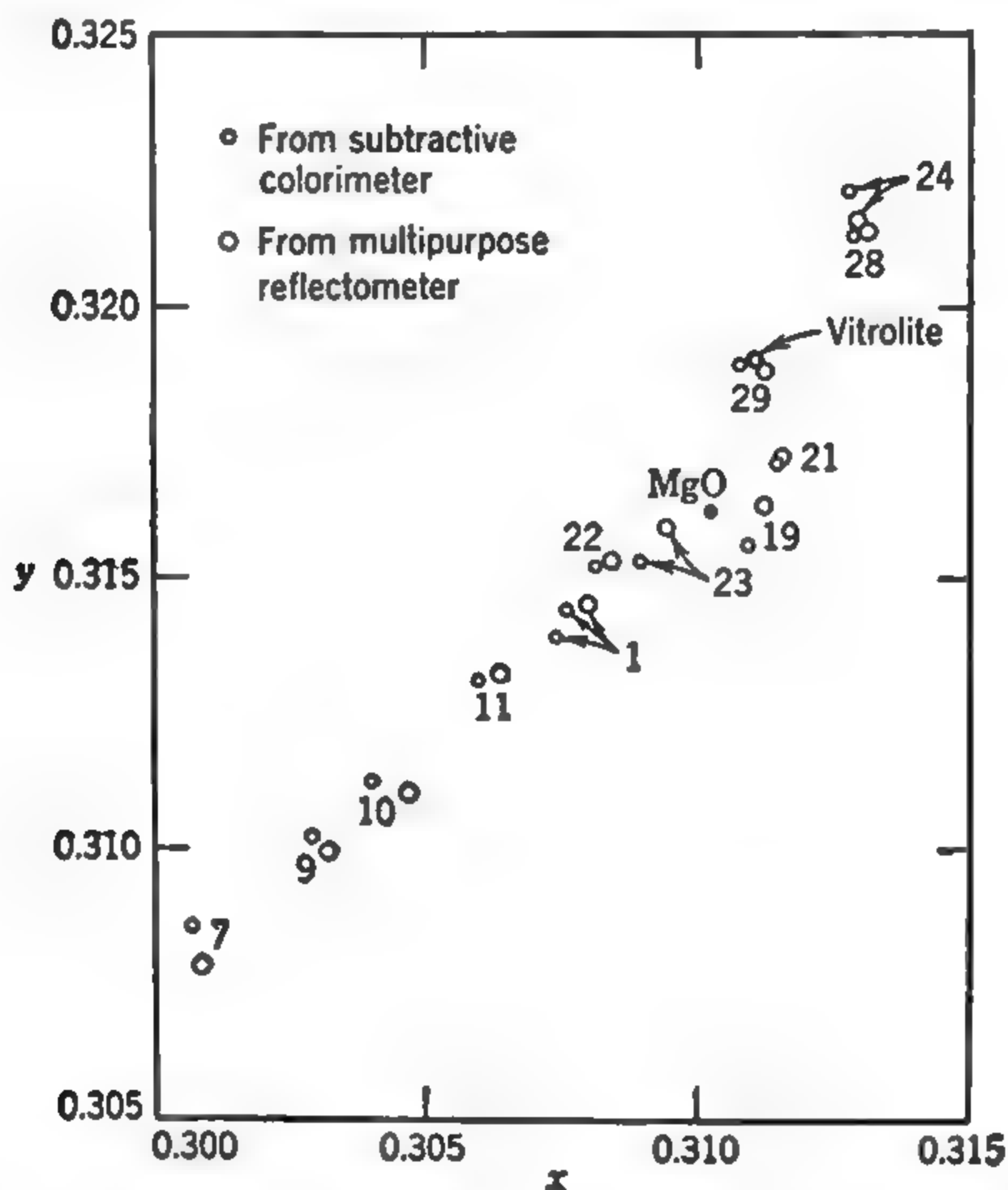


FIG. 45. Agreement between multipurpose reflectometer and the chromaticity-difference (subtractive) colorimeter for near-white vitreous-enamel specimens measured relative to magnesium oxide (after Hunter, 1942).

proved to be impossible to find a single filter having the required double-peaked curve of spectral transmittance.

Approximate chromaticity coordinates x , y may be found from these approximate tristimulus values in the usual way ($x = X/(X + Y + Z)$, $y = Y/(X + Y + Z)$); see equation 7. Figure 44 shows the discrepancies that this use of the Hunter filters introduces. These discrepancies are roughly proportional to the distance from the point representing the magnesium oxide standard (common to both plots) and are frequently larger than 0.02 in x or y , that is, more than 10 times a reasonable chromaticity tolerance for most colorimetric work.

For the comparison of specimens exhibiting essentially nonmetameric differences (such as near-white surfaces), however, the degree

of duplication of the CIE standard observer shown in Fig. 43 is amply sufficient. Figure 45 refers to the small rectangle near the center of Fig. 44 and indicates the degree of agreement between measurements of near-white vitreous-enamel specimens by means of the chromaticity-difference colorimeter and photoelectric tristimulus colorimetry by

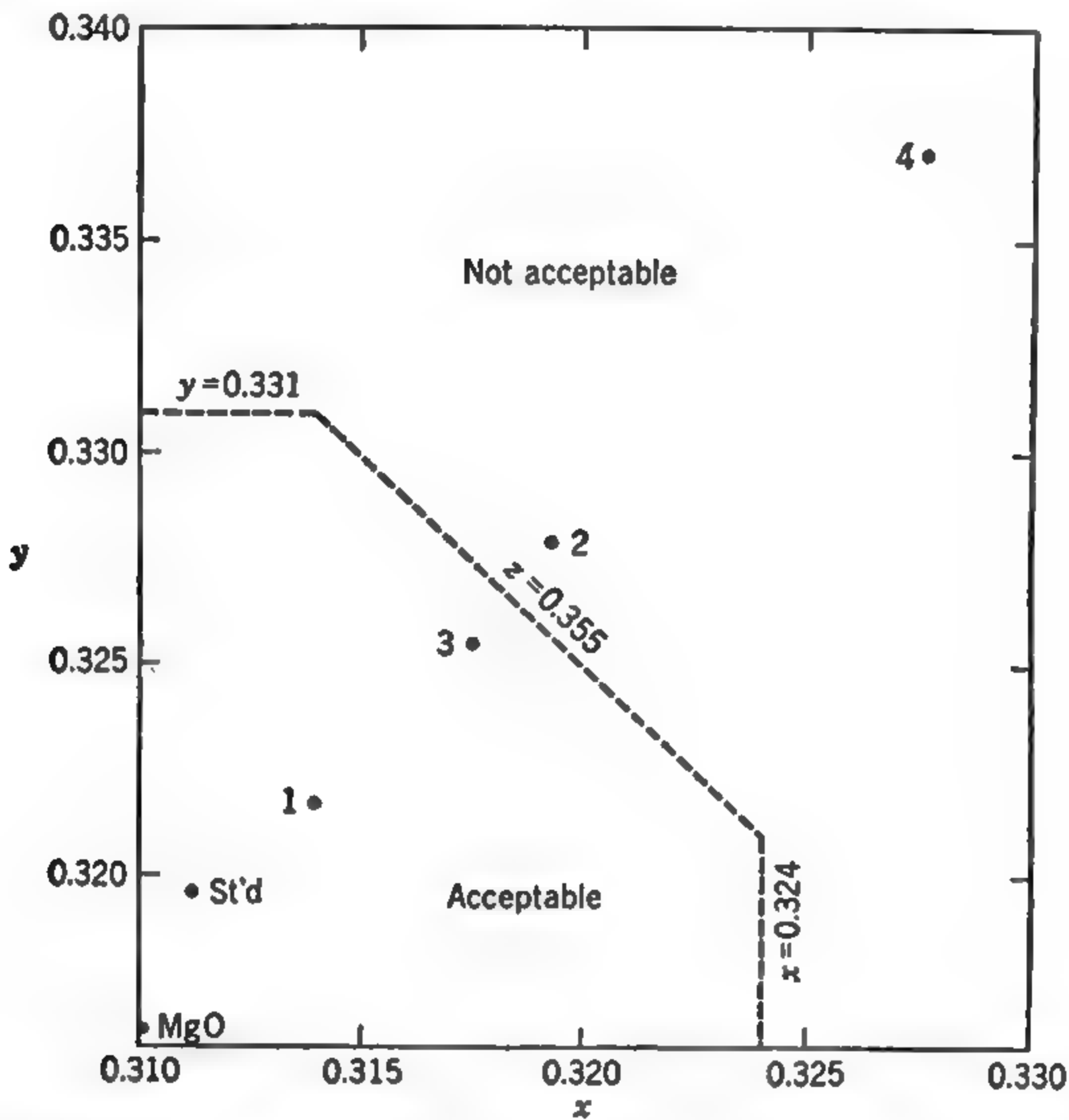


FIG. 46. Chromaticity requirement for white casein paints from Federal Specification TT-P-23a (after Hunter, 1942). The Hunter multipurpose reflectometer has been used to test such paints for conformity to this requirement.

means of the multipurpose reflectometer (Hunter, 1942). Figure 45 indicates that the discrepancies are less than 0.001 in x or y for these comparisons. Figure 46 illustrates an application of photoelectric tristimulus colorimetry to near-whites in which the instrument is used to test white casein paints for one of the requirements of Federal Specification TT-P-23a. Another application of considerable commercial importance is evaluation of the yellowness index:

$$\text{Yellowness index} = (A - B)/G \tag{17}$$

This index yields a scale of yellowness increasing from zero for the standard magnesium oxide surface or any equally nonselective sur-

face to positive values for yellowish surfaces and negative values for bluish surfaces. Photoelectric tristimulus colorimetry is listed as Method 425.2 in Federal Specification TT-P-141a dealing with test methods for paint, varnish, lacquer, and related materials.

If the limitations of photoelectric tristimulus colorimetry are appreciated, the method is most useful in product-control colorimetry

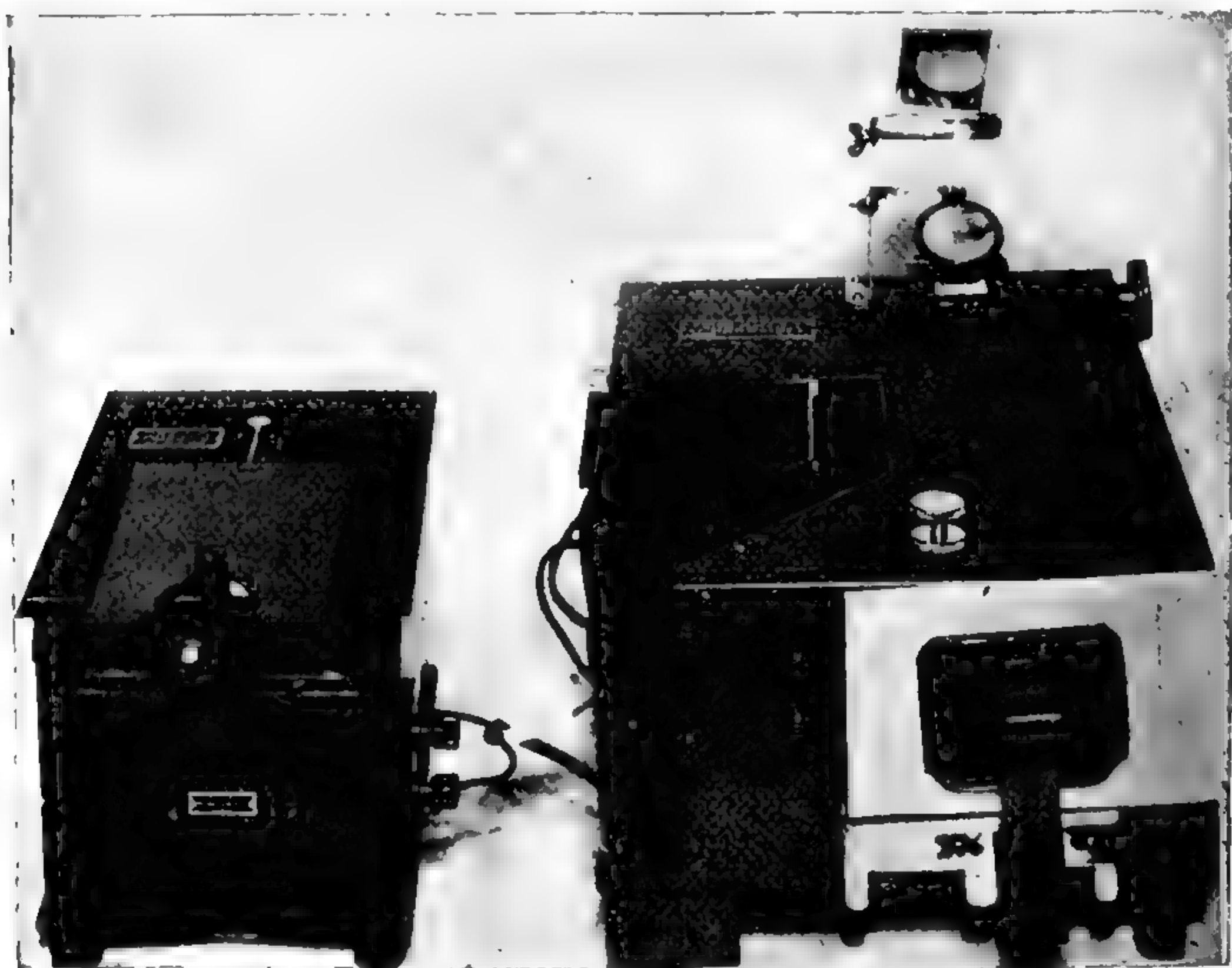


FIG. 47. General view of factory-control reflectometer and galvanometer. A sample of paper is in position for measurement. (Courtesy of Henry A. Gardner Laboratory.)

of nonfluorescent specimens by difference from a working standard. Two sets of permanent standards of 45° normal directional reflectance made of vitreous enamel on metal have been developed at the National Bureau of Standards for the calibration of photoelectric colorimeters. One of these is a set of 11 gray plaques ranging from black to white and nearly nonselective in spectral reflectance. The other is a set of 10 plaques representing the commercially important colors selected by the National Retail Drygoods Association as standard for Kitchen and Bathroom Accessories (NBS Commercial Standards CS62-38 and CS63-38). These standards apply to the colorimetry of spectrally similar specimens under the standard angular conditions of 45° illumination and viewing along the perpendicular to the surface. With

barrier-layer cells the precision of the method is comparable, though perhaps not quite equal, to the best that can be done by visual colorimetry. No unusual qualifications or extended special training is required for an operator, and, compared to visual colorimetry, or to indirect colorimetry by the spectrophotometer, the results are obtained very rapidly.

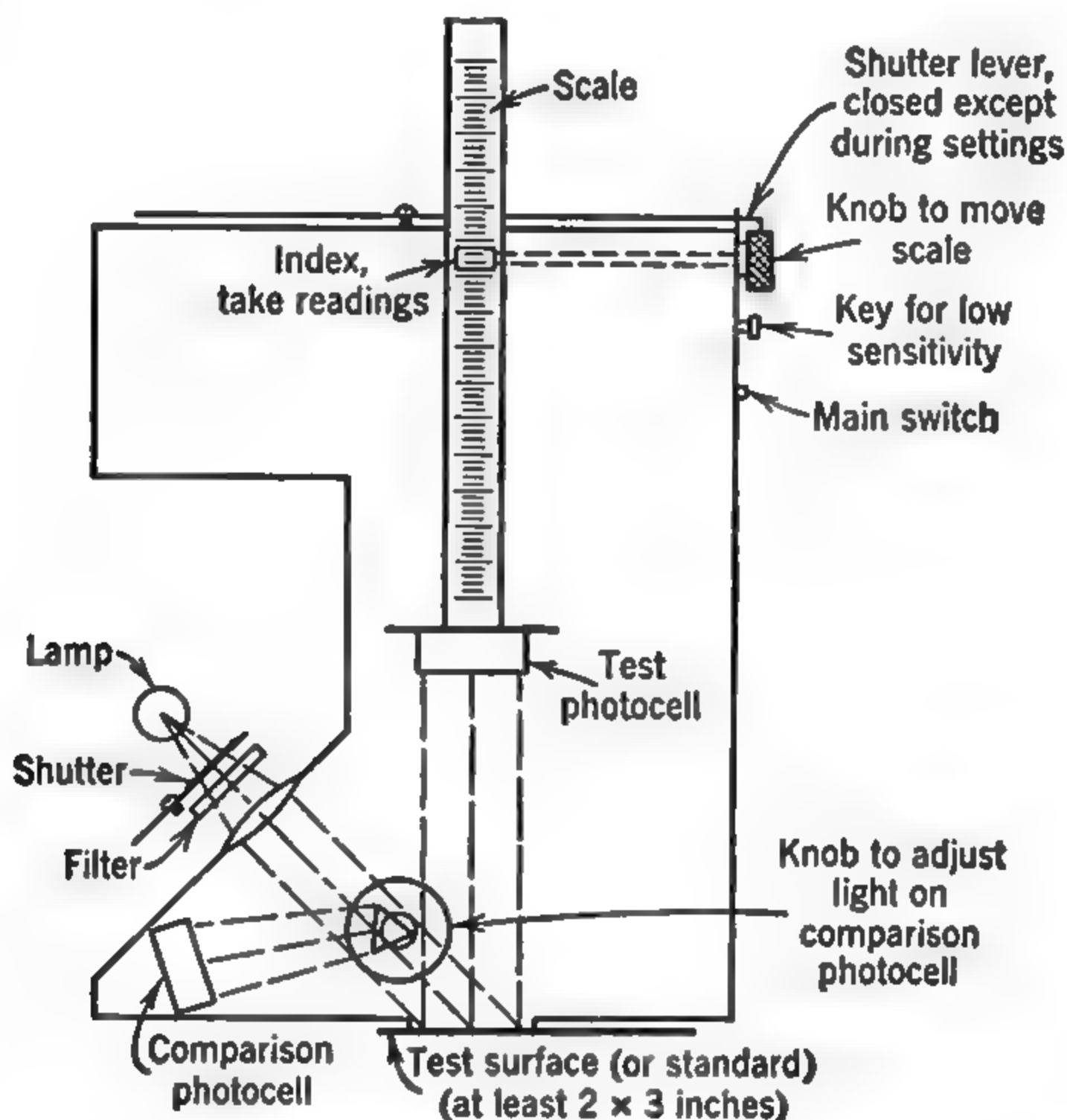


FIG. 48. Diagram of the Hunter factory-control reflectometer, showing the arrangement of parts. A small mirror takes light from the center of the main beam to illuminate the comparison photocell. (Courtesy of Henry A. Gardner Laboratory.)

Photoelectric reflectometers and transmissometers of proved practical value based on barrier-layer cells are commercially available, as are also the filters separately for those who wish to improvise their own devices for photoelectric color measurement and control. A photoelectric transmissometer supplied by the Photovolt Corporation under the trade name Lumetron has been used successfully with the Hunter tristimulus filters to evaluate the colors of petroleum products by comparison with permanent color standards of glass (Lykken, 1948). These standards duplicate closely the spectral transmittance of petroleum products and are obtainable from the National Bureau of Standards together with a certificate showing their tristimulus values.

Figure 47 shows a factory-control reflectometer using the Hunter filters and manufactured by the Henry A. Gardner Laboratory, Inc.; Fig. 48 is a diagram of it. Figure 49 is a diagram of the optical portion of a reflectometer using the Hunter filters and manufactured by the Photovolt Corporation. These photoelectric reflectometers have

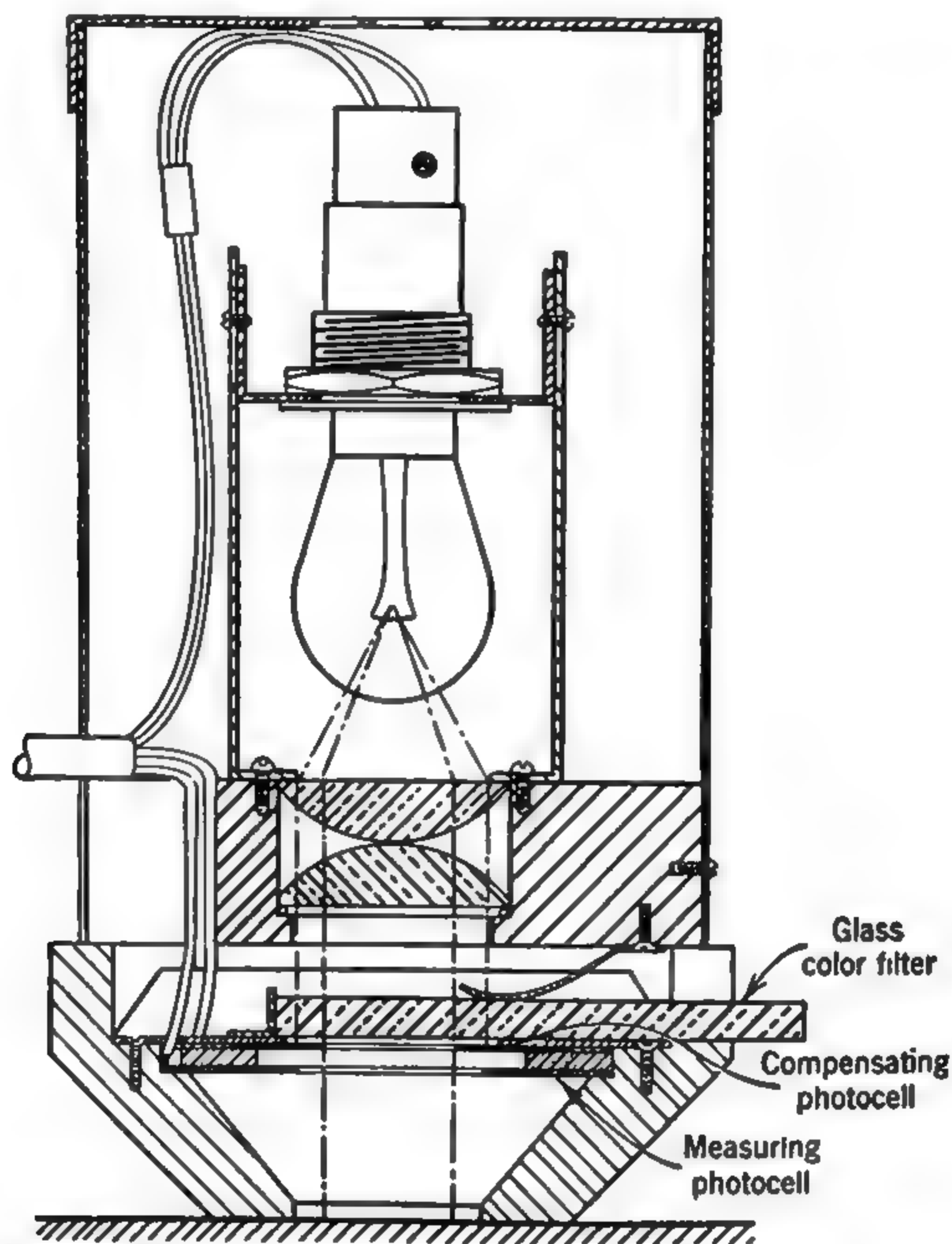


FIG. 49. Photoelectric reflection meter. The control box with current-measuring equipment is not shown. (Courtesy of the Photovolt Corporation.)

been successfully applied to painted panels (Eickhoff and Hunter, 1942), papers, textiles, ceramics, agricultural products, plastics, and a wide variety of fabricated specimens.

It should be noted that these photoelectric colorimeters do not illuminate the specimens with the illuminant of interest (daylight). The source is an incandescent lamp some of whose radiant flux is passed through one of the Hunter tristimulus filters before striking the specimen. The filters are designed so that the color of the specimen as it would appear in daylight is evaluated, provided that the specimen is

nonfluorescent. Attempts to evaluate fluorescent specimens by means of these colorimeters nearly always result in important error. Their use must, therefore, be restricted to specimens known not to be importantly fluorescent.

It should also be noted that much of the simplicity and convenience of these photoelectric colorimeters arises from the fact that they are

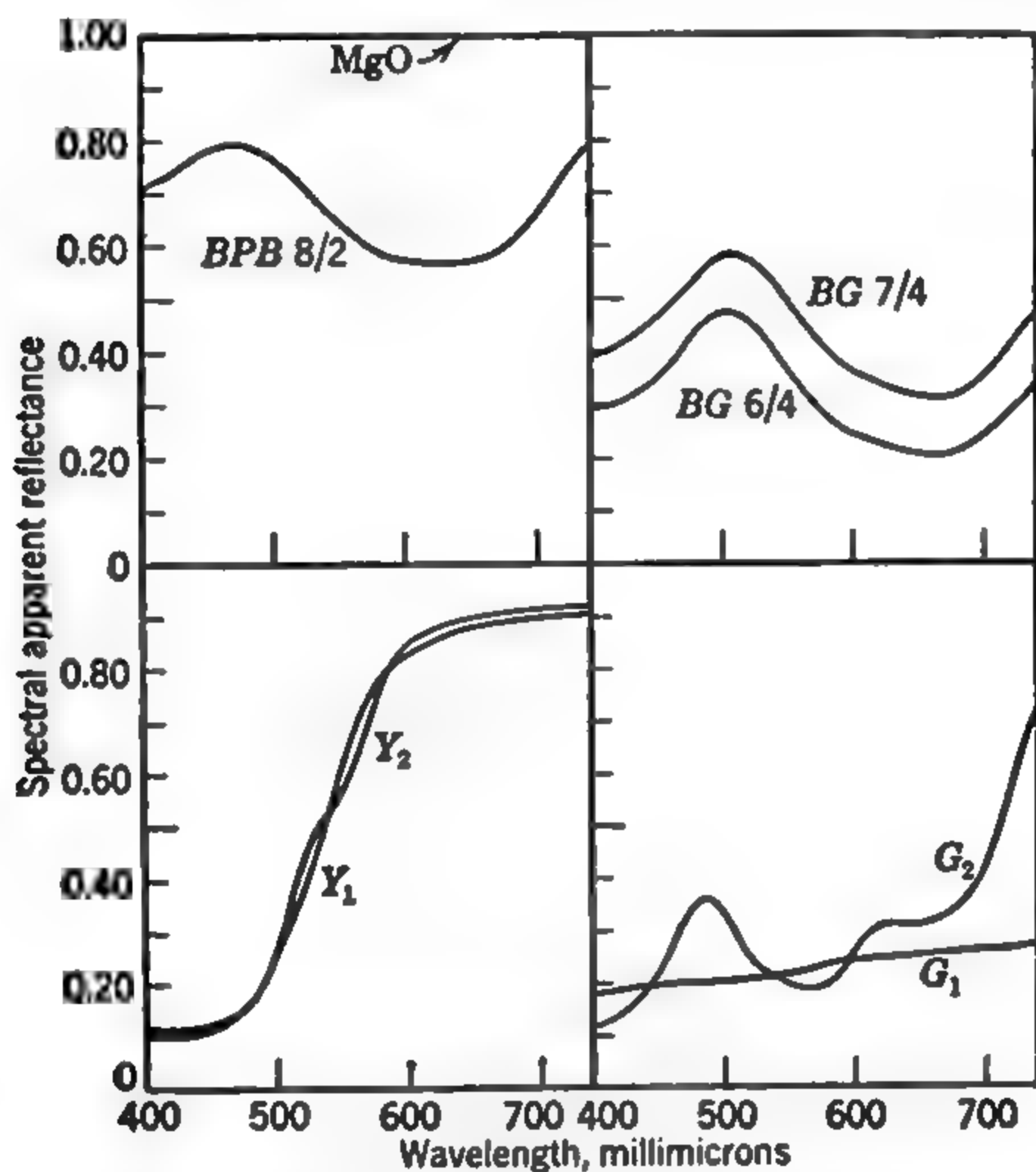


FIG. 50. Spectral reflectances of pairs of samples exhibiting various degrees of metamerism (after Hunter, 1942). Two upper pairs differ considerably in color but show little or no metamerism. Two lower pairs are near-matches; left, moderately metamerism; right, strongly metamerism.

able to operate on the unamplified output of barrier-layer cells. Because of insufficient sensitivity these instruments cannot be applied conveniently to specimens of very dark color, which necessarily reflect only a small fraction of the light incident on them. This drawback has been overcome by a well-designed photoelectric colorimeter based on the photomultiplier tube and built for the Pittsburgh Plate Glass Company. This instrument is now commercially available from the Instrument Development Laboratories, Needham Heights, Mass., and is known as the PPG-IDL Color Eye.

In general, satisfactory accuracy can be expected from such photoelectric tristimulus colorimeters for the determination of small color

differences between nonmetameric pairs. And even for measurement of fairly sizeable nonmetameric color differences, such as those analyzed spectrophotometrically in the upper portion of Fig. 50 (BPB 8/2 vs MgO, BG 7/4 vs BG 6/4), and small chromaticity differences with a moderate metameric component, such as shown in the lower-left portion of Fig. 50 (Y1 vs Y2), the discrepancy is in the neighborhood of 0.002 in x or y , which is negligible for many purposes. However, for highly metameric pairs, such as shown in the lower-right portion of Fig. 50, the discrepancy may be expected to be in the neighborhood of 0.02 in x or y , just as it is for large chromaticity differences. Errors of this size can rarely be tolerated; so the operator of a photoelectric colorimeter must constantly be on guard lest the standards used by him differ too drastically in spectral selectivity from the unknown to permit reliable measurement.

A valuable, detailed study of various photoelectric colorimeters has been made by the Institute of Paper Chemistry (1937, 1938).

COLOR STANDARDS

Because of the convenience of material standards of color, they are often used in commerce in preference to specification according to the more fundamental CIE system. Material standards may be carried from place to place, and, if the colors are sufficiently closely spaced in the neighborhood of the unknown color, the nearest match may be found by visual comparison, or, even better, a fairly reliable intermediate value may be estimated by direct visual interpolation. Thus, the inspector may record that the unknown is closest in color to standard 13 but departs in the direction of the color of standard 28 by about one-third of the difference. This highly practical method has been given a scientific name; it is known as the ratio method (Newhall, 1939; Richardson, 1929, 1932).

Color standards may be classified by how generally they can be applied. A single paint chip used to specify the color required for a consignment of paint has no generality at all. It applies simply to one color, and often only to one order. Thousands of these are selected every year, serve their purpose well, and are set aside. Then there are limited sets of color standards showing a one-dimensional variation. Hundreds of such sets of color standards, sometimes known as color scales, are in constant practical everyday use. A set of solutions, each solution having the same coloring constituent but in different concentration, used as the final step in a chemical test, is a characteristic example. A set of glass standards duplicating such a series of colors is another important example. Scales for the color grading

of petroleum products, sugar solutions, or rosin, showing the colors corresponding to various degrees of refinement or purity, are other examples. These sets of color standards showing a one-dimensional variation are more general in application because they represent a series of colors instead of only one color. And finally, we have collections of color standards intended to represent a considerable fraction of all colors. From such a set it should be possible to find a reasonably close match for any color within the gamut of the collection. These, too, find considerable use in American industry; but important collections of this kind can be counted pretty much on the fingers. Since color itself is tridimensional, such collections must necessarily show tridimensional variation so as to supply adequate sampling of the colors represented in the fraction of the color solid covered.

Systematic Sampling of the Color Solid. If a reasonably small number of specimens (say 1,000) is to cover the range of all possible colors (at least 10 million are detectably different) so that for any color chosen at random a fairly close duplicate can be found in the set, the specimens of the set must be selected according to a system or plan. Otherwise the collection will be found to have some large groups of colors all nearly alike, leaving relatively wide ranges of the color solid unsampled. The internal construction of systematic collections of color standards was for years imperfectly understood until Carl Foss (1949) pointed out that there are only three basic plans to be used.

The most common method is to start with a limited number of pigments or dyes and develop the color gamut by compounding these materials in systematically varied proportions. Such a system may be called a *colorant-mixture system*. Examples are the Plochere color system and the Martin-Senour Nu-Hue systems.

Another method is to copy with colorants the color mixtures produced by means of a tristimulus colorimeter or rotating-sector disk by varying systematically the tristimulus values of the colors. The systems resulting from this method are thus developed by appeal to instruments and may be called *color-mixture systems*. Examples are the Ridgway charts and the Ostwald color system.

An intermediate method is to produce the colors by systematic variations of screening in the screen-plate process of printing. It is intermediate because in so far as the printed dots do not overlap the results are the same as by rotary mixture, and in so far as they overlap the results correspond to colorant mixture. Examples are the *Maerz and Paul Dictionary of Color* and the *Villalobos Colour Atlas*.

Finally, it is possible to develop a systematic collection of color chips simply according to the perceptions of the colors by an observer

with normal color vision. The aim might be to prepare a set of color chips, each one of which appeared to differ from each of its nearest neighbors by a constant amount and so give a uniform sampling of the psychological color solid (never yet achieved), or it might be to provide uniform color scales exemplifying the psychological attributes of color perception: hue, lightness, and saturation (as in the Munsell color system). This is development by visual appraisal, and the resulting system may be called a *color-appearance system*.

Colorant-Mixture Systems. The basic idea of a colorant-mixture system is to show what can be done with the base colorants. The collection of chips must show what colors can be produced by them; this is the color gamut. And it may also indicate about how much of the base colorants are used to produce the various mixtures. The concepts illustrated by such a systematic collection of chips are necessarily colorant concepts. Thus, if three of the base colorants are a white paint, a black paint, and a red paint, one sequence of colors that it would be logical to show is the sequence produced by various admixtures of the white with the red paint (called an extension with white, or a letdown); another is the sequence produced by admixtures of the black with the red paint. It should not be supposed that these admixtures necessarily result in colors perceived to have constant hue. This is almost never true. Alizarin crimson, for example, mixed with white paint produces colors perceived to have red hues very much on the purplish side. A primary purpose of a colorant-mixture system involving any red pigment might be to show the more or less purplish red hues produced by admixture with white paint. Nor should it be supposed that the admixture of the black paint with the white paint necessarily produces grays. Nearly always the colors so produced are perceived to have blue hues and low saturations and to be, therefore, bluish grays. Thus, the concepts exemplified in a colorant-mixture system of chips are colorant concepts. The color sequences by colorant mixture in general depart importantly both from those produced by color mixture (that is, additive mixtures of light) between the same end points and from the sequences judged by visual appraisal to be the most direct between the same end points. The spacing along these sequences, in terms of uniform variation in the proportions of the colorants (by weight or by volume), departs enormously both from the spacings found by equal steps in luminous reflectance and in CIE chromaticity coordinates and from the spacings judged by visual appraisal to be equal. The Munsell Color Company supplies at nominal cost three different developments of the red-white-black gamut to show

how large these differences may be. One of these is on the colorant-mixture basis, one on color-mixture, and one on color-appearance.

To understand a colorant-mixture system requires a knowledge of the plan by which it was developed. Failure to understand the plan may lead to false conclusions from use of the chips and misapplications of them. A description of a few of the more complete colorant-mixture systems used in American industry follows together with a statement of the basic plan of each.

1. *Nu-Hue Custom Color System*, The Martin-Senour Company, 2520 Quarry St., Chicago, Ill., 1946. The Nu-Hue Custom Color System developed by Carl Foss contains 1,000 painted cards, mat finish, available in two forms, 3-by-5-inch cards in a plastic case, and disks mounted in systematic array between transparent cover plates. This is a colorant-mixture system developed from 8 base paints, 6 chromatic, one near-black, and one white. Nine different admixtures with white paint are shown in chart form. At the periphery of the darkest chart there are 54 hues. On this level there are 9 rings around the neutral center, each ring of painted disks corresponding to successive admixture of the near-black paint to the 6 chromatic base paints. Each succeeding lighter chart has one less ring, decreasing systematically from 9 rings at the bottom level to zero rings at white. This organization is shown schematically for one base chromatic paint in Fig. 51. The 1,000 color chips in the system illustrate the extent to which paint mixtures of these particular 6 chromatic paints can be developed by admixture of these particular gray and white paints. For each of these 1,000 colors the amounts by weight and volume of the 8 base paints are known, and it is possible by weighing or measuring out these amounts to mix a satisfactory dry match for any of the colors shown. The base paints are chosen for permanence, and some restriction in gamut for the red-purple to blue hues has resulted. However, the Nu-Hue color system includes the colors commonly used in painting interiors and exteriors of houses and has been very successful in promoting the sale of prescription-mixed paint.

2. *The Colorizer*, Colorizer Associates, P.O. Box 1322, Salt Lake City, Utah, 1947. The Colorizer consists of 1,298 chips, $1\frac{1}{4}$ inches square, and 24 near-white chips, $1\frac{1}{4}$ -by-4-inch rectangles, making 1,322 chips in all, spray painted in mat finish. The chips are mounted on removable strips, convenient for color comparison, and the strips are attached to heavy cardboard sheets bound in the form of an album. There are 12 chromatic base paints, and pairs of them of neighboring hue are mixed in equal parts to form 12 secondary hues. Mixtures in equal parts across 2 and 4 hue steps yield 2 degradations

approximately of the 12 main hues, and equal-parts mixtures across 3 hue steps yield 1 degradation approximately of the 12 secondary hues. Each of these 12 base paints is shown in 6 extensions with a white base paint, as are also the 12 secondary hues and the 36 degra-

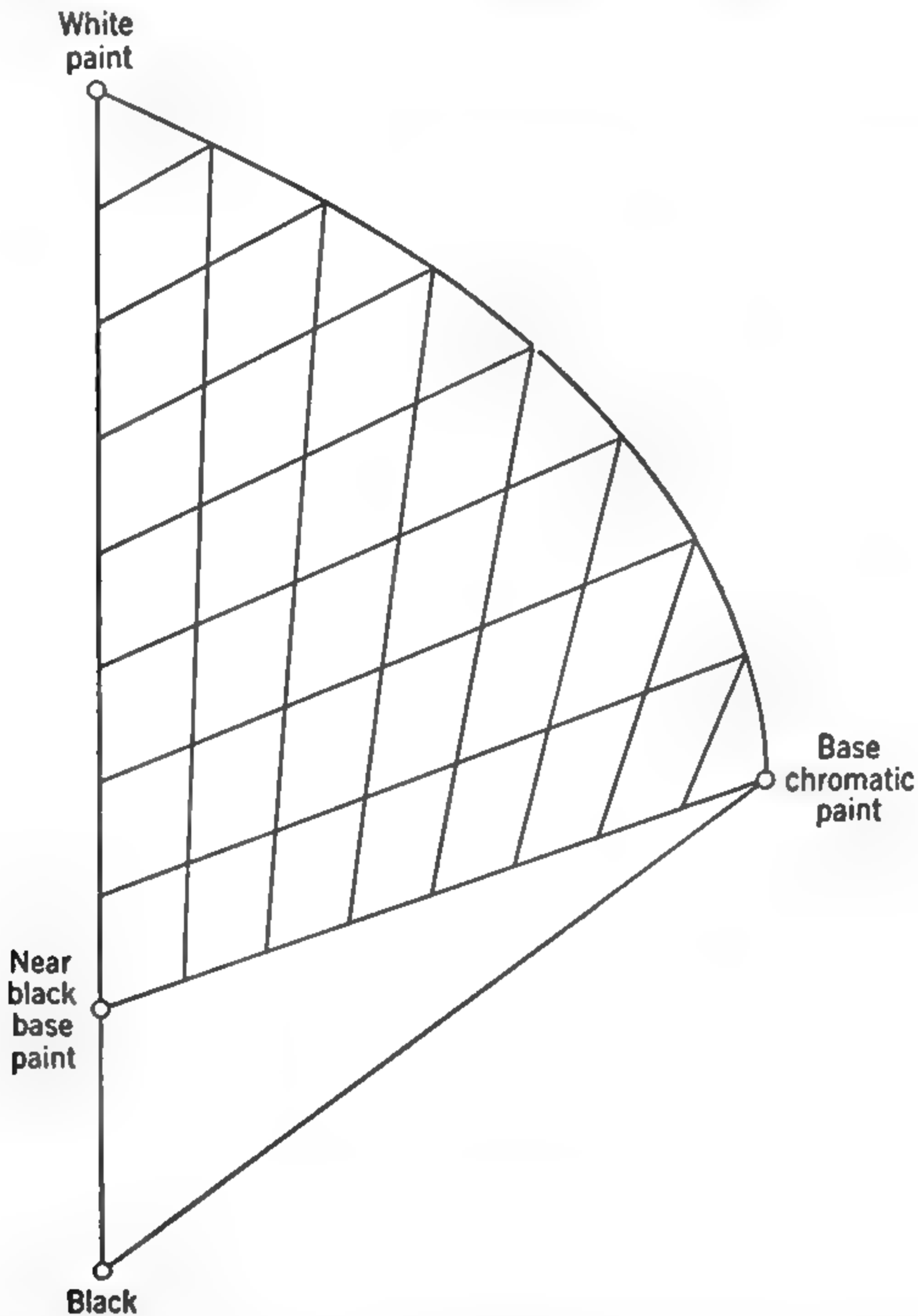


FIG. 51. Organization of the colors derived from one of the base chromatic paints of the Nu-Hue Custom Color System by admixture of the black and white base paints.

dations; and all of these 60 colors are shown in 5 extensions with a gray base paint, making $360 + 300 = 660$ color chips. This organization of the 12 main hues is shown schematically in Fig. 52. Four auxiliary base paints, a light yellow, a pale orange-yellow, a brownish pink, and a medium gray, their equal-parts mixtures with each other, and their equal-parts mixtures with each of the 12 chief chromatic paints give $4 + 6 + 48 = 58$ auxiliary secondaries which are also

shown in 6 extensions with the white base paint and in 5 with the gray, making $58 \times 11 = 638$, completing the 1,298 nonnear-white colors.

The introduction of the auxiliary base paints results in particularly good coverage in the important yellow to red hue range. The propor-

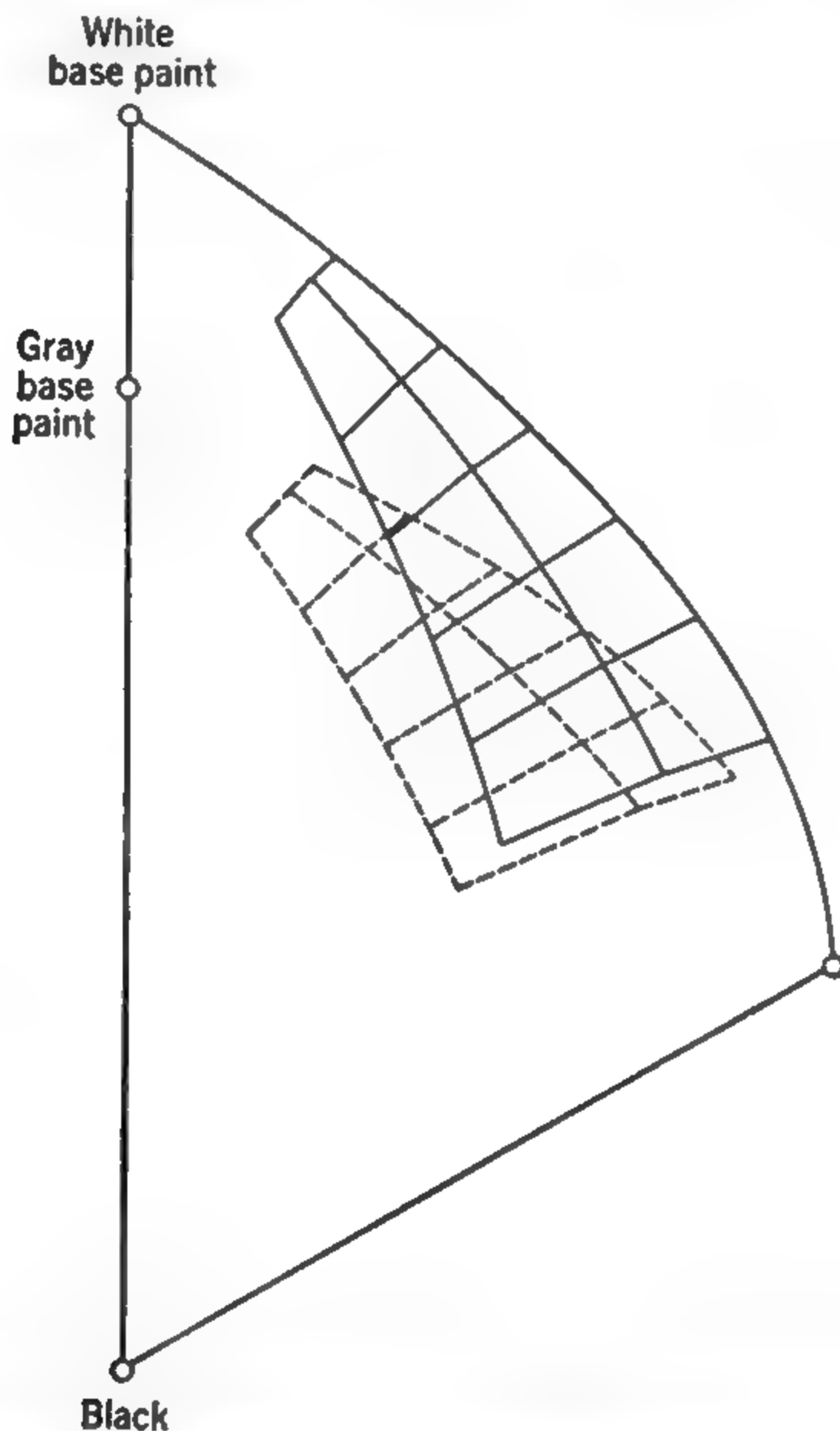


FIG. 52. Organization of the colors shown in the Colorizer of approximately the same hue as one of the 12 base chromatic paints. The departures toward middle gray are derived by cross-mixture of the neighboring chromatic paints, and each such departure is further developed by extensions with the white and the light gray base paints.

tions of the mixtures used in the extensions with the white and gray base paints are shown for each color chip, and these color sequences are shown on the removable vertical strips, each double sheet showing such strips for the entire 12-step hue circuit, the intermediates being shown on another double sheet. This method of presenting the chips tends to obscure the plan of arrangement and makes it difficult to find the nearest match for an unknown color; so the Colorizer is not par-

ticularly useful as a color atlas. However, the large hue contrast between each successive strip on the sheet presents the colors in a very favorable way, tending to make the customer like them, and the clear directions for compounding these mixed paints from the base paints makes it easy for the dealer to supply exactly the right quantities of them to produce the color of the customer's choice. The Colorizer is intended primarily for use by paint dealers to assist them to sell these particular base paints which are widely available.

A set of 648 "Deep Tone" colors is also available from Colorizer Associates to supplement the 1322 chips of the Colorizer.

3. *Plochere Color System*, Gladys and Gustave Plochere, 1820 Hyperion Avenue, Los Angeles 27, California, 1948. The Plochere Color System consists of 1,248 rectangles (3 by 5 inch) of mat-finish spray-painted cards, each identified by a serial number. The same colors are also available in a 52-chart loose-leaf binder, each chart showing 3 series of 8 paper rectangles (about $\frac{3}{4}$ by 2 inch), each identified by name and serial number. There are 26 hues corresponding to the Ostwald 24-point hue scale plus two additional at $24\frac{1}{2}$ and $1\frac{1}{2}$ on this scale. For each of these 26 hues there are 5 shades produced by adding progressive amounts of mixtures of base paints until an approach at constant hue is made to a near-black. For each member in these six-point series ($26 \times 6 = 156$ mixed paints) there are 7 tints produced by adding progressive amounts of white paint making up the $156 \times 8 = 1,248$ paints of the system; see Fig. 53. In the book accompanying the box of cards there are given for each serial number the "hue" notation, shade and tint, and the proportions by volume of the paints used, and, although the sampling near white is excessive compared to dark and saturated colors, the system is of some use as a set of color standards for general purposes and is widely used by decorators and interior painters. The Munsell notations of these 1,248 colors have been published by Middleton (1949). The Plochere color system is intended as an aid to color matching and selection of color harmonies. The 3-by-5-inch cards are widely used as reference material simply because they are at the same time inexpensive and technically good.

4. *Nu-Hue Color Coordinator*, Martin-Senour Company, 2520 South Quarry St., Chicago 8, Illinois, 1949. The Color Coordinator developed by Carl Foss is based on a pentagon-shaped chart showing as circles around the edge the colors of 10 toners of high saturation, and showing in the interior also as circles 5 toners of moderate saturation at the apices of an inner pentagon, and 1 grayish brown at the center, making 16 toners in all. Between the 35 neighboring pairs of the 16 starting colors of the base paints (circles) are rectangular chips show-

ing the colors produced by mixing the starting colors in equal parts. Between the 20 triads of the starting colors (shown as circles) are 20 more equal-part mixtures of the 3 components shown as triangles, making 71 color chips in the base chart. Six additional charts are

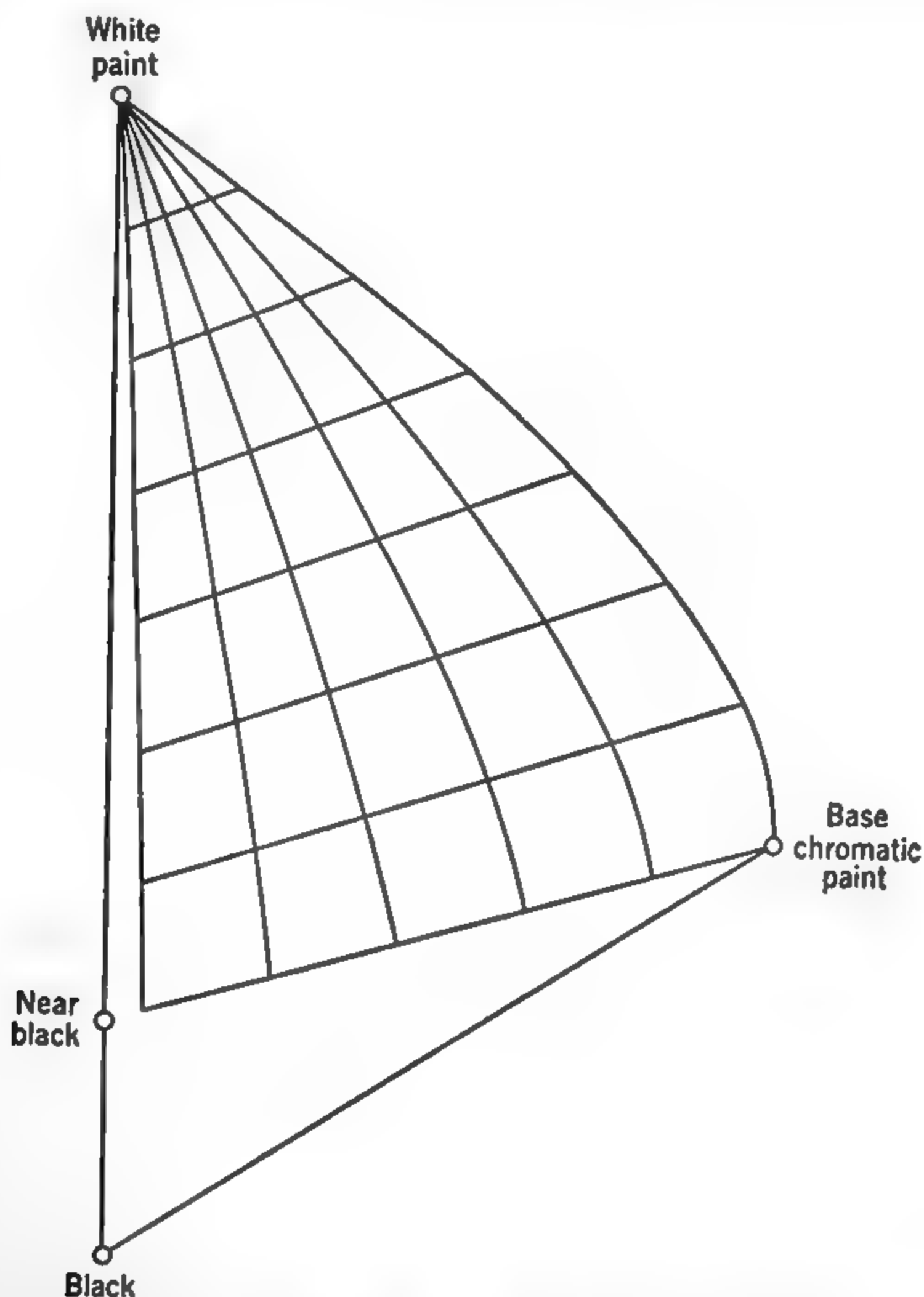


FIG. 53. Organization of the colors derived from one of the 26 base chromatic paints of the Plochere color system. The Nu-Hue Color Coordinator has a similar organization.

produced from the same toner ratios as on the base chart by adding successive amounts of white paint, making $7 \times 71 = 497$ chips in all. These extensions with white paint give an organization similar to that of the Plochere color system; see Fig. 53. These colors are also available in a sample book of 3-by-5-inch chips. The 10 high-saturation toners of this system have been chosen for maximum gamut rather than permanence. Nearly any color desired for interior or exterior

painting can be produced by mixture of not more than 3 of the 16 starting paints plus white paint. The particular merit of this collection of color chips is that the equal-parts mixtures shown provide a painter with the tinting-strength information about these particular base paints that he requires to make from them a rapid and accurate interpolation by mixture to the desired color. An abridgement of the Nu-Hue Color Coordinator is sold under the name of Professional Color Guide.

Color-Mixture Systems. The basic idea of color-mixture systems is to show in the form of material standards the sequences of colors related either to manipulation of the controls of a tristimulus colorimeter or to variation in simple ways of the proportions of sector areas on a Maxwell disk. There are several reasons why these color sequences are of interest. In the first place, the tristimulus colorimeter is the instrument on which the CIE standard coordinate system for colorimetry is based. The colorimetric variables associated with that system (daylight reflectance, chromaticity coordinates, dominant and complementary wavelength, purity) are used for expression of that most fundamental color datum, the spectrophotometric curve. Anything as fundamentally simple as this is explored out of pure curiosity. Constant chromaticity, for example, corresponds to keeping the tristimulus values in the same proportion.

But there is more than mere scientific curiosity back of color-mixture systems. Long before adoption of the CIE coordinate system the Maxwell disk was used as a basis for developing color systems. Colors of constant dominant wavelength, for example, can be obtained on the Maxwell disk from any chromatic sector (like red) combined with an achromatic sector (white, gray, or black) merely by varying the proportion of the two sectors. Chips painted to match the colors so produced can also be easily checked for self-consistency on the Maxwell disk.

Another reason for interest in colors of constant chromaticity is that color sequences of this sort form a large part of our everyday experiences. The shape of objects is rendered apparent to us largely through shadows and shading. The part of the object toward the light is the most strongly illuminated, the part turned somewhat away from the light source is less strongly lighted, and that turned completely away is scarcely illuminated at all, receiving only light reflected from other objects in the room. These three areas of an object, the light, the half-light, and the shadow, correspond closely to colors of constant chromaticity, if the surface of the object is uniformly covered with the same mat paint. Thus, many times a day, when we perceive the shape

of a solid object in our field of view, we are responding to a complex stimulus made up of aperture colors of nearly constant chromaticity. Such colors are sometimes said to form a shadow series. If a color system were based upon groups of such shadow series, we might at least expect such color sequences to take on a familiar appearance since we see them constantly.

Finally, a more practical reason for being interested in color systems organized on the basis of mixture by averaging on the Maxwell disk is its connection with color printing by the half-tone screen process. Colors produced by combinations of a chromatic ink with a black ink correspond to this kind of mixture. The 3 colors so averaged are that of the chromatic ink, the black ink, and the white of the paper. There is no subtractive mixture because the black dots completely hide any part of the chromatic dots on which they fall. Furthermore, as has been shown by Hardy and Wurzburg (1948), even screen-plate printing by means of the so-called subtractive primaries, magenta, yellow, and cyan, can be analyzed in terms of an eight-primary mixture by averaging. The 8 primaries are the white of the paper, the magenta, yellow and cyan parts of the paper covered by only one layer of ink, the red, blue and green parts of the paper covered by pairs of layers (magenta-yellow, magenta-cyan, and yellow-cyan), and the black parts of the paper covered by 3 layers of ink (magenta-yellow-cyan). Color-mixture systems thus apply directly to printing by the half-tone screen process.

1. *Color Standards and Color Nomenclature* by Robert Ridgway, A. Hoen & Co., Baltimore, Md., 1912. *Color Standards and Color Nomenclature*, now out of print, contains approximately 1,000 named color samples, each sample being a 1-by- $\frac{1}{2}$ -inch rectangle of mat-painted paper. The samples are arranged on each page with light samples at the top, grading down from white through 8 steps to black at the bottom. Each column shows 7 chromatic colors of constant dominant wavelength obtained by painting matches for 3 rotary mixtures of a chromatic color with white, and 3 mixtures of the same color with black. These 7 colors are usually seen as having about the same hues. There are 35 different dominant (or complementary) wavelengths represented, the wavelengths being so chosen that the hue spacing is perceived to be approximately uniform. The first series of columns shows colors of maximum purity, then there are 4 similar series, each stated to be developed by disk mixture with black and white from colors themselves progressively dulled by disk mixture with neutral gray. However, there is some evidence (Foss, 1949) that colorant mixture was used instead of strict reliance on the color-

mixture basis stated. Figure 54 shows by straight lines the intended organization. The color names are listed alphabetically, and the corresponding sample located by giving the plate number, column number,

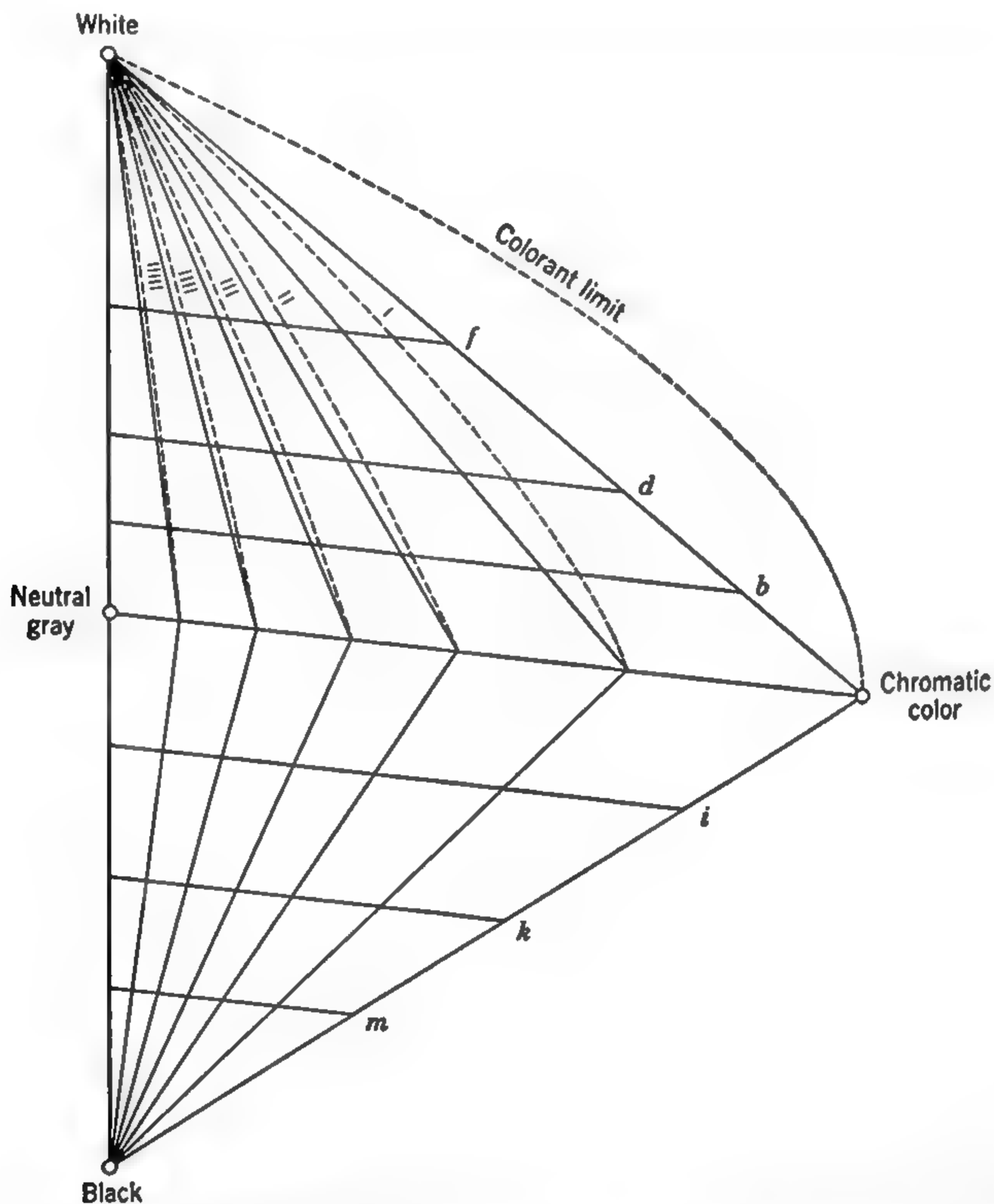


FIG. 54. Intended organization of the colors of one of the 35 dominant (or complementary) wavelengths of the Ridgway collection shown by straight, solid lines. There is some indication, however, that the actual chips of the collection follow the curved dotted lines instead.

and row letter. Munsell notations have been published (Hamly, 1949). There are no near-whites shown, but the coverage of near-grays is excellent. These charts have been widely used by naturalists for the specification of the colors of rocks, soils, plants, flowers, insects, and birds.

2. *Color Harmony Manual*, Color Standards Department, Container Corporation of America, Chicago, Ill., 1st Ed., 1942; 2nd Ed., 1946 (no longer available); 3rd Ed., 1948. In the second edition of *Color Harmony Manual* the 680 chips (1 inch square) are mounted on 12 double-triangle charts, one triangle showing colors of constant dominant wavelength, the other those of the complementary wavelength. Each sample is identified by the Ostwald notation in which dominant (or complementary) wavelength is indicated by a number on the Ostwald hue scale (1 to 24), and the black content, white content, and full-color content are indicated by double-letter symbols (*pa*, *na*, *pe*, and so on). The chips labeled *pa* are pigment approximations to the Ostwald semi-chromes. There are 28 colors of each dominant (or complementary) wavelength which together with the 8-sample gray scale make the $(24 \times 28) + 8 = 680$ colors. The color chips have been prepared by applying a pigmented film to a base of clear transparent cellulose acetate, so that each chip has a dull side and a glossy side, both of which are given the same Ostwald color designation. The colors have been chosen not only to have constant dominant wavelength for each color of the same number on the Ostwald hue scale but also so that the chips in any vertical column have constant purity. These vertical color series, being of constant chromaticity, are known as shadow series. The series connecting each *pa* chip with black and white, however, do not follow the laws of disk mixture, but were made by Foss (1944) to cover the maximum gamut possible with the pigmented coating used; see Fig. 55. The colors of the dull sides of the chips have been measured spectrophotometrically at the Electrical Testing Laboratories (Granville and Jacobson, 1944) and have been shown to correspond to the Ostwald theory more closely than any previous collection of standards. The chips are easily removable from the mounting to facilitate comparison with unknown colors and for combination with each other in the engineering of color harmonies. The double-triangle form of the Manual emphasizes the important single-hue and complementary-hue harmonies.

In the third edition the chips take the form of $\frac{7}{8}$ -inch hexagons. Charts of 6 additional dominant wavelengths (Ostwald hue numbers $1\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$, $12\frac{1}{2}$, $13\frac{1}{2}$, and $24\frac{1}{2}$) have been provided to improve the coverage in the commercially important color ranges. Extra shadow series of 7 steps each are provided for 12 charts (Ostwald hue numbers 1, 2, 3, 5, 7, 10, 13, 15, 19, 22, 24, and $24\frac{1}{2}$), and extra equal-black-content series, just darker than the light clear series, are provided for 4 charts (Ostwald hue numbers $24\frac{1}{2}$, 1, $1\frac{1}{2}$ and 2) for the same reason, bringing the total number of colors in the third edi-

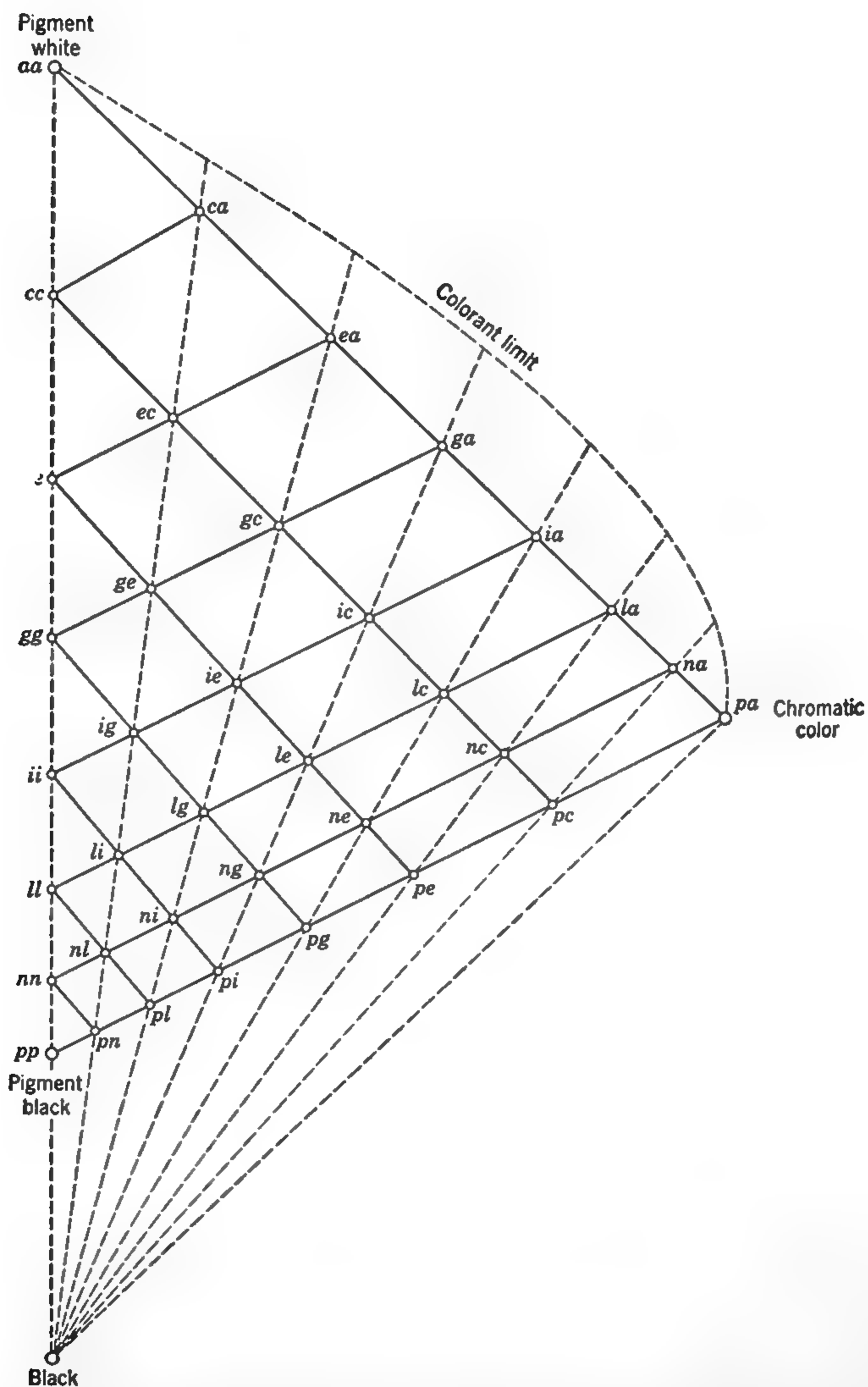


FIG. 55. Organization of the colors of one of the 24 Ostwald dominant (or complementary) wavelengths shown by straight, solid lines. The curved dotted line indicates approximately the deviations introduced into the *Color Harmony Manual* to extend the gamut to higher purities for a given daylight reflectance.

tion to 943. Each chip of this edition is also available separately. The colors of these 943 chips have been measured spectrophotometrically, both glossy and dull sides, by Granville, Foss, and Godlove (1950).

The Manual is designed, as the name implies, chiefly to promote the knowledge and study of color harmony and color coordination in design (Jacobson, 1948). Because of the fact, however, that there is a shiny and a dull side to the chips, both of which are easily available for comparison with unknown colors, the Manual is exceptionally well suited to serve also as a set of color standards for general use. The glossy sides of the chips extend the color gamut to much darker and more saturated colors than is possible with mat chips. The acetate lacquer used as a binder has made the light colors of this collection exceptionally free from yellowing with age. Foss has determined the proportions of the Martin-Senour Nu-Hue Custom-Color toners required to match each chip of this collection, and this information has been made available to paint dealers by the Container Corporation of America, together with daylight reflectance of each chip. The master standards of the first and second editions, measured spectrophotometrically by Granville and Jacobson (1944), are on file at the National Bureau of Standards.

Screen-Plate Printing Color Systems. By using progressive amounts of coverage by the screen-plate printing process a series of colors extending from the white of the paper to the color of the ink, printed solid, may be produced. This series corresponds to color mixture by juxtaposition of small dots unresolved by the eye. By printing rows of such series of a second colorant on top of columns of such series by the first colorant, a two-dimensional array of the mixtures of the two colorants with the white of the paper can be produced. This general method of developing by screen-plate printing systematic color variations has been used countless times. Perhaps the most notable older examples of printed charts are those of Huebner which show all combinations of screen densities for three primary inks and also include all combinations when a fourth (black) ink is used. Ives prepared similarly complete charts with all colors related so as to yield a sampling of color space. The color sequences so produced correspond to colorant mixture in so far as the dots overlap, but the sequences produced by nonoverlapping dots illustrate color mixture. The screen-plate printing color systems thus illustrate a combination of colorant-mixture and color-mixture concepts.

Some of our most valuable color charts are produced by printing. They provide reference color materials at reasonable cost that are still

sufficiently uniform and permanent to be of practical use. All the currently used charts are variations of the idea originally carried to its logical conclusion by Huebner and Ives.

1. *A Dictionary of Color*, by A. Maerz and M. Rea Paul, McGraw-Hill Book Co., New York, 1st Ed., 1930; 2nd Ed., 1950. This dictionary contains 7,056 different color samples printed on semi-glossy paper, 6,048 of which are $\frac{1}{2}$ -by- $\frac{5}{8}$ -inch rectangles, and 1,008 of the darker of which are $1\frac{1}{16}$ -by- $\frac{5}{8}$ -inch rectangles. There are 8 chromatic and 7 base gray pigments, and the mixtures are accomplished by half-tone screen printing. The hue circuit is covered in 8 intervals: purplish red to red, red to orange, orange to yellow, yellow to green, green to blue-green, blue-green to blue, blue to purplish red, and purple to purplish red, the last interval giving again the hues shown in the last part of the preceding interval, apparently because the gamut can be considerably increased toward lighter and more saturated colors in this way. Each hue interval is represented by a series of 8 charts. The first chart in a series shows the colors resulting from two printing impressions on white paper. One of the impressions results in 12 vertical columns of identical colors, the first column being unprinted, and successive columns being printed with increasing coverage of one base ink over to the twelfth column which is printed solid. The other impression corresponds to 12 horizontal rows of identical colors, the first row being unprinted, and successive rows being printed with increasing coverage of the base ink next in the hue series down to the twelfth row which is printed solid. By this method the upper-left corner is white unprinted paper (daylight reflectance 86 percent), and the lower-right is the color produced by printing solid one base ink over the other. The samples intermediate on the chart represent colors that are intermediate, and the coverage of the half-tone screen plates has been so adjusted that the color steps between successive samples are, in general, about equal. Each chart shows a progression of hues from the upper-right to the lower-left corner, whereas the hues of the samples along the straight lines through the upper-left corner are roughly constant. The second chart in a series differs from the first by being laid down on a solid printing of the first base gray ink (daylight reflectance about 74 percent); the third chart is laid down on a darker gray solid printing; and so on down to the seventh and eighth charts of the series showing the dark colors. The seventh and eighth charts differ from the preceding six by showing only 6 columns of 12 samples each instead of 12 columns. The first two hue series (purplish red to red, red to orange) differ from the remaining series by showing only 6 horizontal rows instead of 12. Thus, there are $7 \times 6 = 42$ plates of 144 colors each,

and $7 \times 2 = 14$ plates of 72 colors each, making up the $6,048 + 1,008 = 7,056$ colors. Figure 56 shows the organization of colors of the 8 base chromatic inks.

The daylight reflectances of the series of gray base inks are given as 74, 67, 48, 38, 28, 20, and 10 percent, respectively. The differences in

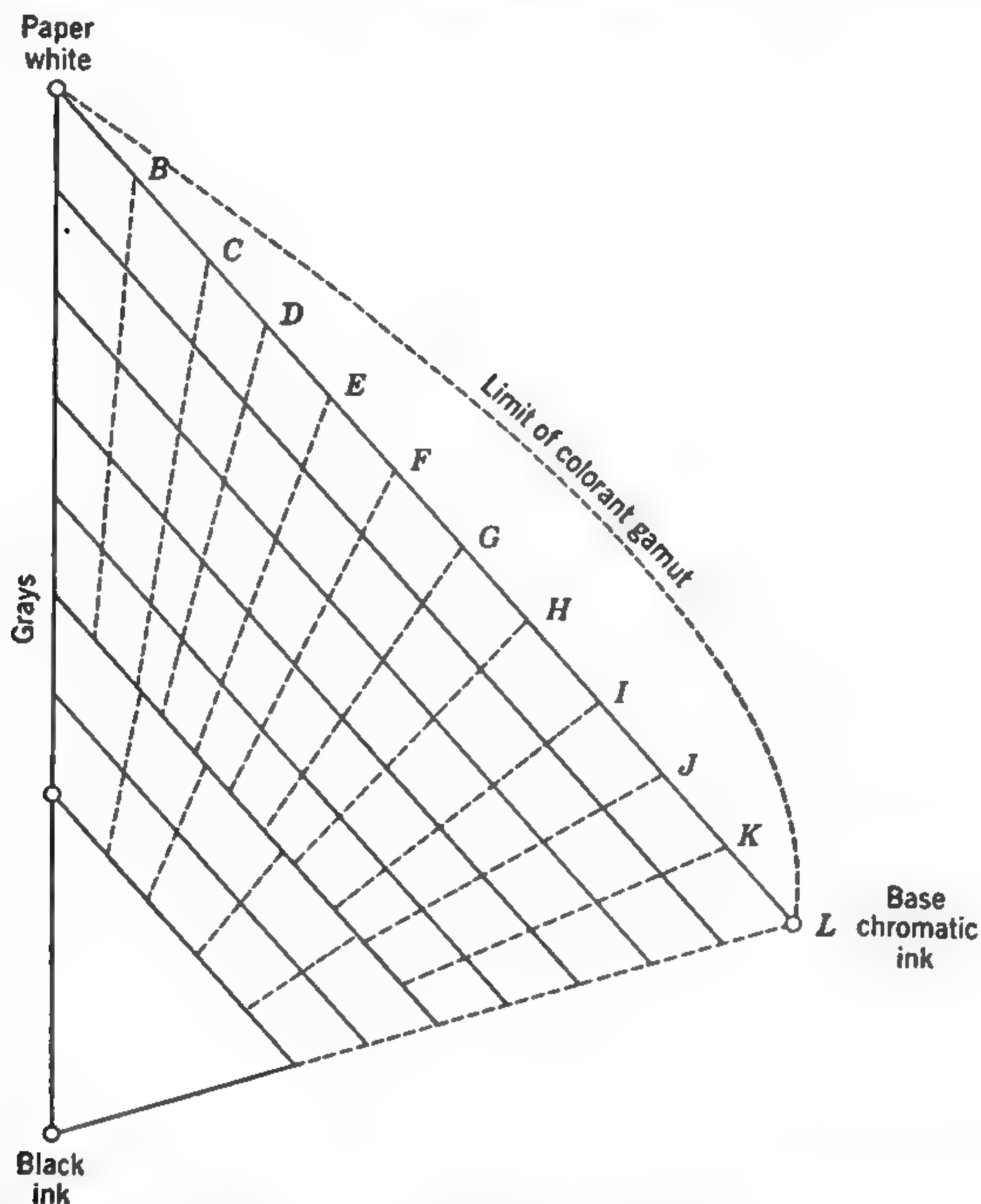


FIG. 56. Organization of the colors derived from one of the 8 base chromatic inks of the *Maerz and Paul Dictionary* by printing it in progressive amounts of coverage on each of a series of gray papers, ranging from white to dark gray.

lightness from chart to chart ordinarily perceived between the dark charts are considerably greater than those between the light charts, and the purities of the light colors achieved by this method of development are somewhat inferior to those possible by other methods. Furthermore, many of the dark red and purple samples exhibit a different hue by specular reflection (so-called bronziness, Foss, 1938; Buc, Kienle, Melsheimer, and Stearns, 1947). These are minor defects,

however. The sampling of the strongest colors is excellent, and the systematic arrangement makes it easy to find the color wanted. An alphabetical list of about 4,000 color names is given, together with a key (plate number, row number, column letter) by means of which each corresponding sample may be found in the charts. These samples are also identified by name in the charts themselves. Munsell notations of the red to yellow colors have been published by Nickerson (1947). The large number of the samples makes the steps between successive colors so small that interpolation often is not necessary. The *Maerz and Paul Dictionary of Color* may be used as a collection of practical color standards in nearly every field. Furthermore, the scholarship and thoroughness of color-name treatment have given this work a wide reputation as the foremost authority on color names.

2. *Villalobos Colour Atlas*, El Ateneo, Buenos Aires, Argentina, 1947. *Villalobos Colour Atlas* is available in the United States through Stechert-Hafner, New York. It shows 7,279 color samples, 1 cm square, with a 4-mm hole cut out of each to facilitate color comparisons. The samples are produced by multiple impressions with half-tone screen plates. There are 38 base chromatic inks adjusted so that the hue intervals are approximately equal, and the colors produced by inks 19 hue steps apart are intended to be optical complementaries. The color series from each chromatic ink to white and to black and that from black to white are produced in 2 series each, the intermediate points being made by bulk mixture of printing ink toned closely to constant hue; see review by Foss (1949). This plan gives a considerably increased gamut and makes the dot pattern produced by the half-tone plates very inconspicuous. Furthermore, the multiple overprinting increases the gloss, thus further increasing the color gamut. All the colors produced from each base chromatic ink are shown on the same chart and are such as are perceived under usual conditions to have nearly the same hues. There is a gray scale ranging from black to white by means of 20 steps well adjusted to perceptual equality, and for each of the 38 hues 19 series of samples are shown, each sample in the series having closely the same daylight reflectance as the corresponding gray. Except for the samples of very high and very low reflectance these series consist of 12 samples each, ranging from the equivalent gray up to the maximum purity obtainable with the base chromatic ink at that reflectance by the method used. All these 191 samples of nearly constant hue are shown on a single chart (see Fig. 57) which together with the 21-sample gray scale makes up the total of $(38 \times 191) + 21 = 7,279$ colors.

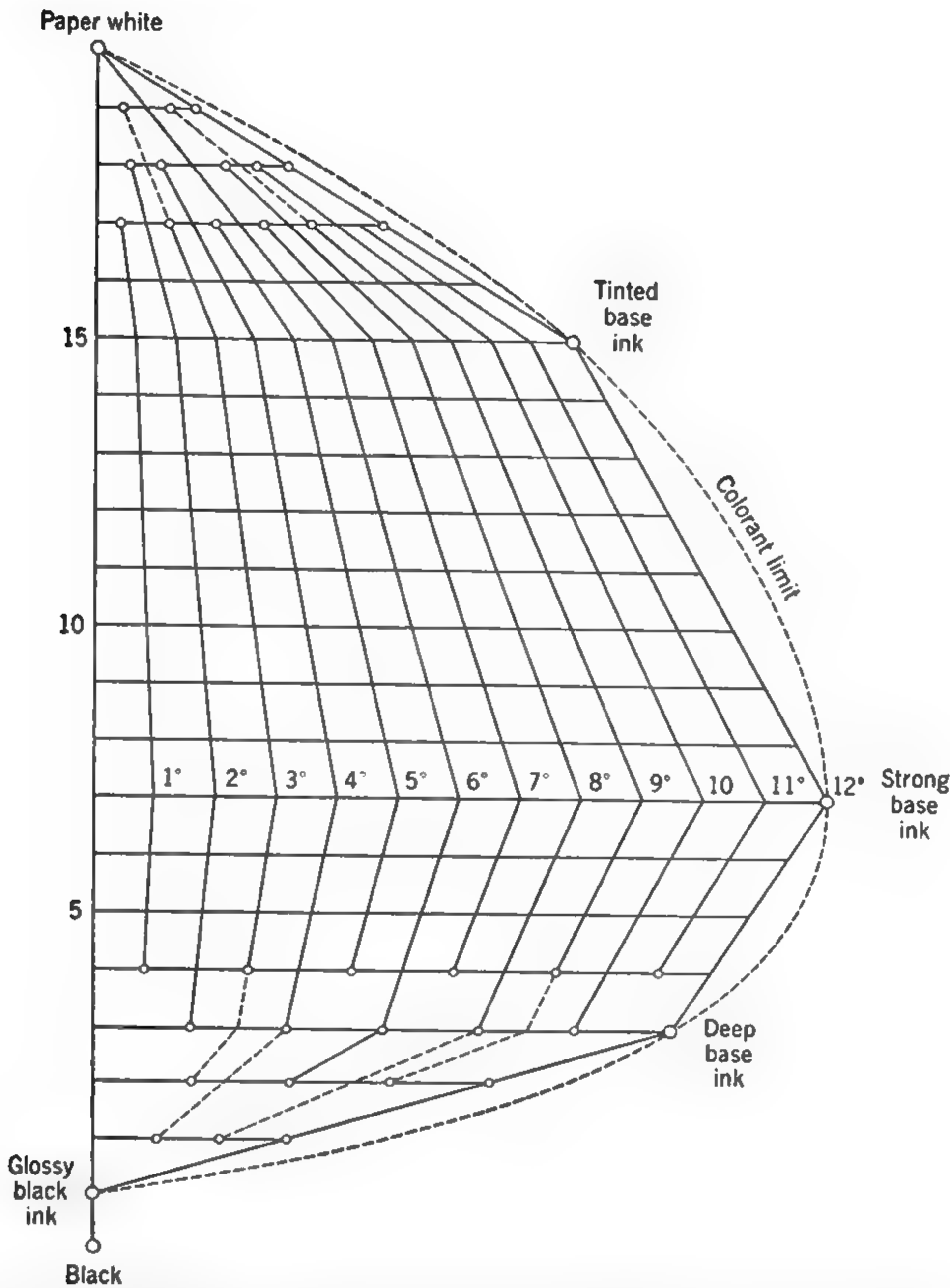


FIG. 57. Organization of the colors derived from one of the 38 base chromatic inks of the *Villalobos Colour Atlas* by two toned extensions with white ink combined with multiple impressions in progressive amounts of coverage of black and gray ink.

Each chart is identified by letters derived from English hue names (ruby, scarlet, orange, yellow, and so on). Each color is identified by the hue letters, by a number called "lightness value," indicating the member of the gray scale to which it is equivalent (or "isovalent"), and by a number called "degree of chromaticity," indicating its position on the horizontal scales of constant-hue and constant-lightness value. Each copy of the *Atlas* is numbered and stamped by the author, C. Villalobos-Dominguez, as a guarantee that the charts have been verified as identical to those of any other copy equally guaranteed. A few of the lightest colors have become noticeably too yellow to accord with their intended hue. These have been crossed off with a black mark, and occasionally a sample in the same neighborhood has been cut out.

The large number of samples and the excellent uniformity of the spacing usually make it possible to find a close match for any unknown surface color so that interpolation is seldom necessary. Furthermore, a highly precise specification by interpolation is facilitated by the uniformity of the spacing. Precision of the result is limited chiefly by the small size of the samples. This collection of color samples amply fulfills the requirements for an atlas of colors and deserves to find wide application. The explanatory text is given both in English and in Spanish.

3. *Basic Color*, by Egbert Jacobson, Chicago, Paul Theobald, 1948. *Basic Color* includes two complete renderings of the 24-hue Ostwald system, 28 colors to the hue, plus 8 grays including black and white, making $(24 \times 28) + 8 = 680$ colors in each rendering. The printed areas are $\frac{5}{16}$ -inch squares.

4. *The American Colorist*, by Faber Birren, The Crimson Press, 40 Gorham Ave., Westport, Conn. This pamphlet contains 550 colors, including a 10-step gray scale, and 45 colors of each of 12 hues organized on the Ostwald system. Although these charts serve as an aid in the design of effective color schemes and provide a means of describing colors in everyday terms, they were prepared primarily for horticulturists and members of garden clubs.

Summary of Colorant-Mixture and Color-Mixture Systems. Each of the color systems so far discussed is organized on a different basis. The methods of developing various colors systematically from a chromatic colorant combined, either additively or subtractively, with a black and a white colorant have been shown graphically on schematic diagrams in which color mixtures are indicated by straight lines between the points representing the colors of the components; see Figs. 51 to 57. Combination of colorants by screen-plate printing is ap-

proximately equivalent to color-mixture combination and is also represented on these diagrams by straight lines; likewise colorant mixtures involving black paint. Colorant mixtures involving white paint, however, are considerably lighter and more saturated than the corresponding color mixtures. They are shown by lines curved so as to be convex upward. On these diagrams the spacing from black to white corresponds approximately to perceptually uniform spacing. Spacing in other directions has no precise meaning on these diagrams but is shown approximately uniform to correspond with the presumed intentions of the makers to have the color spacing perceptually about uniform, unless precise information to the contrary is given by them. The diagrams, therefore, do not show how well the makers have succeeded in this spacing; they show only the basic limitations springing from the method of developing the systems. Note also that the actual arrangement of color spots on the page of the color chart itself is in every case quite different from the arrangements of intersections of lines on these diagrams.

Figure 51 indicates that the plan of organization of Nu-Hue Custom Color System permits the avoidance of overcrowding in any part of the color solid sampled. Figure 53 shows that the Plochere Color System has unnecessarily close spacing for near-whites. The Nu-Hue Color Coordinator is organized along similar lines and has the same defect. All three of these colorant-mixture systems fail to show any very dark colors, but this is because the chips are mat finish instead of glossy.

Figure 54 shows by solid lines the organization said by Ridgway to be followed in the construction of his color standards. Note that there cannot be any very near-whites or very near-blacks represented in a collection organized in this way. Probably the gamut of light, saturated colors in the Ridgway collection, is better than is indicated by the straight lines in Fig. 54, because there is evidence that some use of colorant mixture (dotted lines) was made.

Figure 55 shows by straight lines the organization of the Ostwald system to which the color charts in Jacobson's *Basic Color* conform closely, and on which the *Color Harmony Manual* and Birren's *American Colorist* were based. The color series generated on the Maxwell disk by keeping white content constant are shown as straight lines such as *nn* to *na*, or *ee* to *ea*; and those corresponding to constant-black content are shown by such straight lines as *cc* to *pc*, or *gg* to *pg*. The shadow series correspond to disk mixture with an ideal-black surface of zero reflectance such as might be obtained by rotating a disk with an open sector in front of an enclosure lined with black velvet. These are also shown by straight lines, the dotted lines passing through the

point representing ideal black. Note that the pigment black, *pp*, of the Ostwald system is supposed to be far from the ideal black; its reflectance is intended to be about 3.5 percent instead of zero.

The *Color Harmony Manual* was developed on a plan involving purposeful deviations from the Ostwald system to increase the color gamut. Thus, in the Manual only the shadow series are intended to correspond to the straight lines shown in Fig. 55. The other series are such as would be represented by somewhat curved lines between the same end points (Foss, 1944), such as the curved dotted line showing the upper colorant limit.

Figure 55 indicates that a collection of color chips based upon the Ostwald system may be expected to have rather sparse coverage of light colors and near-whites compared to the coverage near the basic black colorant. This is evident in all color charts developed by a symmetrical application of the Ostwald plan. The third edition of the *Color Harmony Manual* largely corrects this defect by insertion of extra shadow series and extra equal-black-content series.

Figure 56 shows that the *Maerz and Paul Dictionary of Color* avoids overcrowding pretty well in any color region but is deficient in near-blacks and in light, saturated colors. Figure 57 shows that the *Villalobos Colour Atlas* has excellent coverage and is well organized to avoid overcrowding anywhere in the constant-hue plane.

Color-Appearance Systems. It should be noted that all the colorant-mixture and color-mixture systems so far considered, even the well-designed *Villalobos Colour Atlas*, sample the color solid nonuniformly; that is, certain color regions are represented by chips whose colors differ only slightly, and others are scarcely, if at all, represented. One of the basic ideas of a color-appearance system is to provide uniform sampling of the psychological color solid. Another is to exemplify the psychological attributes of color perception—hue, lightness, and saturation; that is, to supply series of chips all of whose colors are perceived under ordinary conditions of illumination and view to have the same hue, or the same lightness, or the same saturation. An overwhelming urge to organize systems of color chips particularly on the basis of constant hue is felt by the designers even of colorant-mixture systems. Most of the collections of chips so far considered follow this plan. Of course, such mixtures between a chromatic color and white or black result in only an approximation to constant hue. In a color-appearance system the attempt is made to follow this urge to its logical conclusion and produce groups of chips that will look right with no apologies needed.

In the *Villalobos Colour Atlas*, there have been produced, by screen-plate printing, color chips illustrating series of colors very nearly of constant hue and very nearly of constant lightness. Furthermore, the spacing is perceptually nearly uniform. This accounts for the superior utility of the color notation derived from this Atlas; it is far more than the usual arbitrary color designation corresponding to the screen densities. Thus the Villalobos "lightness value" of a color chip correlates closely with the lightness of the color perceived to belong to the chip under usual observing conditions. Similarly Villalobos hue correlates well with the perceived hue. At any one hue and lightness value a similar correlation exists between Villalobos "degree of chromaticity" and saturation of the perceived color, but the same correlation does not extend in general to more than one scale of "degree of chromaticity" at a time. Thus 12 degrees of chromaticity at lightness value 19 usually indicate a color perceived to have much less saturation (by a factor of 3 or 4) than 12 degrees at lightness value 10. "Degree of chromaticity" is partly a colorant concept because 12 degrees always correspond to the extreme of the printing-system gamut at that hue and "lightness value." The *Villalobos Colour Atlas* thus falls somewhat short of being a color-appearance system.

1. *Munsell Book of Color*, Munsell Color Co., 10 East Franklin St., Baltimore, Md., Vol. I, 1929; Vol. II, 1943. Two editions, library and pocket size, of the *Munsell Book of Color* are available, both in loose-leaf binding. The pocket edition has smaller chips but is arranged for practical color-specification work. Volume I consists of 20 charts, one for each of 20 different hues with equal visual spacing, showing colors varying in lightness (Munsell term: value) and saturation (Munsell term: chroma). Volume II consists of 20 similar charts of the intermediate hues. The samples of the pocket-edition charts are $\frac{1}{2}$ -by- $\frac{5}{8}$ -inch rectangles of painted paper nearly mat and are arranged in rows and columns. It was intended that the samples in any one row should be perceived under ordinary viewing conditions (daylight illumination, middle gray to white surroundings) as equally light, and that the samples in any one column should be perceived under the same conditions as equally saturated. The colors progress from very light at the top of each chart to very dark at the bottom by steps that are intended to be visually equal; and they progress from achromatic colors, black, gray, or white, at the left side of each chart to saturated colors at the right by steps that are also intended to be visually equal. Each sample is identified by three symbols—the first indicating Munsell hue; the second, Munsell value; and the third, Munsell chroma, such as 3YR 5/10, indicating a Munsell hue of 3 yellow-red, a Munsell

value of 5/ (equally separated from black and from white), and a Munsell chroma of /10 steps away from the gray ($N\ 5/$) of the same value. The notation of the Munsell system, because it is based on uniformly spaced scales of color, is a useful tool for the formulation and solution of many colorimetric problems.

The samples of Volume I have been measured spectrophotometrically both at the Massachusetts Institute of Technology (Glenn and Killian, 1940), and at the National Bureau of Standards (Kelly, Gibson, and Nickerson, 1943). The samples of Volume II have been similarly measured at the Interchemical Corporation (Granville, Nickerson, and Foss, 1943), and many samples produced since 1943 have been measured by the Philadelphia Navy Yard in cooperation with the U. S. Department of Agriculture. From these measurements the luminous reflectance, Y , and chromaticity coordinates, x , y , for each chip have been computed (Nickerson and Wilson, 1950) for various light sources (daylight, incandescent, fluorescent). These results show that all chips of the same Munsell value have closely the same luminous reflectance for standard source C, and they permit similarly important conclusions to be drawn regarding the meaning of the chromaticity coordinates, x , y , to be discussed in detail later.

The Library Edition shows exactly the same colors as the Pocket Edition but has larger samples, $\frac{5}{8}$ by $\frac{7}{8}$ inch, and it shows them arranged in charts of constant chroma and charts of constant value, in addition to the charts of constant hue. It is more elaborate than required for determination of the Munsell notation of an unknown color, but it serves admirably to teach the organization of the psychological color solid and is considerably used in color education.

Separate charts from either the Library or Pocket Editions may be purchased; also complete or partial files of papers of each separate color in the form either of sheets or of disks slit along a radius with center hole for use as Maxwell disks for rotary mixture. There are other forms of charts, both with mounted and with removable chips.

The master standards of the *Munsell Book of Color* that have been measured spectrophotometrically have been deposited at the National Bureau of Standards. The Munsell Color Company also supplies the Bureau with a sample of each repainted color, each interpolated color, and each special color that is produced.

2. *Uniform sampling of the color-perception solid.* It is appropriate to show the organization of a color-appearance system on a graph in which each straight line corresponds to the locus of colors having the smallest number of just noticeable steps between the colors represented at the end points. Figures 58 and 59 are of this type of graph and

show the organization of the Munsell constant-hue chart and Munsell constant-value chart, respectively. On Fig. 58 the locus of colors producible by colorant mixture with black and white pigments would be indicated by curved lines, such as the dotted line shown. The Munsell

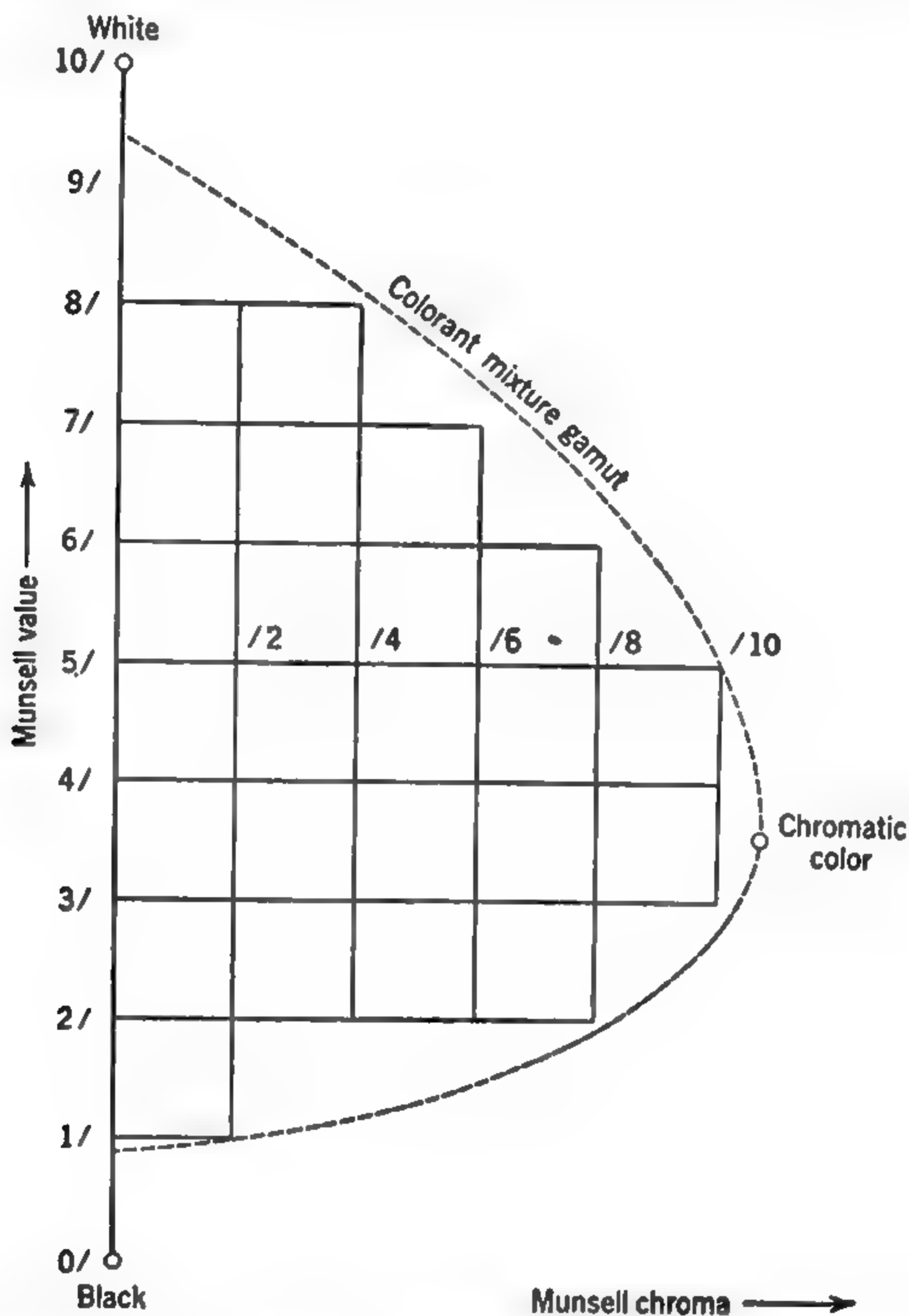


FIG. 58. Organization of the colors of constant Munsell hue in the *Munsell Book of Color*.

color system is built up at each hue by adding progressive columns of chromatic colors to the 9 neutrals of the same Munsell value, each column of chromatic colors departing from the equivalent neutral by two additional steps of Munsell chroma. Each row of colors of constant value is extended until the next interval of two chroma steps would extend beyond the colorant-mixture gamut. It is obvious from Fig. 58 that the sampling of the interior of the color solid in the con-

stant-hue planes is nearly ideal in the Munsell color system. The sampled points make a uniform rectangular array. It is equally obvious that charts developed by large uniform saturation increments can never hope to show the extremes of the pigment gamut of color. Only by chance, such as 5/10 and 1/2 in Fig. 58, will an even number

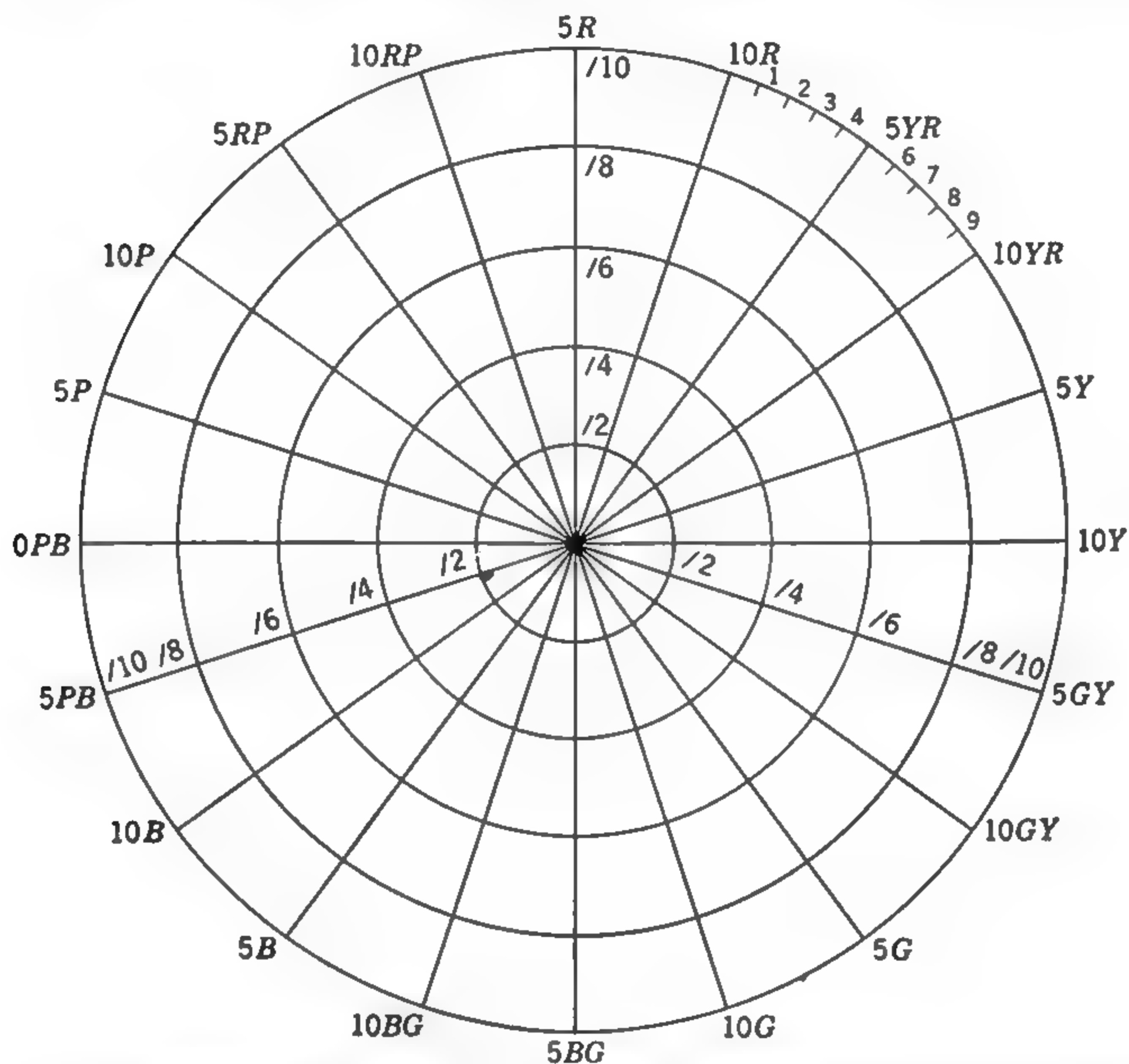


FIG. 59. Organization of the colors of constant Munsell value in the *Munsell Book of Color*.

of chroma steps coincide with the limit of the pigment gamut. This is a necessary defect of all collections of color chips built up by accretion of discrete steps from an interior point or points in the color solid.

Figure 59 shows the organization of the Munsell constant-value charts. The 100-point Munsell hue scale and notation are shown around the outer circle. Colors of constant Munsell hue are shown by the radial lines intersecting at the point representing neutral, or chroma /0. The hue scale is built up of 10 segments of 10 hues each, such as the segment from 1YR to 10YR, and the fifth hue of each of these segments is alternatively notated by the letters alone, thus 5YR is often written simply YR. The Munsell chroma scale up to chroma

/10 is shown along any radial line in Fig. 59, and the colors of constant Munsell chroma are shown by each of the 5 concentric circles. It will be noted that the sampling of the color solid near the black-white axis (chroma /0) is, by this plan of development, much more dense than that of the saturated colors represented far from the axis. This is a necessary defect of any collection of color chips organized on the radial plan; it is a defect possessed by most collections of color chips now commercially available, such as those referred to in Figs. 52, 53, 54, 55, and 57.

Our consideration of the relative merits of the various collections of color chips has so far been on the basis of extent of the gamut of colors and uniformity of the sampling within it. Most of the collections considered have shown groups of colors of approximately constant hue, and we see that this one feature of their organization alone has prevented the achievement of uniformity in the sampling. The chromaticity coordinates, x , y , of the standard CIE system form a nonradial basis for a collection of color chips at constant luminous reflectance, and even though the chromaticness spacing in the (x, y) diagram is far from uniform there would be some advantage in using the rectangular coordinate system of this diagram as a base for a system of color chips instead of the overworked polar system of coordinates. If we could get a rectangular coordinate system with nearly uniform chromaticness scales, lighter and darker colors could be formulated on this basis and an approximation to uniform cubic sampling of color space could be developed. This method of sampling still would probably not quite satisfy the ultimate condition that each color of the collection should contrast equally with each of its nearest neighbors. Note that each point in a cubic array is surrounded by 26 nearest neighbors, only 6 of which are distant by the length of the cube edge. Twelve are distant by a diagonal of the square forming the faces of the unit cube, and 8 are distant by a diagonal of the unit cube itself. In actual space the distances from each point to each of its nearest neighbors are thus far from equal. Note, for example, that the diagonal of a cube exceeds its side in length by 73 percent. In color-perception space it is to be expected that a cubic array would lead to corresponding discrepancies. We must look for another method of arraying points in space.

If equal-size spheres are close packed on a plane, their centers form a continuous triangular array of points. Each point is surrounded by 6 equally distant points, forming the apices of a regular hexagon. On top of the spheres whose centers form this hexagon, 3 other spheres can be fitted snugly in each of 2 positions, one position differing from

the other by 60° rotation about the vertical axis passing through the central point. By adding spheres, either all in the one position or all in the other, a second layer of spheres is packed onto the first layer. The centers of the spheres of this second layer form a uniform triangular array of points having the same spacing as the first layer. Similarly a third layer may be added, but now which position is chosen makes a difference. We may choose the position in which each sphere in the third layer is exactly over that in the first, or we may choose the other position. If every other layer in a hexagonal array is an exact repetition, we shall call the array of points two-layer close packing. The second we shall call three-layer close packing. Every third layer is an exact repetition. In three-layer close packing each point in space has 12 nearest neighbors, all equally distant.

The development of a collection of color chips modeled as closely as possible on three-layer close packing might proceed as shown in Fig. 60. Select as a starting color a middle gray (N) such as Munsell $N\ 5/$. This is easy. Then choose a yellow color appearing equally light as the gray and illustrating a step from this gray of some convenient size such as 2 steps of Munsell chroma. This is harder. It involves the judgment whether two colors of different chromaticity are equally light. However, we can take advantage of the colors of the Munsell book that already have been adjusted closely so as to appear equally light and choose Munsell $5Y\ 5/2$. These are steps (a) and (b) on Fig. 60. Then choose a color of hue toward reddish yellow appearing equally light that also contrasts equally with the first two. This is still harder. It involves a judgment of relative size of differences along divergent color paths, but it can be done with some precision, and so we get color YR as in step (c). Similarly by step (d) we derive a red color R contrasting equally with the neutral gray N and the color YR , and so on through steps (e) and (f) until in step (g) the green color G is chosen. At this point a geometrical difficulty of color perception may arise.

The color G has been chosen to be equally different from the neutral gray N and the bluish color B . But we hope that it is also different from the yellowish color Y by the same perceptual amount. Experiments so far conducted, however, indicate that this happy result will usually not quite be realized. The color difference YG will appear larger than all the other color differences (NY , NR , PB , BG) that have been adjusted to equality. This proves (Silberstein, 1942) that the geometry of color-perception space is not Euclidean. In color-perception space the ratio of the circumference of the hue circle to its diameter is greater than $\pi = 3.14 \dots$. We see by this result that it

is not going to be easy to make a collection of color chips each one of which differs in color from each of its nearest neighbors by color differences perceived to be equal.

Perhaps we cannot make this ideal collection of color chips. But we can make a far better approximation to this ideal than has ever been made before. So, putting aside the temptation to fly off into

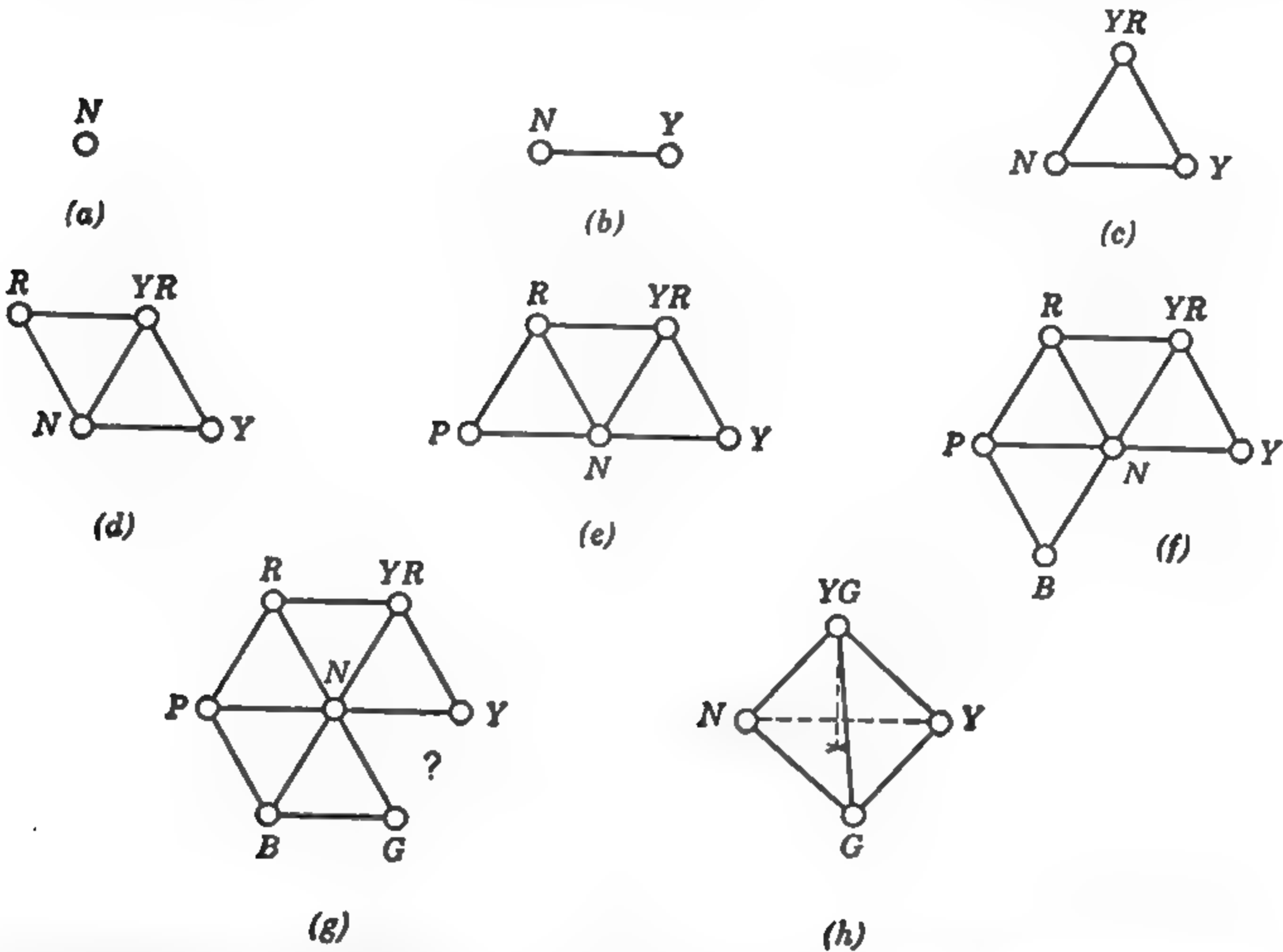


FIG. 60. Steps in the production of a collection of color chips organized according to three-layer close packing.

Riemannian geometry and curved space, we proceed as follows: Each of the steps, *BG*, *BP*, *PR*, and so on, must be lengthened slightly so that all of them are equal to the remaining step *GY*. All of them are therefore slightly larger than the radial distances *NR*, *NP*, *NY*, and so on. We go on by this method to all colors of the same lightness, fudging our way throughout the gamut of colors of this lightness producible by the pigments at our disposal. Then we are ready for step (h). We must choose a color lighter than the previous colors such that all its contrasts with *N*, *Y*, and *G* are equal. We represent these colors at the corners of a regular tetrahedron. Step (i) would be to find a lighter color equally different from *N*, *P*, and *B*; and step (j) would be based similarly on colors *N*, *R*, and *YR*. Thus alternate triangles in the base layer serve to generate the colors of the second layer.

By three-layer close packing, the triangles of the base layer not used to generate lighter colors would be used to generate darker colors. By continuing this process the entire gamut of the pigments at our disposal can be sampled with approximate uniformity. That is, the entire gamut except for an outer shell less than one step thick.

No collection of color chips is commercially available with this three-layer close-packing inner construction, but the idea of the color space to be so sampled is basic to the thinking of those who use the Munsell notation (Foss, 1949) with understanding. The color chips of the Munsell charts illustrate another kind of sampling of the same color space. Balinkin (1950) has used the Munsell colors in accord with an approximation to three-layer close-packing construction as an aid to the selection of harmonious colors for painting prefabricated houses. It may be that the next great advance in the systematic sampling of the color solid will be a collection of color chips organized on the three-layer close-packing principle.

Applications of the Various Color Systems. Table 14 summarizes the applications of the systematic collections of color chips used in American industry. The outstanding collections are the *Villalobos Colour Atlas*, the *Color Harmony Manual*, and the *Munsell Book of Color*.

The *Munsell Book of Color* is preeminent for color specification and for teaching what is meant by uniform color space. The Munsell color chips have been measured, and specifications for chips on the ideally smoothed Munsell system have been published (Newhall, Nickerson, and Judd, 1943). The continued availability of the Munsell samples, both separate and in chart form, and closely approaching these ideal specifications, is ensured by the existence of a nonprofit corporation, the Munsell Color Foundation, set up to hold the stock of the Munsell Color Company and to "further the scientific and practical advancement of . . . knowledge relating to standardization, nomenclature, and specification of color." Visual interpolation along the color scales of the *Munsell Book of Color* is easy, and the Munsell notation is easy to understand and convenient for recording the results of such interpolations. This notation is recommended in American Standard Z58.7.3/15 (1951).

The preeminence of the *Munsell Book of Color* for specification results from shortcomings in the other systems. Thus, the *Villalobos Colour Atlas* provides excellent hue and lightness scales, but the saturation scales, individually uniform, show steps of different sizes, depending on the hue and the lightness. The meaning of the Villalobos notation is thus not immediately clear but has to be learned. Furthermore, the *Villalobos Colour Atlas* cannot be expected to remain avail-

Table 14. Applications and Principal Defects of Systematic Collections of Color Chips Used in American Commerce

Color System	Principal Use	Secondary Use	Principal Defects
Nu-Hue Custom Color	Sale of prescription-mixed paint	Restricted gamut in red-purple to blue hues
Colorizer	Sale of paint	Aid to color matching by customer	System rather complicated
Nu-Hue Color Coordinator	Aid to color matching by painter	Aid to color matching by customer	Nonuniform color spacing
Professional Color Guide	Aid to color matching	Too few colors
Plochere	Inexpensive color chips	Aid to color matching	Nonstandardization of paint components
Maerz & Paul	Definition of color names	Color specification	No light, saturated, or near-black colors
Villalobos	Color specification	Selection of harmonious colors	Small size of color chips
Ridgway	Color nomenclature in biology	Color specification in biology	No near-black or near-white colors
Color Harmony Manual	Selection of harmonious colors	Color specification	Ostwald notation used not adapted to convenient interpolation or to easy visualization of the color
Munsell	Color specification	Color education	Many of the strongest colors not shown

able with the present colors because of the difficulty of maintaining the same colors in any new production of the entire collection. Munsell repaints, on the other hand, are done color by color. The Ostwald notation used by the *Color Harmony Manual* is not purely a color notation but is rather a notation indicating color relationships within a particular collection of standards. Note that the same Ostwald notation is used for both sides of the chips of the *Color Harmony Manual*, importantly different in color, and also that the colors of the third edition are intended to agree only approximately with those of the first and second. The Ostwald notation has meaning only in relation to the color chosen at each Ostwald hue to occupy the *pa* position, and these colors vary from one collection to another, depending on the colorants chosen for that collection. The Ostwald notation thus specifies a color only in the sense of identifying a particular chip in a par-

ticular collection. Furthermore, it is not easy to visualize from the Ostwald notation what color is meant. Finally it should be remarked that the Ostwald notation is not suited to the recording of color notations found by visual interpolation among the colors shown in the Manual beyond the half-way points provided by use of alternate letters of the alphabet to notate the color chips shown.

Not all the advantages in color specification, however, lie with the *Munsell Book of Color*. Because of its inner construction (see Fig. 58), and because of the mat finish of the chips, many of the strongest colors used in commerce cannot be notated by interpolation along the color scales of the *Munsell Book of Color* but must be found by spectrophotometry or estimated with some uncertainty by visual extrapolation of the Munsell scales. Furthermore, the chips of the regular editions of the *Munsell Book of Color* are not removable. Only since 1950 have loose-chip editions of the *Munsell Book of Color* been made available. It is a great advantage to be able to remove a chip from the collection and send it along with an order as a color standard if the supplier does not have the whole collection. The *Color Harmony Manual*, the Nu-Hue, and Plochere collections also have this advantage.

The *Color Harmony Manual* is preeminent as an aid to selection of harmonious colors. Like the *Villalobos Colour Atlas* and the *Munsell Book of Color*, it shows two types of constant-hue series important in color harmony: first, that of increasing lightness and saturation (constant-white content), and second, that of increasing lightness and decreasing saturation (constant-black content). The preeminence of the Manual lies in the fact, however, that every color finds its place not only on these two series but also on two other color series of even greater importance to color harmony: the shadow series and the Ostwald isovalent series. None of the other collections of color chips listed in Table 14 show both these series or make it at all easy to find approximations to them. Pairs or triads of colors taken from any shadow series not only are harmonious but also are well liked because we see them so constantly. Colors taken from the isovalent series have two important properties: first, no member of this hue series will injure (make grayish) any other member by contrast, and second, the lightnesses of the colors vary with hue in the so-called natural sequence, yellow being lightest, purplish blue darkest, and red and green intermediate.

Arguments about the relative merits of various collections of color chips for various purposes have gone on for years. Up to about 1940 these arguments were bitter and involved considerable unfair sniping. Since that time the arguments have become more good natured as it

has become generally recognized that the various collections serve different purposes and to a large extent supplement each other. These collections are important tools for the solution of color problems. They are available by purchase to anyone, and it is important to know what each is good for.

Two-Dimensional Color Scales. Each single chart of the various color atlases, dictionaries, and systems, already discussed, shows a two-dimensional array of colors. Sometimes a single chart will serve for some restricted purpose and is used separately. For example, many of the constant-Munsell-hue charts are sold separately for such restricted application.

Fischer Color Chart, Pinkham Press, Boston, 1932, available from C. W. Brown, Secretary, New England Gladiolus Society, Ashland, Massachusetts. The Fischer color chart shows 6 specimens of each of 18 hues (*R*, *O*, *Y*, *G*, *B*, *V*, and intermediates) arranged radially about a central black, making $(6 \times 18) + 1 = 109$ colors in all, rendered by screened printing plates. The colors shown around the edge are the lightest of the series, though no near-whites are included. It is thus a two-dimensional colorant-mixture system showing only colors represented by points near the outer surface of the color solid. It is intended for the specification of the coloring of flowers. In addition to the hue names, the terms light, lighter, pale, dark, and darker are used for designating these colors.

Munsell Value Scales for Judging Reflectance, available from the Munsell Color Co., Inc., 10 East Franklin St., Baltimore 2, Md. In the Munsell value scales there are 20 scales, 8 chips each, comprising two scales for each of the 10 basic hues of the Munsell system, and there is a 19-chip neutral value scale, making 179 chips in all. For each hue there is a value scale of near-neutral colors (chroma /2), and a value scale of the colors of maximum chroma for that hue and value. The scales of this latter sort, taken together, give, like the Fischer Color Chart, a two-dimensional representation of the colors near the outer surface of the color solid. The scales of the near-neutral colors, taken together, give a two-dimensional representation of the colors on a right-circular cylinder of the color solid centered on the neutral axis. The color chips are rectangles, $\frac{1}{2}$ by $\frac{5}{8}$ inch, of mat-finish painted paper. They are mounted on the edges of 11 cards for easy comparison with surfaces whose reflectance it is desired to estimate. Each chip is identified by its Munsell notation, and beside each chip is printed its percentage reflectance (Kelly, Gibson, Nickerson, 1943) for each of three light sources (CIE standard source A representative of incandescent-lamp light, CIE standard source C

representative of average daylight, and a source S representative of light from the clear blue sky). These scales were developed as a result of the activity of the delegates of the Illuminating Engineering Society to the Inter-Society Color Council during compilation of the color section of the *IES Lighting Handbook*.

One-Dimensional Color Scales. Tests analogous to the comparison of a solution of unknown concentration with a series of suitably prepared standard solutions to find the unknown concentration are numerous. In these comparisons the colors of the unknowns exhibit a one-dimensional change with concentration. A suitably spaced series of standards over this range of colors will yield the desired concentration, either by actual match with one of the standards or by visual interpolation among them. Such a series of standards is often said to constitute a color scale. The ideal material from which to make the standards is the unknown constituent itself; in this way there is guaranteed not only a perfect color match at some point along the scale but also a perfectly nonmetameric match so that variations of the light source or variations from observer to observer are of no consequence.

If the unknown is impermanent, however, it may become necessary to try to duplicate the desired colors in a more permanent medium. Thus, haemoglobin scales showing the colors produced by blood collected on filter paper have to be rendered in another medium, such as paint, because blood itself rapidly discolors in contact with oxygen. Glass is a frequent choice for color scales because of its generally superior permanence. Some degree of metamerism has then to be tolerated in nearly every case because the permanent colorants introduced into the standard rarely have exactly the same spectral character as the impermanent unknown. It is also rare that a perfect job of color matching is done for any standard light source and observer. The observer is then faced with what is often a difficult, and sometimes an impossible, task. He must estimate the position of the unknown color on the scale, and often it will seem to him that the unknown color is not equal to any of the standard colors or intermediate between any two of them. The concepts in terms of which the observer perceives these color differences then come into play. He sees a color difference between the two luminous areas presented to him by the comparator. If he should judge this color difference in terms of hue, brightness, and saturation, as is fairly common, he could estimate the position of the unknown on the color scale as the point on the scale yielding the same hue, or as that yielding the same brightness or the same saturation; or he could try to estimate the point on the scale yielding the closest chromaticity match. The determination becomes

an estimate based on what criterion of equivalence is used by the observer, and it depends upon his mental furniture in a way hard to specify precisely. In spite of these drawbacks, a good color scale is a useful timesaver, as long as it is not used in attempts to provide a one-dimensional solution to what is essentially a multidimensional problem.

Judgments of position on the color scale according to equality of brightness can be expected to correspond to luminous transmittance of a light-transmitting specimen or to luminous reflectance of a light-reflecting specimen, as the case may be. It is also possible to predict fairly well the results of settings made on the basis of hue equality, or of saturation equality, or of closest chromaticity match. The bases for these predictions are to be discussed later. But, if the colors of the scale vary importantly both in luminance and in chromaticity and the observer makes the setting on the basis of the nearest color match, there is as yet no reliable way of predicting the result. According to the OSA Colorimetry Committee (OSA, 1944), "the complete experimental clarification of this problem is one of the major programs yet to be undertaken in the field of colorimetric research."

One-dimensional color scales may be defined unambiguously in terms of the CIE system. Osborne and Kenyon (1946) have given detailed instructions for doing this.

Color Temperature. Perhaps the most widely used one-dimensional color scale is that of color temperature for classifying light sources. The *color temperature* of a light source is the temperature at which the walls of a uniformly heated furnace must be maintained so that light from a small hole in it shall yield the chromaticity of the source to be specified. It would be logically more precise, therefore, to call it *chromaticity temperature*; but in this case we shall defer to history and let logic go by the board. The color scale thus consists of the series of lights producible by closed-cavity radiation and is specified by temperature on the absolute scale (degrees Kelvin).^{*} The temperatures at which furnaces have been maintained steadily probably do not much exceed that of the melting point of iridium (2,716° K) achieved by the induction furnace, but the color temperature scale is extrapolated to infinity by all-out reliance on the Planck radiation law which states that the density, W , of radiant flux issuing from the furnace door is distributed according to wavelength, λ , as follows:

$$W = c_1/\lambda^5(e^{c_2/\lambda T} - 1) \quad (18)$$

^{*} To convert from the Celsius scale (formerly known as the centigrade scale) add 273. The temperature of absolute zero is close to -273 on the Celsius scale.

where T is the temperature on the Kelvin scale; e , the base of Napierian logarithms, is $2.71828 \dots$; c_2 is 14,380; and c_1 is a constant depending on the units used for the spectral emittance, W .

The first thing to notice about Planck's law is that the temperature of the furnace wall is the only variable in it. Nothing is said about the color of the furnace walls. Would not a furnace with blue walls radiate a greater proportion of short-wave energy than one with red walls at the same temperature? This is a fairly natural first thought, but observations of the light coming from such furnaces reveals identical colors; so we must think again. Consider two opaque balls of identical size, one white, the other ideally black, suspended in a red-hot furnace maintained at constant temperature. Radiant energy is incident on both balls at the same rate, P_0 , because they are the same size. All this energy incident on the black ball is absorbed and causes the black ball to heat up rapidly. Part of this energy, $P_0 R_w$, incident on the white ball is reflected, but the remainder, $P_0(1 - R_w)$, is absorbed; so the white ball, too, heats up, though more slowly. Eventually, both balls will get red hot and will come to equilibrium when they heat up to the temperature of the furnace walls. By equilibrium we must mean that the rate of incidence of radiant energy, P_0 , on each ball is exactly the same as the rate at which radiant energy is leaving the balls, which can be written:

$$P_0 = P_0 R_b + P_b$$

$$P_0 = P_0 R_w + P_w$$

where P_b is the rate at which the black ball emits energy because of its high temperature, R_b is the reflectance of the black ball, and P_w and R_w refer to the white ball. Now, since the black ball is ideally black, $R_b = 0$; and we see that $P_0 = P_b$, which merely states that the black ball is reflecting nothing but is radiating because it is red hot at a rate equal to the flux incident on it. The white ball must be radiating energy at a much slower rate because a large part, $P_0 R_w$, of the incident flux, P_0 , is being reflected and we see that:

$$P_w = P_0(1 - R_w) = P_b(1 - R_w)$$

or a body of reflectance, R_w , radiates more slowly than a black body at the same temperature in proportion as its reflectance approaches 1.* This is known as Kirchhoff's law. Written another way we find it:

* It is interesting to consider what would happen if a body either having an ideally white coating or coated with an ideal perfect mirror surface ($R_w = 1$) were

$$P_w/P_b + R_w = 1 \quad (19)$$

and the name "emissivity" is given to the ratio, P_w/P_b , of the emittance of any body expressed relative to that of a black body at the same temperature. If the reflectance is a function of wavelength, the emissivity must vary in the way shown in equation 19 and is then called the spectral emissivity.

There are several interesting consequences of this consideration of the temperature equilibrium between the furnace walls and objects in it. First, all such objects must be invisible if the furnace door is a negligible fraction of the furnace-wall area. This accounts for the difficulty of seeing objects in a red-hot furnace after they have become heated nearly to the temperature of the furnace. The only way we can see them at all through a small peephole is through small failures of the temperatures to be exactly equal. Second, the red or orange color comes from the temperature alone. It does not depend on whether the object reflects much or little in any part of the spectrum. This is what makes a heated enclosure a suitable basis for a color standard. Third, there is a direct connection between the color of an object in daylight and the departure of the color of the light that it emits when hot from that of a black body at the same temperature. This connection is given in equation 19. A blob of red glass taken in a plastic state from a furnace glows greener and more dimly than a similar blob of black glass; and it is greener and brighter than a similar blob of clear glass which scarcely glows at all.

A body having spectral emittance at all wavelengths less than that of a black body by a constant factor is said to be a gray body. Its spectral emissivity is a constant, less than 1, independent of wavelength, and from equation 19 we see that its reflectance must likewise be constant, independent of wavelength. If, on cooling down, its surface character remains unchanged, we should expect it to appear gray by daylight, hence, the name, gray body.

The ordinary incandescent lamp has a tungsten filament heated by an electric current. When cool the tungsten appears yellowish gray put into a furnace. We see that it would absorb no radiant energy at all, nor would it emit any. Eventually it would heat up to furnace temperature by contact with the hot gases in the furnace; but we could not say that it is red hot because it would not be emitting any radiant energy. What we would see by looking into the furnace through an infinitesimal peephole at a body covered with an ideal mirror surface would be an image of the red-hot walls of the furnace. This image would color-match exactly the walls of the furnace; so the body would blend perfectly with the walls of the furnace, and we should see exactly nothing.

with a metallic luster. We might expect, therefore, that when heated it would glow more dimly and more bluish than a black body heated to the same temperature. This is found to be true. By heating the black body to a somewhat higher temperature than the filament of the lamp it is possible to obtain light of nearly the same chromaticity from a black body. The chromaticity of the light from a tungsten filament, therefore, can be very conveniently specified by color temperature, and, to be sure, the color temperature of the tungsten filament is higher than its true temperature (Forsythe and Worthing, 1925).

An incandescent lamp must have a bulb to prevent oxygen from reaching the hot filament and oxidizing it. This bulb alters slightly the spectral-energy distribution because its spectral transmittance is never exactly a constant for all wavelengths. Furthermore, the filament is coiled so that parts of it may be heated by radiant energy falling on them from other parts, the net effect being more lumens of light flux per watt of electrical input. Sometimes the coil itself is coiled to increase the luminous efficiency still more. Such coiled-coil lamps approach fairly closely the characteristics of a closed-cavity radiator because a large fraction of the energy radiated comes by reflection from an interior coil or from an interior turn of a coil. The temperature of a coiled tungsten filament varies over a considerable range, the outer parts and the ends of the filament being cooler. From the curvature of the Planckian locus (see Fig. 28) it is evident that such temperature variations will produce a color by additive mixture that is too purple to be matched by any closed-cavity radiator at constant temperature. A straight filament of tungsten has been found (Forsythe and Worthing, 1925) to yield a slightly greener color than that of the closed-cavity radiator, and the bulb of the lamp is usually slightly greenish. It will readily be recognized that it would be largely a matter of luck if the radiant energy leaving a modern projection lamp yielded a color exactly like that from the door of a furnace maintained at some constant temperature, but trials have shown no certain color difference between them (Wensel, Judd, and Roeser, 1934; Judd, 1950). The idea of color temperature provides, therefore, not only a convenient but also a precise specification of the chromaticity of coiled-tungsten-filament lamps.

The extension of the color-temperature scale above the melting point of tungsten (about $3,600^{\circ}$ K) is also based on the Planck radiation law. Even though the law is unverified for these high temperatures, the idea of color temperature is convenient for expressing the yellow-to-blue variation in the colors of many common light sources, the chromaticities being either exactly or approximately producible by the ideal

radiator described by Planck's law. This is judged by comparisons on the chromaticity diagram such as Fig. 28.

If the chromaticity of the light source is close to, but not exactly equal to, any of the Planckian chromaticities (see solid curve on Fig.

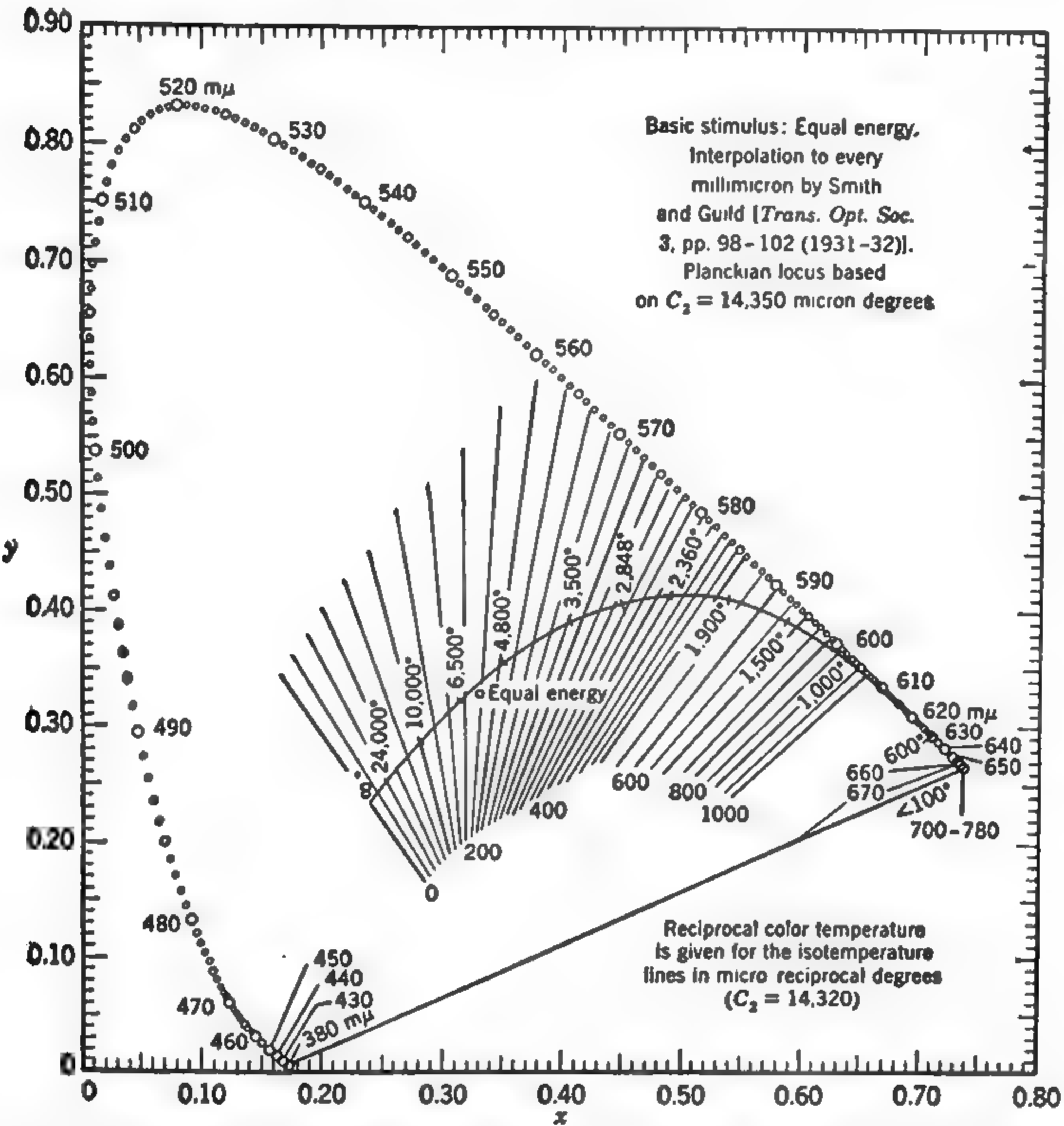


FIG. 61. Isothermperature lines for evaluation of a correlated color temperature for nonPlanckian colors (after Judd, 1936).

28), still it is possible to correlate a color temperature with the source by taking the nearest chromaticity match. Figure 61 shows this correlation (Judd, 1936). All the isothermperature lines, which cut the Planckian locus at varying angles, are such as to be perpendicular to this curve on a chromaticity diagram designed to yield uniform chromaticity spacing, to be discussed later. Table 15 shows correlated color temperatures for a number of light sources (Macbeth, 1928; Gage, 1933; Wensel, Judd, and Roeser, 1934; Nickerson, 1941; For-

Table 15. Correlated Color Temperature of Light Sources

Light Source	Correlated Color Temperature	
	T , Degrees K	$10^6/T$, μ rd
Sun at horizon through smoky air	1,000	1,000
Furnace at melting point of gold	1,336	749
Candle or kerosene lamp flame	1,900	526
Furnace at melting point of platinum	2,042	490
Carbon-filament lamp	2,100	476
Furnace at melting point of rhodium	2,233	448
Vacuum tungsten lamp or acetylene flame	2,360	424
Furnace at melting point of iridium	2,716	368
Gas-filled tungsten lamp, 40-watt	2,760	362
60-watt	2,790	358
100-watt	2,860	350
1000-watt	3,000	333
Standard source A (representative of gas-filled tungsten lamps)	2,854	350
Photoflood lamp	3,400	294
"Warm White" fluorescent lamp	3,500	286
Electric arc, solid carbons	3,750	267
"Cool White" fluorescent lamp	4,500	222
Average noon sunlight	5,000	200
Standard source B (representative of noon sunlight)	4,870	205
Electric arc, "Sunshine" carbons	6,260	160
"Daylight" fluorescent lamp	6,500	154
Heavily overcast sky	6,500	154
Standard source C (representative of average daylight)	6,740	148
Lightly overcast sky	7,500	133
Hazy blue sky	9,000	111
Clear blue sky	25,000	40

sythe and Adams, 1944; Judd, 1950a). It also shows these color temperatures, T , expressed on a reciprocal scale ($10^6/T$) whose units are micro reciprocal degrees (μ rd) sometimes also written "mireds." The isothermperature lines on Fig. 61 are identified in terms of reciprocal color temperature. There are two advantages of reciprocal color temperature (Priest, 1933) over color temperature itself. First, reciprocal color temperature yields nearly uniform chromaticity spacing; one μ rd corresponds closely to a just detectable chromaticity difference over the whole scale from 0 to 1,000 μ rd. Second, an ideal filter (amber or blue) required to change reciprocal color temperature by an amount, $\Delta 10^6/T$, in one part of the scale changes reciprocal color temperature by nearly the same amount anywhere on the scale (Gage, 1933), and this prop-

erty furnishes a convenient way to specify the color-temperature-altering power of actual filters.

Since color temperature specifies only the chromaticity of a light, there are many spectral compositions corresponding to the same color temperature, any two of which form a metameric pair. Color temperature of an illuminant is, therefore, an incomplete and unreliable indication of the rendering of colors of objects illuminated by it, or of the photographic effect of the illuminant. Correlated color temperature is still less reliable. For example, the "Warm White" fluorescent lamp and the electric arc are shown in Table 15 to have similar values of correlated color temperature; but it is not true that they give similar colors to the objects that they illuminate. To make color temperature a perfectly valid basis for comparing two lights, it must first be shown that they are spectrally similar. Thus, incandescent lamps with nearly colorless bulbs may be usefully compared by means of color temperature without reservation; and two fluorescent lamps may be usefully compared by means of color temperature, provided that they have about the same phosphors and about the same admixture of mercury spectrum. But comparison of incandescent lamps with fluorescent, by correlated color temperature, has no precise meaning in terms of object-color rendition and seldom serves a useful purpose; it is merely an indication of the appearance of the lamps themselves on a yellow-to-blue scale.

Working standards of color temperature may consist of an incandescent lamp operating at a fixed voltage combined with a series of amber or blue filters, like the Davis-Gibson (1931) liquid filters, or the Lovibond glasses; but by far the most common way of producing these chromaticities over moderate ranges of color temperature is by variation of the voltage applied to an incandescent lamp. Such standards with an equation showing color temperature over the range 2,000 to 2,854° K as a function of applied voltage may be purchased from the National Bureau of Standards. They are used not only as color standards but also as standard illuminants (for example, CIE source A, see Fig. 27), and as approximate standards of the spectral distribution of radiant flux on the fairly well-supported assumption that this distribution is given between 400 and 700 $m\mu$ by Planck's law (equation 18) by setting T equal to color temperature.

Lovibond Grading of Vegetable Oils. The Lovibond glasses have been used extensively in the grading of vegetable oils in this country. McNicholas (1935) has shown, from the spectral transmittance of representative samples of refined cottonseed oils and oils of sesame, peanut, soybean, corn, rape, and olives, that these oils owe their colors

to independently varying amounts of two groups of coloring matters commonly found in plants, a group of brown pigments and a group of green (chlorophyll compounds). For commercial purposes the colors of these oils have been graded by the number of Lovibond red units required in combination with Lovibond 35-yellow to color-match a 5¼-inch layer of the oil. Figure 62 shows (McNicholas, 1935) the

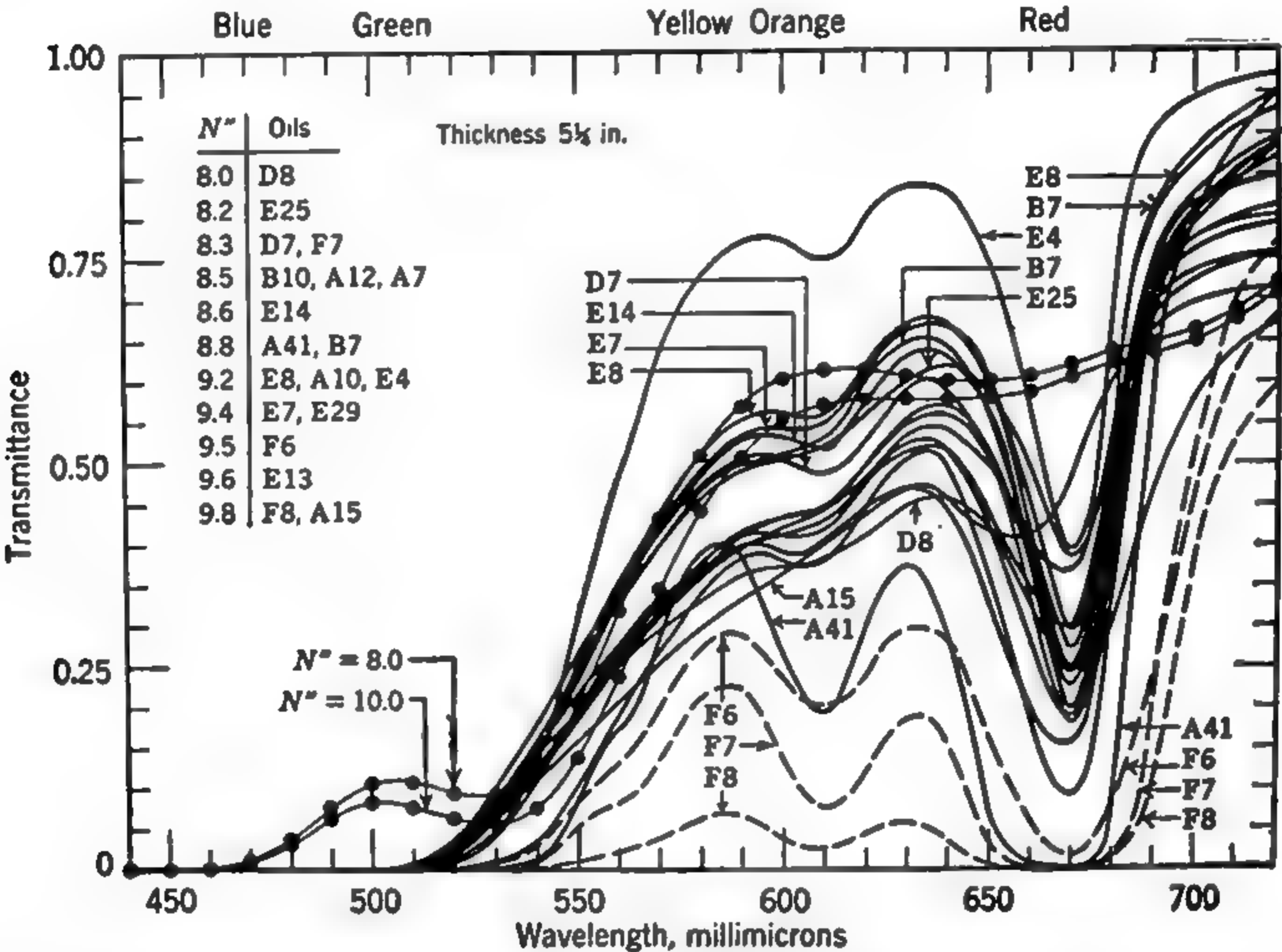


FIG. 62. Spectral transmittance of vegetable oils compared to that of the Lovibond glass combinations (solid circles) giving the nearest chromaticity match (after McNicholas, 1935).

spectral transmittance of a group of cottonseed oils (solid curves) and other vegetable oils (dotted curves), all of which have color grades between 8.0 and 10.0 Lovibond red units on the 35-yellow plus red scale. Figure 62 also shows the spectral transmittances of these limiting Lovibond glass combinations (solid curves with solid circles). It will be noted first that all the oils make metameric pairs with the Lovibond glass combinations. A standard light source is thus required, and a certain amount of individual observer difference may be expected in grading oils by this method. Since there are two main groups of pigments in these oils, it is also to be expected that no one-dimensional grading system, like the scale of 35-yellow plus some amount of red, will yield perfect color matches. Computation of chromaticity coordinates from the spectral-transmittance curves and plotting on a

chromaticity diagram (see Fig. 63, due to McNicholas, 1935) reveal this spreading of the chromaticity points over an area instead of along a single line and show further that the Lovibond 35Y plus *R* locus is too far from the spectrum locus to be a good match for the center of the area except for Lovibond red less than 3 units. There is consider-

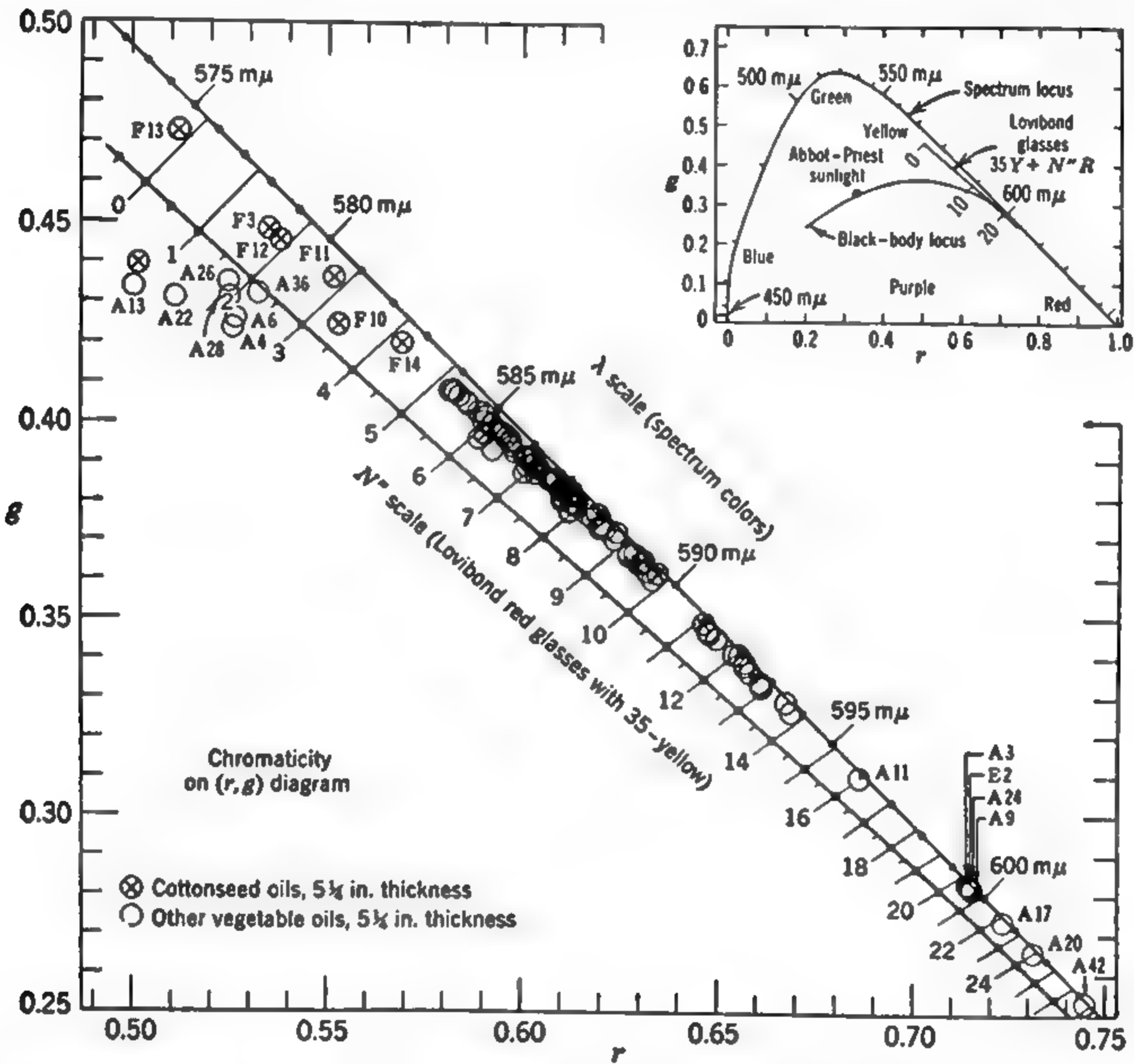


FIG. 63. Chromaticities of vegetable oils compared to those of the Lovibond glass combinations used in grading them (after McNicholas, 1935).

able doubt, therefore, what Lovibond grade corresponds to a given spectral-transmittance curve of Fig. 62. McNicholas chose to base the decision on nearest chromaticity match and made use of a uniform-chromaticity-scale diagram to be discussed later. The short straight lines on Fig. 63 intersecting the Lovibond 35Y plus *R* locus at somewhat oblique angles would be perpendicular to the corresponding locus on this uniform-chromaticity-scale diagram. In spite of these difficulties the method is well suited to indicate the quality and salability of vegetable oils, and it has been used for more than 30 years. Because of the difficulties, however, and the further disadvantage of being de-

pendent upon a foreign source of glass standards, there have been fairly continuous agitation and research work intended to lead to another method (Thomson, 1947), and in 1950 a spectrophotometric method based on transmittance at four wavelengths (460, 550, 620, and 670 $m\mu$) was tentatively introduced. But the Lovibond color scale, 35Y plus R, is so ingrained in the purchase and sale of vegetable oil that this spectrophotometric method has been chosen so as to give as good a correlation as is possible with the Lovibond reading. The color index chosen is $1.29 \log (1/T_{460}) + 69.7 \log (1/T_{550}) + 41.2 \log (1/T_{620}) - 56.4 \log (1/T_{670})$, which gives a correlation coefficient of 0.993 with readings in terms of Lovibond red.

Grading of Petroleum Products by Union Colorimeter. For more than 20 years the color of lubricating oils and petrolatum has been graded by comparison with the colors of 12 glass standards made up of combinations of Lovibond red, yellow, and blue glasses. Crude oil is black, nearly opaque, and as it is refined the color changes through reddish black, dark reddish brown, brownish orange, orange, yellow, pale greenish yellow, to crystal clear. The purchaser of partially refined oil can judge fairly well by its color how much it will cost to refine it to the degree required for his own purpose. He feels cheated, and with good reason, if the delivered oil is darker in color than represented. The Union colorimeter (ASTM Method D 155-45T) with its 12 glass standards of color thus have considerable importance in the purchase and sale of petroleum products.

The petroleum product, in a 33-mm inside-diameter bottle, and the standard are illuminated by artificial daylight produced by combining an incandescent lamp of color temperature approximately 2,750° K with a filter of Corning Daylite glass specially selected to have spectral transmittances within specified tolerances and further to have for standard source A luminous transmittance, T_w , and chromaticity coordinates, x , y , z , within the limits:

T_w	0.107 to 0.160
x	0.314 to 0.330
y	0.337 to 0.341
z	0.329 to 0.349

The specimen holder, the magazine containing the glass standards, the artificial daylight assembly, and a viewing diaphragm defining the direction of view are mounted together to form a portable instrument known as the Union colorimeter. Figure 64 is a photograph of the Union colorimeter now supplied by Fisher Scientific Company.

Table 16 gives the Lovibond analysis of the glass color standards (ASTM Method D 155-45T), the luminous transmittance, T_{10} , and chromaticity coordinates, x , y , for standard source C (Scofield, Judd, and Hunter, 1941), and the color names used by the National Petroleum Association. Figure 65 shows by large circles on the (x, y) dia-

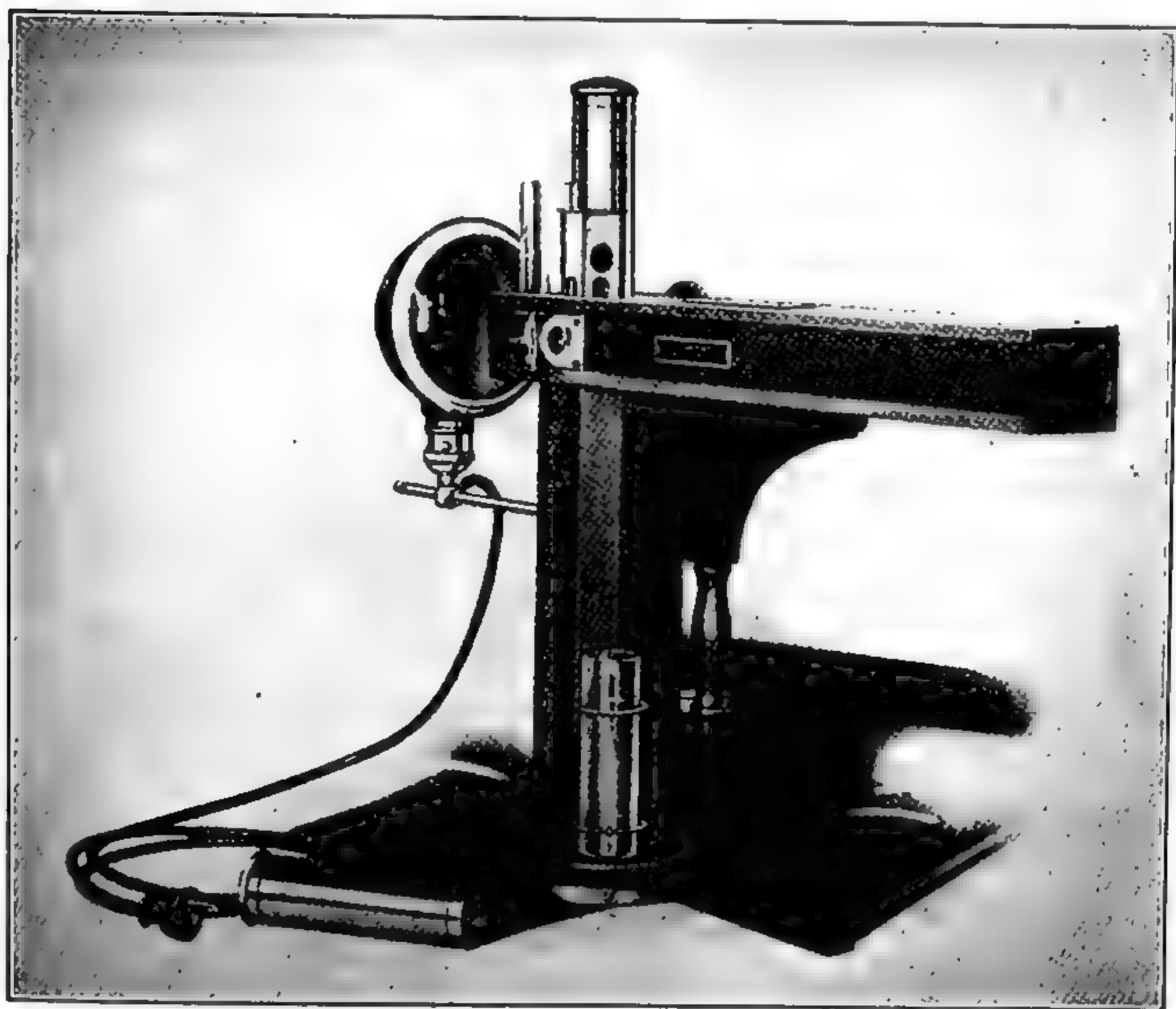


FIG. 64. General view of the FISHER-TAG Union colorimeter. (Courtesy of the Fisher Scientific Company.)

gram the chromaticities of these glass color standards compared to those of 22 petroleum products (small circles) measured by Diller, De Gray, and Wilson (1942). It will be noted from Fig. 65 that the Union colorimeter glasses yield good chromaticity matches for the petroleum products represented except for ASTM color numbers $1\frac{1}{2}$ to $3\frac{1}{2}$ in which range the oil points fall between the spectrum locus and the Union locus. This has been corroborated by more recent studies (Judd, Plaza, and Belknap, 1950). Note also that the oil points and the Union locus above ASTM color 5 fall on the spectrum locus; that is, the standard observer would find that each such glass standard and each such oil color match some part of the spectrum

Table 16. ASTM Union Colorimeter Standards, Lovibond Analysis, Luminous Transmittance, T_w , and Chromaticity Coordinates, x , y , for Standard Source C, and NPA Color Names

ASTM Color Number	Lovibond Analysis			Luminous Transmittance, T_w	Chromaticity Coordinates		National Petroleum Association Names
	Red	Yellow	Blue		x	y	
1	0.12	2.4	0.751	0.349	0.382	Lily white
1½	0.60	8.0654	.400	.446	Cream white
2	2.5	26.0443	.472	.476	Extra pale
2½	4.6	27.0365	.498	.457	Extra lemon pale
3	6.9	32.0287	.525	.440	Lemon pale
3½	9.4	45.0211	.556	.423	Extra orange pale
4	14.0	50.0	0.55	.096	.591	.400	Orange pale
4½	21.0	56.0	0.55	.065	.620	.376	Pale
5	35.0	93.0036	.653	.347	Light red
6	60.0	60.0	0.55	.017	.676	.323	Dark red
7	60.0	106.0	1.8	.0066	.684	.316	Claret red
8	166.0	64.00020	.714	.286

between 605 and 640 $m\mu$. This applies even to Union color 8 which specifies a nearly black oil. It is possible that some improvement could be achieved by choosing glass standards of higher excitation purity for the colors 2, 2½, 3, and 3½; such a change would make it easier for an observer to decide which glass standard is closest in chromaticity to such oils as are represented on Fig. 65. Research Division IX of ASTM Committee D-2 has suggested such a change, together with a change in spacing of the colors to make the color intervals between standards perceptually more uniform (Judd, Plaza, and Belknap, 1950). The Union colorimeter, however, does a generally satisfactory job in spite of occasional important ambiguity of setting; and the Union color scale is well known in the petroleum industry.

United States Color Standards for Rosin. Gum rosin has been graded by color for more than 50 years. Up to 1914 the color standards were made of rosin itself in spite of the relative impermanence of its color, and from 1914 to 1936 standards composed of combinations of Lovibond glasses were used. Brice (1940) has described the selection of 12 official standards composed of 2 components of colored glass com-

bined with 1 component of clear glass, all 3 cemented together with Canada balsam. The various combinations are given letter designations denoting the grades of rosin delimited by them and have legal status under the Naval Stores Act. The cemented face of the clear glass in each combination is fine ground so as to duplicate the slight

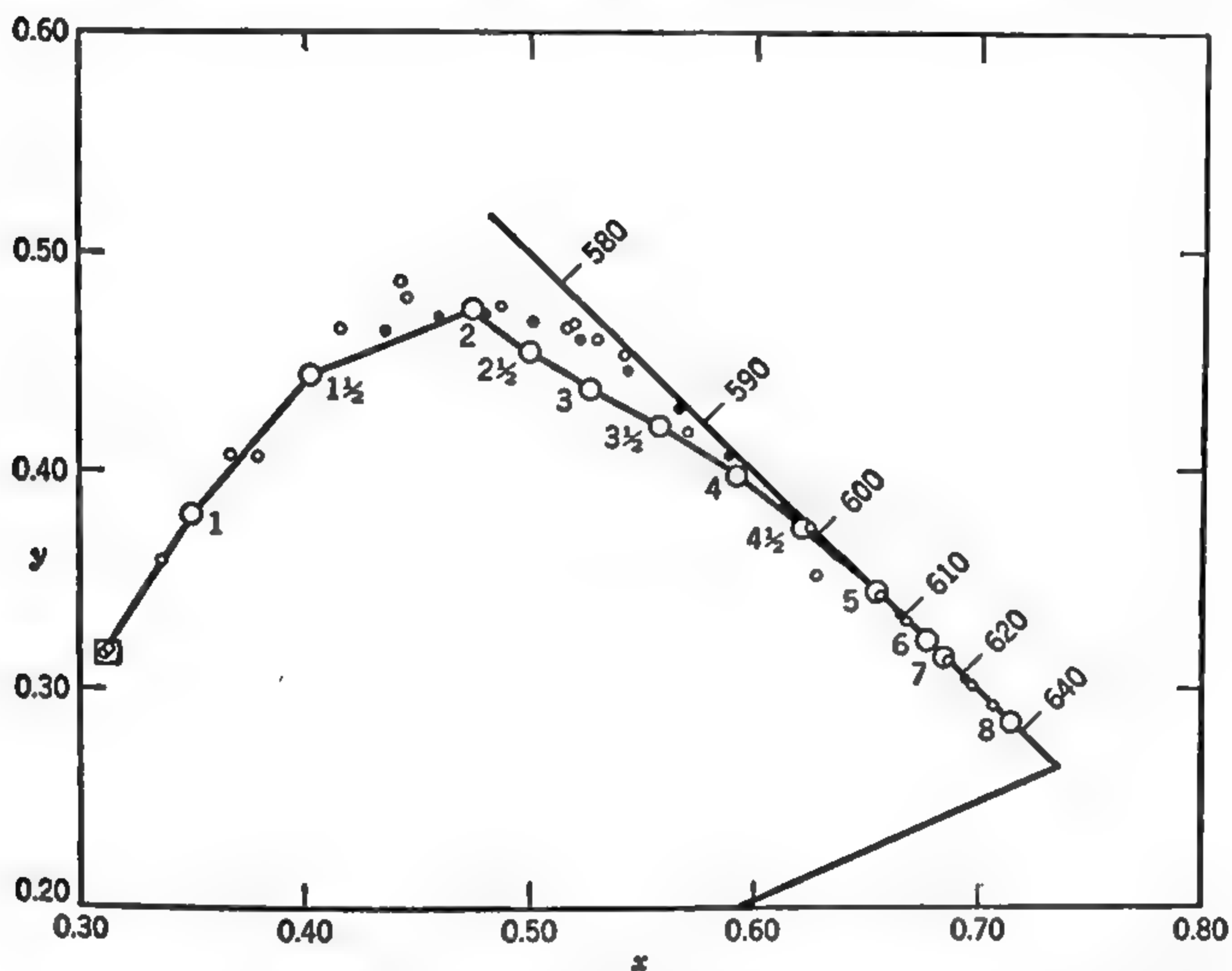


FIG. 65. Chromaticities of the glass color standards of the Union colorimeter (large circles) compared to those of petroleum products (small circles), and to those of the United States color standards for rosin (solid circles). Wavelengths of the long-wave portion of the spectrum locus are indicated in millimicrons.

(From *Analytical Absorption Spectroscopy*, edited by Mellon, 1950.)

turbidity characteristic of molded samples of rosin, which commonly contain traces of fine dirt. The chromaticities are shown by solid circles on Fig. 65, and it will be noted that they cover approximately the same chromaticity range as ASTM Union color 1½ to 8. The chromaticity spacing was adjusted by means of a uniform-chromaticity-scale diagram so as to progress regularly from small steps for yellow rosins to steps of about four times this size for reddish orange rosins.

Table 17 gives the luminous transmittances, T , and chromaticity coordinates, x , y , of these standards for source C, together with the names that are sometimes used instead of the letter designations. The lowest-grade rosin, called Dolly, is also sometimes said to be a Dilly.

Table 17. Luminous Transmittance and Chromaticity Coordinates (for Standard Source C) for 1936 U. S. Standards for Rosin

Letter	Designation Name	Luminous Transmittance,	Chromaticity Coordinates	
		<i>T</i>	<i>x</i>	<i>y</i>
X	Extra water white	0.609	0.4339	0.4663
WW	Water white	.531	.4579	.4732
WG	Window glass	.466	.4785	.4741
N	Nancy	.396	.5001	.4704
M	Mary	.322	.5212	.4619
K	Kate	.245	.5430	.4483
I	Isaac	.178	.5649	.4310
H	Harry	.1114	.5879	.4102
G	George	.0723	.6116	.3874
F	Frank	.0398	.6364	.3632
E	Edward	.0131	.6640	.3358
D	Dolly	.0021	.6943	.3057

Color Standards for Paint Vehicles. Many special color scales have been set up for the specification of vehicles (varnishes, linseed oil, tung oil) into which pigment is mixed to form a paint. A solution of nickel sulfate and iodine (British Engineering Standard, 1936) is used in Great Britain to define the darkest color permissible for spar varnish. A color comparator having 18 glass color standards made by Hellige, Inc., has been used for similar purposes (Gardner and Sward, 1946). The Pfund color grader, made by the Munsell Color Company, compares a variable thickness of the unknown specimen with a variable thickness of a carbon-yellow glass (Pfund, 1925). The standard is wedge shaped, and the cell for the specimen is likewise wedge shaped. The Parlin (or Cargille) color standards consist of a set of 35 solutions. The first 10 are Hazen platinum-cobalt solutions (Hazen, 1892), developed originally to measure the color of natural waters and still used for that purpose under the name of APHA (American Public Health Association, 1936) standards. The remainder of the Parlin color standards are caramel solutions. They have been adopted tentatively by the ASTM (Method D 365-39T) for testing the color of soluble nitrocellulose base solutions. The Pratt and Lambert color standards are varnish mixtures calibrated against the Pfund color grader. The du Pont colorimeter employs 6 glass plates as color standards, together with a wedge of the same glass permitting a continuous variation of color between the standards. The Hellige-Stock-Fonrobert colorimeter has 3 sets of 9 glass disks each. The unit of color is 1 mg of iodine in 100 ml of potassium iodide solution (Gardner and Sward, 1946).

The Gardner color standards consist of 18 combinations of the red and yellow Army solutions. The Gardner Laboratory has determined the Army and Lovibond specification for the nearest matches for all the above-mentioned sets of color standards and has obtained also the nearest equivalents in terms of potassium dichromate solutions (Gardner and Sward, 1946). From these nearest equivalents it is possible to express color specifications given by any of these means in terms of any of the others.

The British Paint Research Station has recommended (Tilleard, 1937) combinations of Lovibond glasses for color grading of oils and varnishes in 1-inch thickness. Some of the combinations involve colorless (or blue) glasses to be combined with the varnish or oil to match red and yellow glasses, and a device facilitating the setting up of such combinations is also recommended. The Lovibond glasses are mounted in a slide, and the two photometric fields to be compared are brought into juxtaposition by mirrors. For oils 2 scales, a reddish and a greenish, are provided; and for both oils and varnishes care has been taken to find glass combinations that yield a luminance as well as a chromaticity match for the average commercial product.

F.A.C. Color Standards. A series of 26 solutions of inorganic salts has been developed by the Fats Analysis Committee of the American Oil Chemists' Society and the American Chemical Society for the specification of the colors of animal fats. These are known as the F.A.C. color standards (Urbain and Roschen, 1949) and are used in the commercial color grading of tallows and greases.

Miscellaneous Color Scales. In numerous chemical tests the end product is a colored solution. Of these probably the test of greatest industrial importance is the one in which a dye is used as an indicator of the acidity or alkalinity (pH) of a solution. Glass color standards for use with 18 indicators covering the pH range from 0.2 to 13.6 are supplied by Hellige, Inc., and are widely used. In addition to these chemical tests, there are a few tests of commercial importance requiring a one-dimensional color scale made up of light-reflecting specimens.

The United States Safety Service Company manufactures for sale a carbon monoxide indicator designed by Shepherd (1947). The indicator is a silica gel impregnated with a yellow silico-molybdate complex which is reduced by carbon monoxide to form the blue tritoxide, Mo_3O_8 . A color scale showing 5 steps from yellow to blue rendered in half-tone screen printing is supplied with the indicator and permits the percentage of carbon monoxide in the gas tested to be read.

Army-Navy Aeronautical Design Standards, AND10376 (approved November 13, 1941), is a humidity indicator chart used to test the

water absorptive power of a drying agent designed to protect bomb sights and other delicate mechanisms from corrosion. The drying agent is a silica gel impregnated with cobaltous chloride that is blue in the active state and turns pink when it is in equilibrium with an atmosphere of 60 percent relative humidity. The four-step color scale, rendered in solid printing, permits a quick reading of the relative humidity of the air inclosed with the gel and so tells when more gel must be added to prevent corrosion.

Ringelmann's Scale for Grading the Density of Smoke, issued by the Publications Section, Bureau of Mines, Department of the Interior, Washington, D. C., consists of four rectangular patterns ($5\frac{1}{2}$ by $8\frac{1}{4}$ inches) in black and white to be used as a color standard for smoke. The patterns when viewed from a distance of 50 feet are equivalent to 20, 40, 60, and 80 percent black, respectively.

Nonsystematic and Incomplete Sampling of the Color Solid. There are thousands of collections of color swatches, painted chips, plastic chips, and manufactured materials of many kinds, in which the choice of colors has no relation whatever to spacing in the color solid. These collections answer such questions as what colors a manufacturer believes will sell, what colors he has stocked and can supply immediately, what colors are used for a particular purpose, and so on. New collections are coming out by the hundreds every year. Government agencies themselves constantly develop and issue new sets of color standards in connection with procurement of goods and materials required to carry out their duties. A few of the more permanent or important collections of such color standards are worth describing.

Textiles. In the highly competitive textile industry, choice of colors varies almost daily in response to fashion trends, and it is hard to say what, if any, color standards are used in the manufacture and sale of textiles by more than one manufacturer at a time. The outstanding collections of color standards for textiles are those issued by the Textile Color Card Association of the United States, Inc.

1. *Standard and Seasonal Color Cards*, issued by the Textile Color Card Association of the United States, Inc., 200 Madison Avenue, New York, N. Y. The samples are $1\frac{7}{8}$ -by- $1\frac{1}{4}$ -inch rectangles of fabric. Each swatch is identified by name and TCCA cable number. The standard cards are revised only infrequently and show about 200 colors for which there is a continuing demand. The ninth edition of *Standard Color Card of America* shows 216 swatches of satin-finish pure-dye silk, the back or mat side being displayed over most of the area of each. The colors of these swatches have been measured in fundamental terms at the National Bureau of Standards (Reimann, Judd,

and Keegan, 1946), and for each the daylight reflectance, Y , chromaticity coordinates, x , y , and Munsell notation are given. At least two seasonal color cards are issued each year, a spring card and an autumn card; they show swatches of crepe-finish rayon. Similar seasonal *Woolen Color Cards* are issued, and there are also seasonal color cards for men's shoe leathers, for women's shoe leathers, for felt bodies for men's hats, and for women's gloves and hosiery. These cards are widely used in the textile and allied industries so that the products of various mills may be color coordinated. They are also used in color specification by government agencies and by industries quite unrelated to the fashion trades.

2. *United States Army Color Card*, 1930-1943, and *Supplement*, 1943, issued by the Textile Color Card Association of the United States, Inc., 200 Madison Ave., New York, N. Y. This card and supplement show 22 U. S. Army colors standardized for the different arms and services. It has been approved and accepted by the Quartermaster General as being in accord with the standards on file in that office. The samples are rectangles of silk, 1 by $\frac{1}{2}$ inch. Their colors have been measured in fundamental terms at the National Bureau of Standards (Reimann, Judd, and Keegan, 1946), and for each the daylight reflectance, Y , chromaticity coordinates, x , y , and Munsell notation are given. This card is used in the purchase of certain textile materials by the Army, chiefly for insignia and decorations.

3. *Standards for Grade of Raw Cotton*, available from Production and Marketing Administration, U. S. Department of Agriculture, Washington 25, D. C. There are 33 grades, 13 of which are defined by actual cotton samples, and the remaining 20 of which are defined by reference to these 13. There are so defined 7 classes of colors: extra white, white, spotted, tinged, yellow stained, and gray, the several grades of each class differing chiefly in lightness. To define any one grade requires 12 cotton bats (about 5 by 7 by 6 inches) showing the color range. These same bats also show the ranges in leaf and preparation. The 12 color standards defining any one grade are put up in a 20-by-20-inch box and are available at \$5.00 per box.

4. *Color Standards for Electrical Insulation*, issued by Bureau of Ships, Navy Department, 1946. The color standards are 22 rectangles of painted paper, $2\frac{1}{2}$ by 5 inches, semigloss finish. They represent light and dark limits for 11 colors used for electrical insulation to facilitate tracing of circuits. A color notation chart accompanying the standards gives the daylight reflectance, chromaticity coordinates, x , y , and Munsell notations of the colors, together with the cable

number of the nearest match from the *Standard Color Card of America* issued by the Textile Color Card Association.

Paint. Most of the collections of color standards listed here are for the purpose of specifying the color requirements of various government agencies in the purchases of paint.

1. *Federal Color Card for Paint*, Federal Specification TT-C-595, 12 January 1950, Color (for) Ready-Mixed Paints, for sale by the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., Price \$4.50. This collection of color standards consists of 187 rectangles, $1\frac{1}{2}$ by $2\frac{5}{8}$ inches, of paint deposited directly on sheets of paper by the McCorquodale process. The sheets are fastened together in a loose-leaf ring binder. This collection is a composite of the separate color cards formerly issued by government agencies and is intended to supersede them. The near-duplicates in the former collections have been omitted. The samples are divided into gloss, semigloss, and lusterless samples, and each is identified by a 4-digit number. The final sheet is a key to these numbers, showing the service using the color and the designation previously used by that service for the color.

2. *Color Standards for Public Buildings*, issued by Public Buildings Administration, Washington, D. C., to contractors. The standards are in the form of 3-by-5-inch painted cards with flat to eggshell finish identified by Munsell color notation. About one-half of the 117 colors are duplicates of some one of those in the Munsell book; the rest are interpolated. They are used as guides in the painting of both interiors and exteriors.

3. *American Standard Gray Finishes for Industrial Apparatus and Equipment*, issued by the American Standards Association, Inc., 70 East 45th St., New York 17, N. Y. Four nearly neutral grays identified by Munsell notation are shown in the form of cardboard chips, $1\frac{1}{2}$ by 3 inches, mounted in a folder. These form the basis for American Standard Z55.1-1950.

4. *Paint Pigment Standards for Color and Tinting Strength*, issued by the National Bureau of Standards at \$3.00 per sample, identified by name and number as follows:

NBS Number	Material	NBS Number	Material
300	Toluidine Red Toner	305	Burnt Umber
301	Yellow Ocher	306	Venetian Red
302	Raw Sienna	307	Metallic Brown
303	Burnt Sienna	308	Indian Red
304	Raw Umber	309	Mineral Red

NBS Number	Material	NBS Number	Material
310	Bright Red	320	Lemon Chrome Yellow
311	Carbon Black (High Color)	321	Medium Chrome Yellow
312	Carbon Black (All Purpose)	322	Light Chrome Yellow
313	Black Iron Oxide	323	Dark Chrome Yellow
314	Iron Oxide, Light Lemon	324	Ultramarine Blue
315	Iron Oxide, Lemon	325	Iron Blue
316	Iron Oxide, Orange	326	Light Chrome Green
317	Iron Oxide, Dark Orange	327	Medium Chrome Green
318	Lampblack	328	Dark Chrome Green
319	Primrose Chrome Yellow		

These standard samples are used to test colored paint pigments purchased in accord with Federal specifications.

Printing Ink. Probably every manufacturer of printing ink makes a practice of displaying his line of inks by printing color charts showing what they will do on various kinds of paper and with various amounts of coverage by screened printing plates. A color chart by the International Printing Ink Company is typical of these and is outstanding because trouble was taken to measure the colors spectrophotometrically.

1. *ipi Offset Color Guide*, issued by International Printing Ink Company, Division of Interchemical Corporation, 350 Fifth Ave., New York 1, N. Y. The guide shows 318 colors produced by 53 inks, each ink identified by company trade designation. There is a solid printing, a medium screening, and a light screening, and all are shown both on coated and on uncoated paper. Daylight reflectance, dominant wavelength, and excitation purity based on a spectrophotometric study are given for each solid printing. Munsell notations based on visual interpolation along the color scales of the *Munsell Book of Color* are also given.

2. *A.M.S. Ink Color Guide*, issued by Army Map Service, Corps of Engineers, U. S. Army, Washington, D. C., to provide a standard basis for the production of A.M.S. maps. Each of the 41 inks identified by color name is shown solid ($5\frac{1}{2}$ by $1\frac{1}{4}$ inches), in six different kinds of crosshatching and half-tone screening (each block $\frac{5}{8}$ inch square), and in various weights of solid lines and sizes and styles of type. There are also 3 fluorescent inks (gold, flesh, and cream) printed ($5\frac{1}{2}$ by $3\frac{1}{2}$ inches) on fluorescent paper.

3. *Standard Colors of Tag Stocks*, page 22, *Manual of Standard Specifications*, issued October 31, 1947, by the Tag Manufacturers Institute (Attention Frank H. Baxter, Executive Director), 370 Lexington Ave., New York 17, N. Y. This chart shows 18 rectangles (1 by 2 inches) of printed or painted tag stocks identified by color name.

Plastics. The molders of plastics display the colors of the plastics that they have formulated by means of plastic chips about 2 inches across, often in a range of thickness to demonstrate the opacity of the material. A manufacturer does not have to be in business many years before his collection of chips runs into the thousands. In an effort to save money otherwise lost by stocking molded articles in a multiplicity of colors, these manufacturers through the Plastic Materials Manufacturers Association set up a commercial standard establishing a limited number of standard colors which they expect will make up a large fraction of their total business.

1. *Colors for Molded Urea Plastics*, available from Manufacturing Chemists Association, Woodward Building, 15th and H Streets, NW, Washington 5, D. C., at \$2.50 per set of 17. Each sample is a 2¼-inch-diameter plastic disk identified by the letters MUP followed by a number. The colors of the samples conform to the CIE specifications given in Commercial Standard CS147-47, U. S. Department of Commerce, obtainable from the U. S. Government Printing Office, Washington, D. C. Munsell notation, nearest match in the first edition of the *Color Harmony Manual*, and nearest hue match from the *Standard Color Card of America* (TCCA) are also given.

2. *Colors for Polystyrene Plastics*, available from Manufacturing Chemists Association, Woodward Building, 15th and H Streets, NW, Washington 5, D. C., at \$2.50 per set of 18. The samples are similar in size and shape to the MUP chips. There are 12 opaque samples and 6 transparent, each identified by the letter *P* followed by a number. The colors of the samples conform to the CIE specifications given in Commercial Standard CS156-49, U. S. Department of Commerce. The same information is given in this standard as for the MUP samples in CS147-47.

Miscellaneous. The use of special selections of colors is widespread in industry and technology. Some are merely sample books showing what goods the manufacturer can supply as is common in the paper industry (Judd, 1940); others are carefully assembled by societies of national scope.

1. *Colors for Kitchen and Bathroom Accessories*, a group of 10 colors, 6 of which apply to kitchen accessories, and 7 of which apply to bathroom accessories, there being 3 colors applying to both. The samples are 3-by-5-inch rectangles of vitreous enamel on metal and are obtainable from the National Bureau of Standards at \$10.00 for the set of 10 colors. These standards are described in two Commercial Standards, *Colors for Kitchen Accessories*, CS62-38, and *Colors for Bathroom Accessories*, CS63-38, obtainable from the Superintendent of

Documents, Washington, D. C., at 5 cents each. The colors were selected by a committee of the National Retail Drygoods Association and have been accepted by producers, distributors, and users of kitchen and bathroom accessories. Sets of these samples calibrated in terms of reflectance for the amber, green, and blue filters of photoelectric tristimulus colorimetry (Hunter, 1942) are available from the National Bureau of Standards at \$75.00 a set for use as standards for color measurement.

2. *Rock-Color Chart*, distributed by the National Research Council, Washington, D. C. This chart was prepared by the Rock-Color Chart Committee representing the U. S. Geological Survey, The Geological Society of America, American Association of Petroleum Geologists, Society of Economic Geologists, and Association of American State Geologists. The chart shows 115 rectangles, $\frac{1}{2}$ by $\frac{5}{8}$ inch, of nearly mat painted paper. The colors are identified by Munsell notation and by the ISCC-NBS color designation, to be discussed later. They are intended to cover the full color range of rocks, wet or dry, except possibly of very rare rocks of high Munsell chroma.

3. *Permanent Glass Standards for Maple Sirup*, available from Phoenix Precision Instrument Co., Philadelphia, Pa., in accord with specifications set by the U. S. Department of Agriculture (Brice, Turner, Southerland, and Bostwick, 1950). These are $1\frac{1}{4}$ -inch glass squares defining the grade "Fancy" and grades A, B, and C. Suspensions of bentonite in glycerin-water solutions are used in conjunction with these glasses to simulate the turbidity as well as the color of maple sirup.

4. *Munsell Soil Color Charts*, prepared and distributed by the Munsell Color Co., Inc., 10 East Franklin St., Baltimore 2, Md. The charts show 264 colors of Munsell hue from 7.5 Red through 5 Yellow, including colors of Munsell chroma /1 for hues 10 Red, 5 Yellow-Red, 10 Yellow-Red, and 5-Yellow. The samples are rectangles, $\frac{1}{2}$ by $\frac{5}{8}$ inch, of nearly mat painted paper. There are two forms, the special form having the samples mounted on heavy gray paper with half-inch diameter holes punched beside each sample. These charts are intended for the use of soil scientists, geologists, and archaeologists. They were made for, and approved in 1946 by, the Soil Survey Color Committee, U. S. Department of Agriculture.

5. *Color Standards for McIntosh Apple Leaves*, Cornell University Agricultural Experiment Station Bulletin 824A, obtainable from the Mailing Room, Experiment Station, Ithaca, N. Y. These standards are used to determine whether McIntosh apple trees are obtaining the correct amount of nitrogen fertilizer.

6. *House and Garden Colors*, issued seasonally by *House and Garden*, The Condé Nast Publications, Inc., 420 Lexington Ave., New York 17, N. Y. In the 1951 set there are 23 colors shown on 3-by-6-inch cards, half of each of which is in the original mat finish, and the other half of which is covered with a clear lacquer. The colors are identified by name, 16 being also called current colors since they are based on the Rahr Color Count of shopper's preferences (Frederick H. Rahr, 9 East 56th St., New York, N. Y.), and 7 being called decorator colors since they are forecasts of the future. Sets of these colors are issued yearly for homefurnishing and housewares as the central part of a nationwide color promotion.

UNIFORM COLOR SCALES

A uniform tridimensional color scale would be an important commercial as well as scientific achievement. It would simplify color specification and the setting of color tolerances, it would clarify the interpretation of one-dimensional color scales applied to off-color samples, it would be an important guide to the preparation of reference color samples as standards (see Fig. 60), and it would aid the selection of harmonious color combinations. Unfortunately, scores of attempts have so far not succeeded in developing such a scale. They have, on the contrary, supported a somewhat dismaying suspicion that a strictly uniform tridimensional color scale cannot possibly be developed. These attempts have, however, indicated that at least a good approximation to ideally uniform color spacing is possible, and much color-spacing information of commercial value has been developed. We shall, therefore, attempt a review of the present status of these studies.

Uniform Lightness Scales. If an observer is given a white chip, a black chip, and a batch of grays and is asked to select one that appears about as different from white as it does from black, he will have a little trouble because judging the relative sizes of two large color differences is after all only an estimate based on subjective impression. This is a special case of what Newhall (1939) has called the ratio method of evaluating color differences—the case of the ratio, 1:1. The average of judgments by a number of observers, however, serves to locate the required gray with any desired precision merely by taking enough judgments and enough observers. Then the color interval between black and mid-gray may be halved by the same procedure, and also that between white and mid-gray, resulting in a uniform lightness scale of 5 colors extending from black to white by steps that are subjectively equal. This was one of the procedures used in the evaluation of the Munsell neutral value scale (Munsell, Sloan, and Godlove, 1933).

Another way to evaluate a uniform lightness scale is to start with black, select a gray just noticeably lighter, then one just noticeably lighter than that, and so on until the white or a near-white chip is selected.

The same two procedures may be used to set up a uniform lightness scale between any light color and any dark color, provided that color chips illustrating with very small steps the intermediate colors are available. The results of all such experiments can be conveniently compared if the number of lightness steps, V , between black and each gray chip is plotted as ordinate against the reflectance, R , of the gray chip as abscissa; and then a formula showing how V depends on R supplies a concise summary of the results.

In the original Munsell system (Priest, Gibson, and McNicholas, 1920) the relation:

$$V = 10R^{1/2} \quad (20)$$

was used for the 10-step scale from black to white.

A subsequent careful experimental study showed (Munsell, Sloan, and Godlove, 1933; Godlove, 1933) that this applies best to observations with a white background and indicated that the true relation would have to take into account the reflectance of the background as well as reflectances of the gray chips themselves. If the background is middle gray ($R_B = 0.191$) the relation:

$$V = 10(1.474R - 0.474R^2)^{1/2} \quad (20a)$$

describes well the experimental data then available.

Later studies by an OSA subcommittee (Newhall, 1940; Newhall, Nickerson, and Judd, 1943) based on more precise reflectance measurements of the samples of the 1929 *Munsell Book of Color* indicated that a small correction to equation 20a should be made. The definition of Munsell value recommended by the subcommittee is:

$$R/R_{MgO} = 1.2219V - 0.23111V^2 + 0.23951V^3 - 0.021009V^4 \\ + 0.0008404V^5 \quad (20b)$$

and this function was used by Newhall to define Munsell renotation value.* The samples of the *Villalobos Colour Atlas* adjusted by independent experiments agree well with equation 20b.

* By equation 20b the tenth step in the scale ($V = 10$) corresponds to $R/R_{MgO} = 1.0256$, the reflectance of the magnesium oxide standard being 0.976 (Preston, 1929).

The gray scale of the *Color Harmony Manual* is based (Foss, 1944) on the Weber law which states that to be just noticeably brighter one half of a photometric field must exceed in luminance that of the other half by a constant fraction. This leads to a logarithmic form such as:

$$\log_{10} R = (4V - 41)/20 \quad (21)$$

in which $V = 0$ corresponds, not to ideal black ($R = 0$), but to a good pigment black ($R = 0.009$), and $V = 10$ corresponds, not to the ideal perfectly reflecting, perfectly diffusing white surface ($R = 1.000$), but to a good pigment white ($R = 0.891$). This relation describes very satisfactorily a uniform lightness scale, provided that the visual appraisal of each small difference is made by an observer completely adapted to a luminance intermediate to those of the two grays defining the step. When the gray scale of the *Color Harmony Manual* is displayed on a white background, the observer is adapted closely to the luminance corresponding to the lightest member of the scale, and only a few members of the scale near the white end appear well spaced. Under these observing conditions the dark grays appear unduly crowded.


Table 18 gives the reflectances corresponding to a 10-step lightness scale extending approximately from black to white by equations 20, 20a, 20b, and 21. It also gives the Ostwald letter designations (Foss, 1944) for the gray specimens of reflectances found by equation 21. Equation 20 corresponds very well to appraisal of the lightness steps by an observer adapted to the luminance corresponding to the white member of the scale. Equations 20a and 20b correspond well to evaluation of the steps by an observer adapted to the middle-gray member of the scale ($V = 5$). Equation 21 corresponds to evaluation of the steps by an observer adapted in turn to each part of the scale being evaluated. These are only moderate changes in adapting luminance (changes by a factor of less than 100), yet we see that the influence on spacing is considerable. For example, the reflectance of the middle gray ($V = 5$) by equation 21 corresponds to that for the third step ($V = 3$) by equation 20.

Adams and Cobb (1922) were able to summarize by one simple equation of hyperbolic form the essential influence of adapting luminance, L_b . By this formula the brightness, B , of a field of luminance, L , would be:

$$B = 10L/(L + L_b) \quad (22)$$

where the subjective estimate known as brightness is expressed on a 10-step scale ranging from $B = 0$, the perception of a completely dark

Table 18. Reflectances of Specimens Making Up Various 10-Step Lightness Scales

Num- ber of Steps from Black <i>V</i>	Reflectance of the Specimen				Ostwald Desig- nation from Equation 21
	Equation 20 (<i>V</i> /10) ²	Equation 20 <i>a</i>	Equation 20 <i>b</i> (Munsell Renotation)	Equation 21 Antilog (4 <i>V</i> - 41)/20	
0	0.000	0.000	0.000	0.009	<i>vv</i>
1	0.010	0.007	0.012	.014	<i>tt</i>
2	0.040	0.027	0.031	.022	<i>rr</i>
3	0.090	0.063	0.066	.036	<i>pp</i>
4	0.160	0.113	0.120	.056	<i>nn</i>
5	0.250	0.180	0.198	.089	<i>ll</i>
6	0.360	0.267	0.300	.141	<i>ii</i>
7	0.490	0.378	0.431	.224	<i>gg</i>
8	0.640	0.522	0.591	.355	
9	0.810	0.712	0.787	.562	<i>cc</i>
10	1.000	1.000	1.026	.891	<i>aa</i>

field with light surroundings, to *B* = 10, which designates the perception of a dark-surrounded field so bright that further increase in luminance would fail to make it appear any brighter. Thus, brightness varies from dark to dazzling, and the appearance of the field of luminance, *L_b*, to which the observer is adapted is (*B* = 5), halfway between the two extremes, *B* = 0 and *B* = 10.

Figure 66 was plotted from equation 22 and shows correctly the essential features of the dependence of brightness *B* on luminance *L* of the field observed for various values of the luminance, *L_b*, of the surrounding field. It will be noted, first, that all fields appear dazzling (*B* = 10) to a completely dark-adapted observer (*L_b* = 0) except a completely dark field (*L* = 0) for which the brightness is indeterminate. This corresponds to the common experience of coming from a dimly lighted cellar into a bright lighted room. At the first instant, all nonblack objects, whether they be white or gray, are indistinguishable; all dazzle the eye. Second, it will be noted that an observer

adapted to a very high luminance (such as by looking at a snow field in bright sunlight) is equally unable to discriminate white objects from gray or black. To such an observer all objects, whether white, gray, or black, in a normally lighted room are indistinguishably dark. If the adapting luminance is only 10 times that of the white member of a gray scale ($L_b = 10$), the appearance of the scale will be as if it

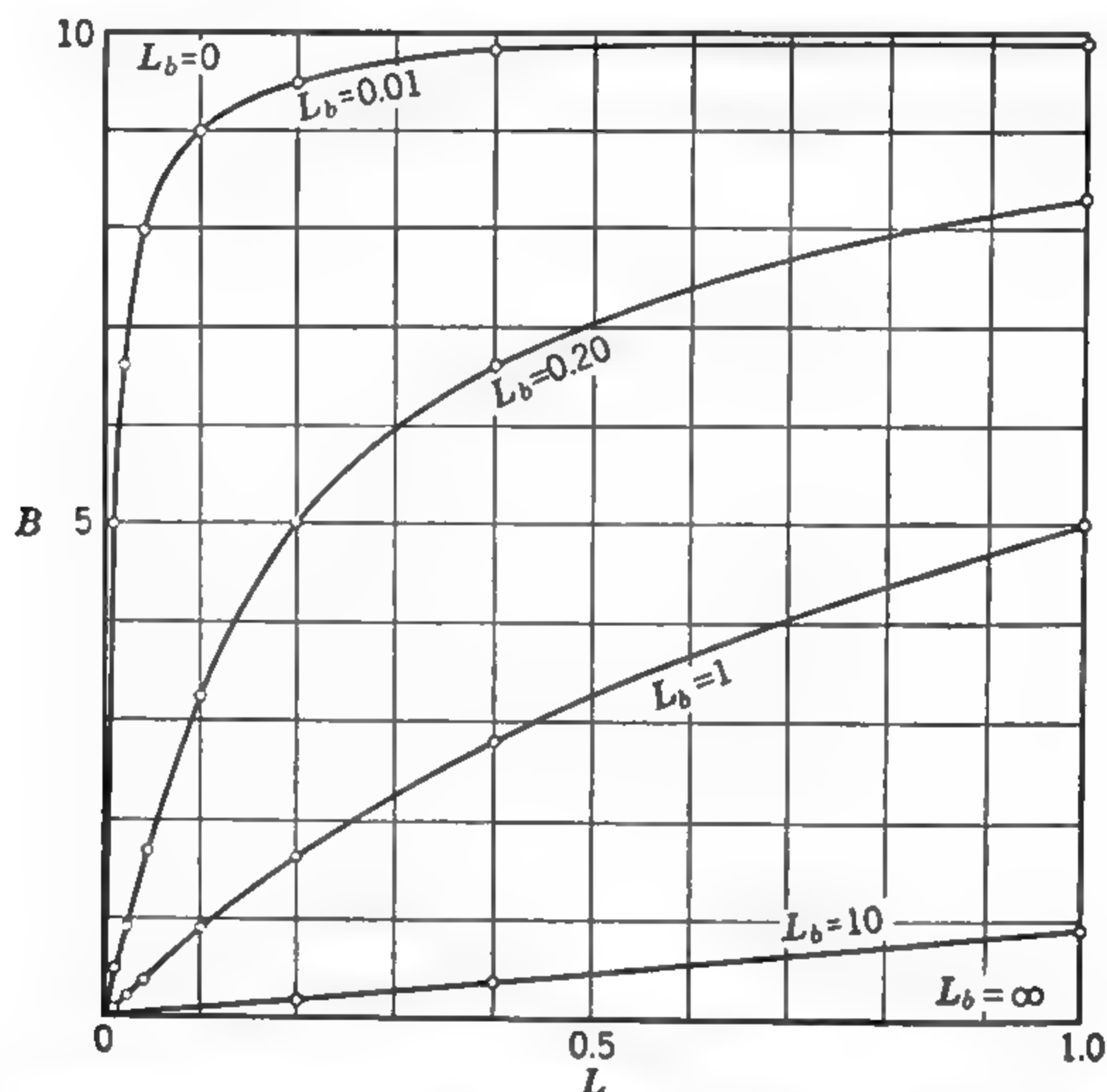


FIG. 66. Theoretical dependence of brightness on luminance (see equation 22) for the observer adapted to various surrounding-field luminances.

were compressed by about a factor of 10. All the members of a lightness scale would appear nearly but not quite alike to an observer with this adaptation.

The other three curves shown on Fig. 66 correspond fairly well to observation of lightness scales ranging from black to white displayed against a black-velvet background ($L_b = 0.01$), a middle-gray background ($L_b = 0.20$), and a white background ($L_b = 1$). We see that, if the scale is to appear equally spaced against a black velvet background, the reflectances of the dark gray members will have to differ but little compared to those at the white end. But, if the scale is to appear uniform against a white background, the spacing of reflectance must be much more nearly uniform, since the curve for $L_b = 1$ has much less curvature.

The Adams formula (22) thus permits us to place the experimentally determined lightness scales in their proper perspective relative to the adaptive state of the observer's eye. By substituting reflectance R for luminance L and adapting reflectance R_b for adapting luminance L_b , the Adams formula may be converted so as to refer explicitly to a lightness scale of equally illuminated gray specimens. It is convenient

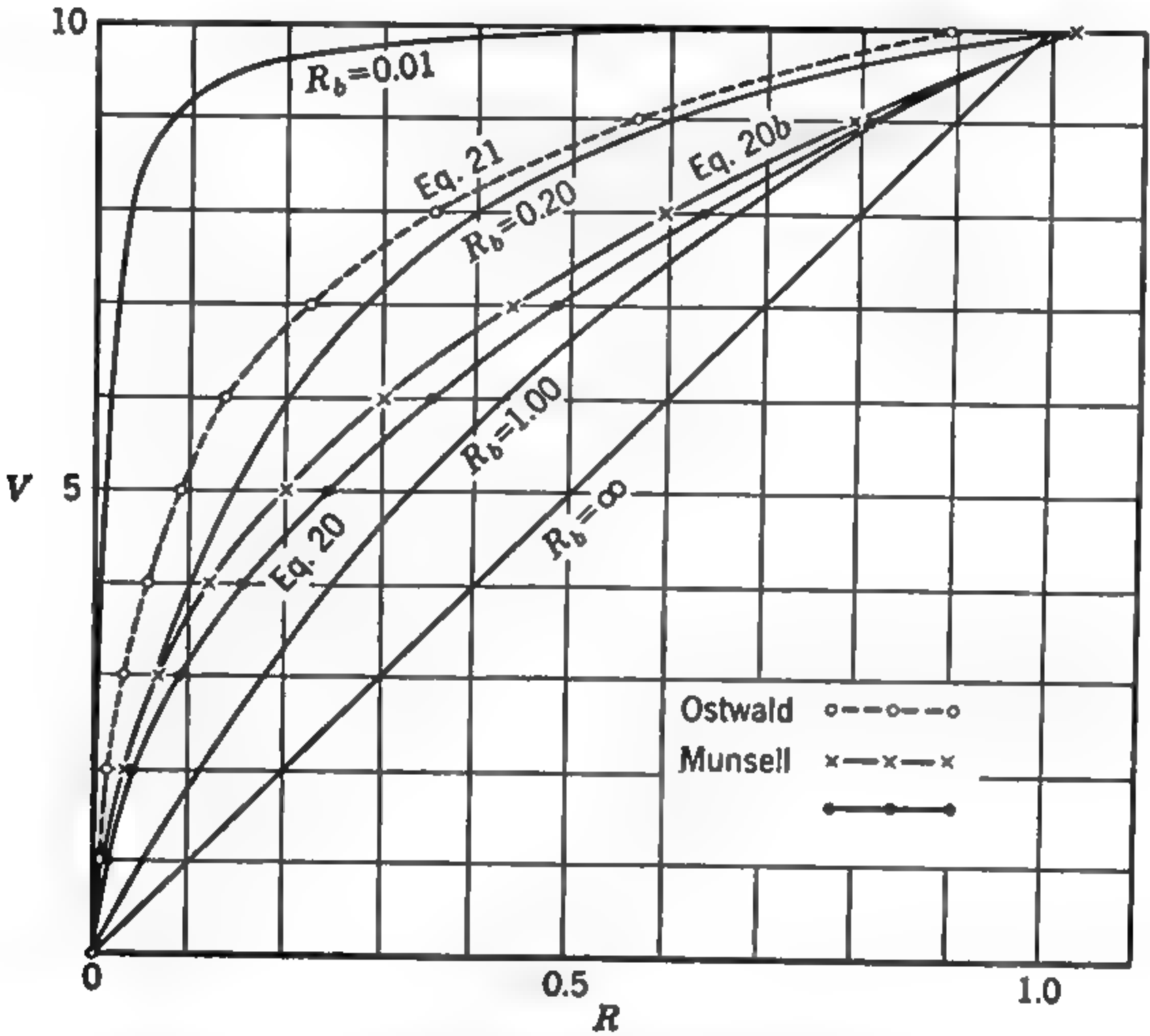


FIG. 67. Dependence of lightness on daylight reflectance according to equation 22 for white, gray and black backgrounds. The dependences defined by equations 20, 20b, and 21 are also shown.

to multiply by the factor $R_b + 1$, much as was done by Judd (1940), so that a perfect white surface ($R = 1$) corresponds exactly to 10 lightness steps from black ($V = 10$), thus:

$$V = 10R(R_b + 1)/(R + R_b) \tag{23}$$

Figure 67 shows the number of lightness steps from black V plotted against reflectance R of the corresponding gray specimen in accord with equation 23 for $R_b = 0.01, 0.20$, and 1.00 , and also shows the curves corresponding to equations 20, 20b, and 21; see Table 18. It will be seen that the original Munsell value scale (equation 20) and the present Munsell renotation value scale (equation 20b) correspond closely to adaptation to white ($R_b = 1$) near the white end of the scale,

and to adaptation to middle gray ($R_b = 0.20$) near the black end of the scale. Thus, the scale of grays defined by Munsell renotation value will appear well spaced when displayed against a white background, the adaptation of the observer being controlled somewhat by the particular gray specimens looked at as well as primarily by the background.

Because the reflectances of the end points defined by equation 21 differ from 0 and 1 by being 0.009 and 0.891 (see Table 18), it is not immediately apparent that the spacing of the gray scale used in the *Color Harmony Manual* corresponds to adaptation of the observer to the particular part of the scale being judged. Nor is it apparent by setting $L_b = L$ in equation 22, for this gives merely $B = 5$. But it may be seen by differentiating equation 22 which gives:

$$dL/L = (L/L_b + 2 + L_b/L) dB$$

The logarithmic relation given as equation 21 comes from the assumption that two halves of a photometric field are just perceptible when the difference in luminance is some small constant fraction of the luminance itself. This assumption may be written in differential notation as:

$$dL/L = k dB$$

where k is a constant. It may be seen by inspection that the term $(L/L_b + 2 + L_b/L)$ approaches the constant value 4 as L approaches L_b . Therefore, the logarithmic law (equation 21) corresponds to the general Adams formula (22) for the observer adapted to a luminance close to those being compared (L_b approaching L).

In summary it may be stated that the apparent spacing of a lightness scale depends importantly on the adaptive condition of the observer's eyes. For a dark-adapted observer a gray of low reflectance will be seen to differ greatly in lightness from the ideal black. A gray of double this reflectance will appear still lighter, but the second step will not seem as large as the first because already the high sensitivity of the observer's eye corresponding to the dark-adapted state has been reduced somewhat. All experimentally determined lightness scales illustrate this law of diminishing returns. Only for the nearly dazzled eye should we expect a scale of uniform reflectance steps to appear about equally spaced in lightness. For ordinary conditions of observation the scale of Munsell renotation value (equation 20b) is usually seen to have uniform lightness spacing. The square-root scale (equation 20) is a convenient first approximation. This known empirical

relation between reflectance and lightness is an important psychophysical tool, second in commercial importance only to the color-mixture functions. It applies to lightness differences perceived between chromatic colors as well as to those perceived among grays. We shall find repeated uses for it.

Uniform Chromaticness Scales. Chromaticness of a color perception is defined by hue and saturation taken together. In the psychological color solid, variations in chromaticness alone are represented by horizontal planes representing constant lightness. A uniform chromaticness scale is thus a two-dimensional array of equiluminous colors, each one of which appears equally different from each of its nearest neighbors. If a large number of color chips of identical luminous reflectance were available, we could proceed to select chips meeting this criterion by the method outlined in steps *a* to *g* of Fig. 60. This method seems never to have been tried.

Munsell Hue and Chroma Scales. Another way to develop this two-dimensional array of equally spaced colors of the same luminance is to (1) prepare a large number of color chips of identical reflectance (say about 20 percent corresponding to the center of the lightness scale for usual observing conditions); (2) from these chips select all those appearing at some convenient fixed departure from middle gray (such as Munsell chroma /5), so that all hues are represented though not necessarily equally spaced; (3) from this hue circuit select a small number of chips (say 5) that appear equally spaced in hue [such as the 5 principal Munsell hues, red (*R*), yellow (*Y*), green (*G*), blue (*B*), and purple (*P*)]; (4) subdivide each of the 5 hue intervals by insertion of a chip from the hue circuit equally spaced between the two terminal hues, thus producing ten equally spaced hues; (5) repeat this subdivision process as many times as desired, the first resubdivision producing 20 equally spaced hues, the second, 40, and so on; (6) for each such chip (such as at Munsell value 5/ and chroma /5) select all chips having the same hue and lightness, so that all departures from gray are represented, though not necessarily, equally spaced in saturation; (7) select from each hue series a chip having a convenient arbitrarily chosen departure from middle gray; (8) from each hue series select a second chip departing from middle gray by twice the amount arbitrarily chosen in step (7); and (9) finally repeat this selection for 3, 4, 5, and so on, multiples of the arbitrary saturation step until the end of the gamut illustrated by the chips available is reached.

So far the method accords exactly with that used in the development of the Munsell color system. The result is a set of chips of constant daylight reflectance (say 20 percent) that can be arranged in polar

coordinates as shown in Fig. 59. This selection of color chips (say at Munsell 5/) is, of course, a small fraction of the total set of chips (of reflectance 20 percent) from which the selection was made. By visual two-dimensional interpolation along the hue and saturation scales so selected it is possible, however, to specify the colors of each of the

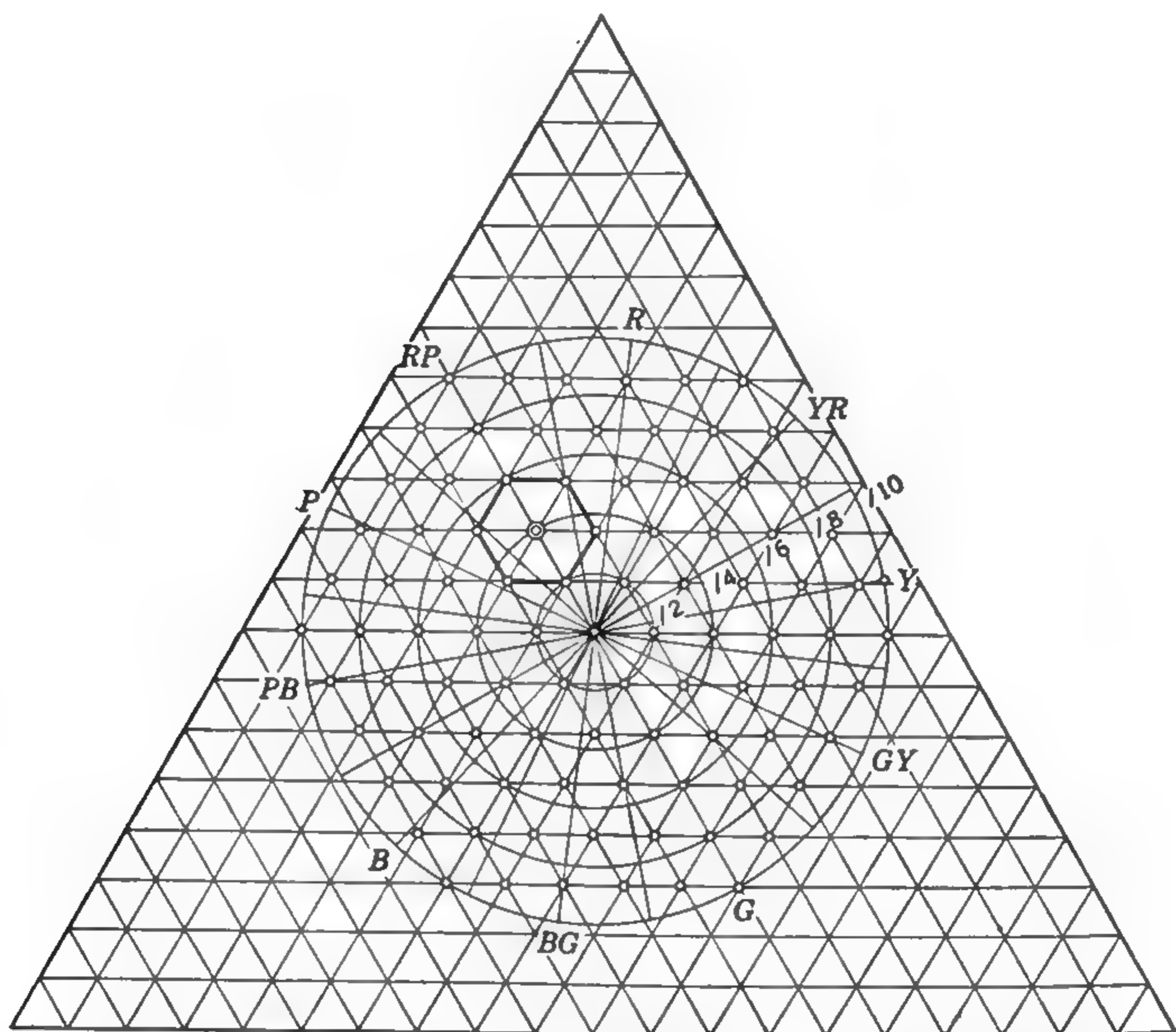


FIG. 68. Derivation of a uniform two-dimensional chromaticness scale from the Munsell data on chromaticness spacing.

remaining chips by a point on the polar-coordinate diagram of Fig. 59, or by giving the Munsell hue and chroma of each. The next step is to superimpose a triangular coordinate grid onto the polar-coordinate grid as is shown in Fig. 68. Then the color chip corresponding to the small circles on Fig. 68 would yield a uniform two-dimensional chromaticness scale of 91 chips, provided that the geometry of color space is Euclidian geometry (not proved). For example, by this method it is to be expected that 6 equiluminous colors equally spaced from Munsell 5RP 5/4 are those represented by the corners of the hexagon shown on Fig. 68. Their Munsell notations may be read from Fig. 68 as approximately: 5RP 5/6, 10RP 5/5.3, 3R 5/3.3,

5RP 5/2, 6P 5/3.6, and 9P 5/5.3. If the geometry of color space is, in fact, not Euclidian, then we do not know how to derive a uniform two-dimensional chromaticness scale of equiluminous colors, and we

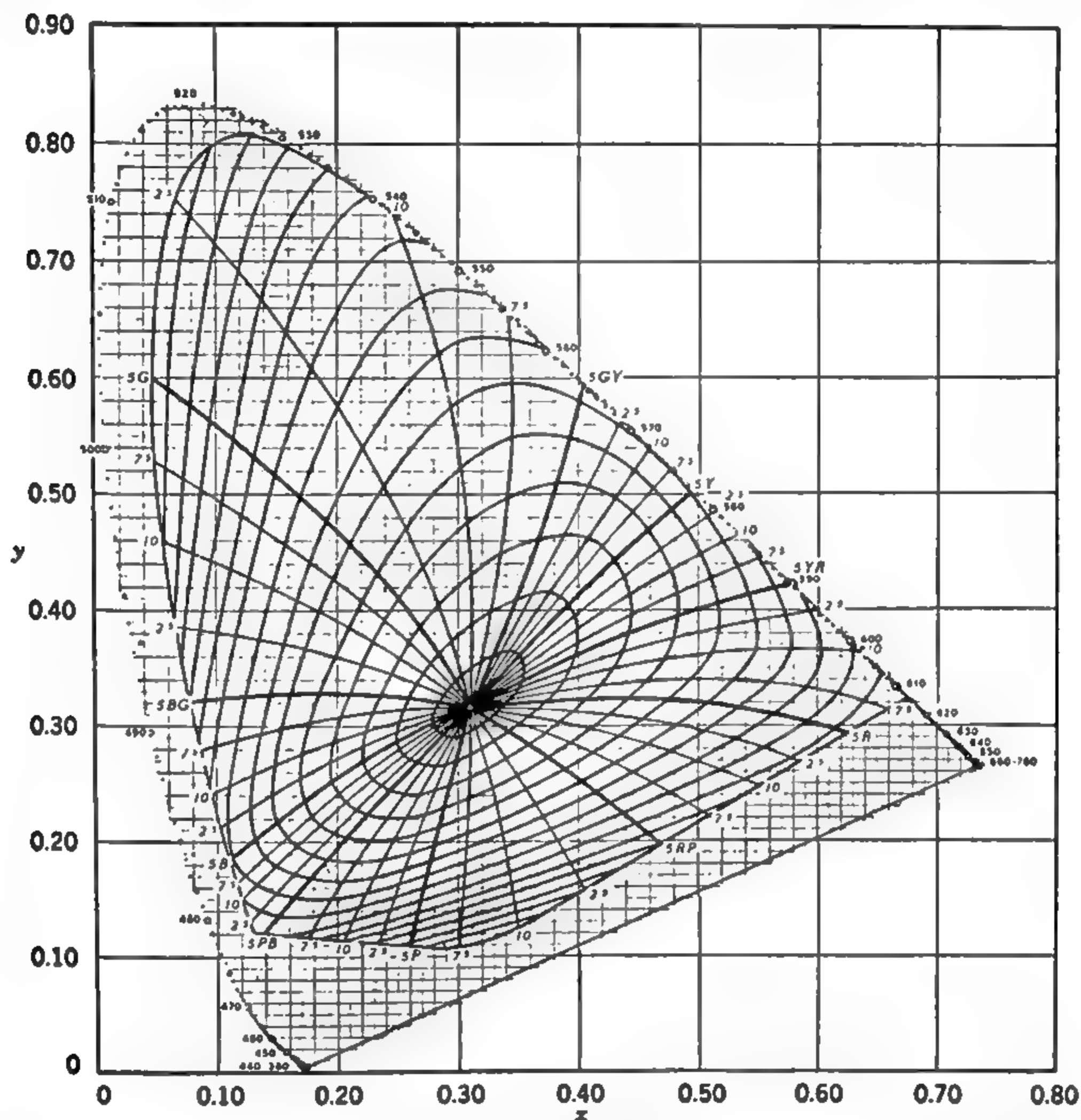


FIG. 69. Chromaticities of ideal Munsell colors, value 5/, shown on the (x, y) -chromaticity diagram. This chart serves to define Munsell notation hue and chroma for colors having $Y/Y_0 = 0.198$. (Prepared by Color Measurement Laboratory, War Food Administration, U.S.D.A.)

should be forced to entertain doubts as to whether there can be any such thing. The method would still be of commercial interest, however, because it should still lead to about as close an approximation to a uniform chromaticness scale as is possible.

A few moments' study of the *Munsell Book of Color* will convince anyone with normal color vision that at least a useful approximation to a uniform chromaticness scale can be developed. Figure 69 shows

the hue and saturation scales of the Munsell system (Newhall, Nickerson, and Judd, 1943) for Munsell renotation value 5/ (chips of reflectance equal to 19.8 percent of that of magnesium oxide) defined by chromaticity coordinates (x, y). Similar definitions have been published by the OSA Subcommittee on the Spacing of the Munsell Colors (Newhall, Nickerson, and Judd, 1943) for Munsell renotation values 1/ to 9/ (see equation 20b and Table 18 for the corresponding reflectances).^{*} These definitions are given not only for the chromaticities of the color chips in the *Munsell Book of Color* but also by extrapolation over the entire gamut of chromaticities producible by any imaginable nonfluorescent pigments (MacAdam, 1935; OSA, 1944); that is, out to the so-called MacAdam limits. Figure 70 shows the main features of these definitions. For the light end of the Munsell value scale (9/ to 5/) the contours of constant Munsell chroma ($/2, /4, /6$, and so on) increase in size only slightly for each darker step in the value scale. The chief difference in the definitions of Munsell chroma for the light end of the value scale is that they cover a much larger fraction of the chromaticity diagram at 5/ value than they do at 9/. The MacAdam limits for 9/ value ($R = 0.79$) show that it is not possible to produce color chips of daylight reflectance as high as 0.79 and still have them depart very much from the equivalent gray ($N\ 9/$), particularly in the blue, purple, and red sense. But for 5/ value ($R = 0.20$) this possibility has extended to cover more than two-thirds of the area of the chromaticity diagram. From value 5/ down to $/1$ we may note from Fig. 70 that the size of each locus of constant Munsell chroma increases rapidly with decreasing Munsell value, so that, at value 1/, the locus for chroma $/2$ already embraces nearly as large a fraction of the chromaticity diagram as the entire gamut of possible chromaticities producible at Munsell value 9/.

The Munsell chroma loci may be found for any intermediate Munsell value between 1/ and 9/ by interpolation among the loci shown on Fig. 70. Between value 9/ and value 10/ (perfect white) the chroma loci may be read reliably by extrapolation. Note how little difference there is in the chroma loci for Munsell values 7/, 8/, and 9/. The loci for value 9/ are closely correct for the whole interval from value 9/ to value 10/, the chief difference being the continued contraction of the MacAdam limits as value 10/ (Reflectance = 1) is approached. For a perfectly reflecting body only one chromaticity is possible, that of the light source itself; so the MacAdam limits collapse to a single

^{*} Packaged sets of these graphical definitions can be purchased for work sheets (10 by 14 inches) from the Munsell Color Company.

point (that representing standard source C) at Munsell value equal to 10/.

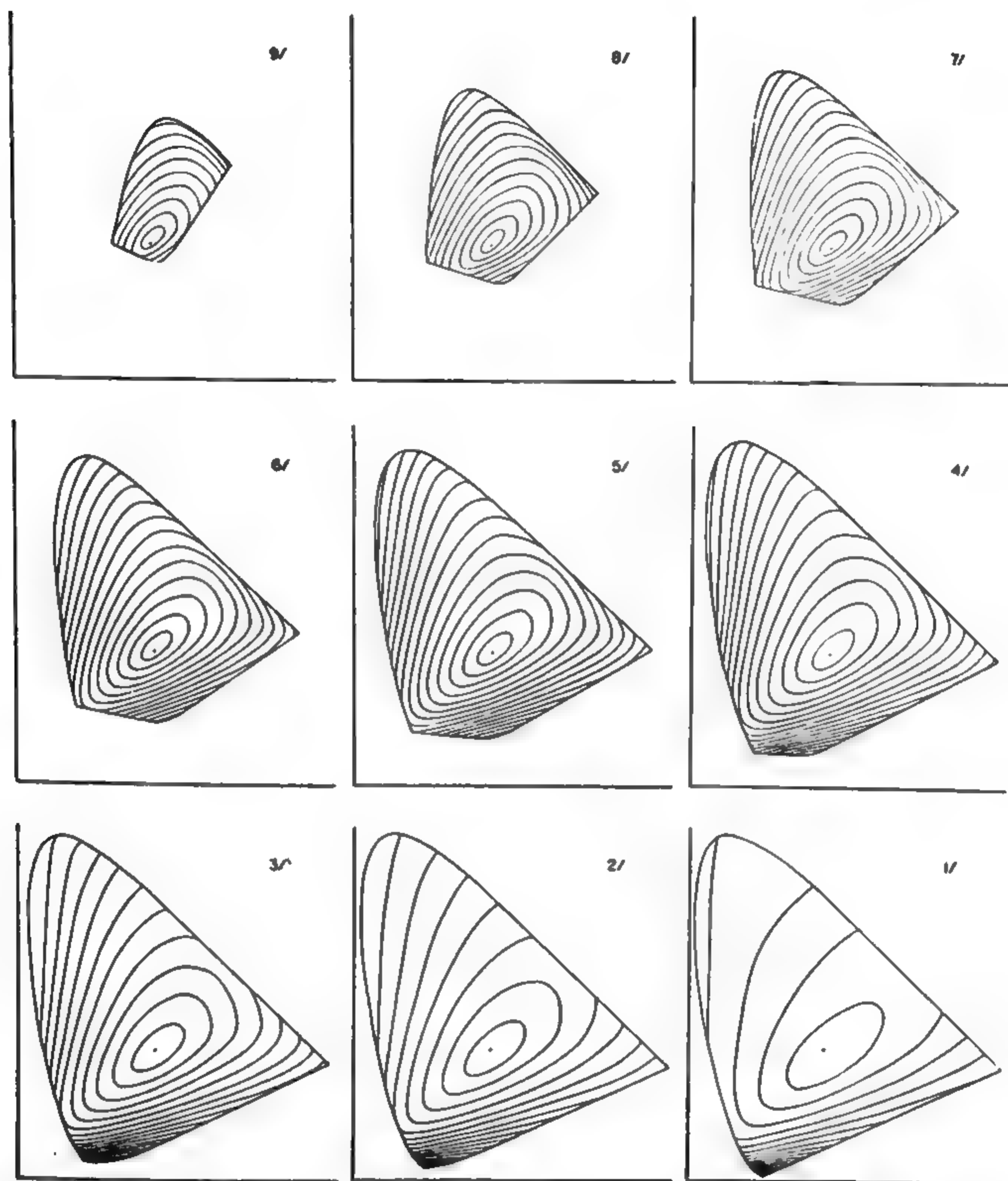


FIG. 70. Chromaticities of ideal Munsell colors, values 1/ to 9/, shown on the (x, y) -chromaticity diagram. Note that the loci of constant Munsell chroma become progressively larger as Munsell value is reduced (after Newhall, Nickerson, and Judd, 1943).

For Munsell values between 0/ and 1/ interpolation of a very unreliable sort is possible because the locations of the Munsell chroma loci for Munsell value 0/ are known; all of them are entirely off the chromaticity diagram. It is evident that color chips of reflectance just slightly above zero (Munsell value also approaching zero) can in

theory, at least, be produced with chromaticities matching all the spectrum chromaticities. We have only to imagine a color chip having spectral reflectance exactly zero for all but one narrow spectral band. Then as the width of this band is decreased we have envisaged a color chip whose daylight reflectance approaches zero while its chromaticity coordinates stay constant at those of some part of the spectrum. But notice that all such colors would have Munsell chromas approaching zero because all of them would be indistinguishable from black. Thus the locus of chroma constant at $/2$, or $/1$, or $/0.1$, or any chroma, however small, would inclose the entire chromaticity diagram for Munsell value approaching zero as a limit. One may ask how it is that a spectrum color cannot be told from the equivalent dark gray. The answer is that these chroma loci refer to observation of color chips displayed on a middle gray to white background, and the somewhat dazzling effect of the relatively high luminance of the surrounding field prevents the detection of even the greatest chromaticity differences between very dark color chips. The Munsell chroma scale, as well as the value scale, is an expression of the color-spacing characteristics of an observer with eyes adapted as they must be for viewing objects in an ordinary room. For a dark-adapted observer the chroma contours would show no such expansion with decrease in Munsell value.

Figure 71 shows 5 vertical sections of the solid figure defined by the MacAdam limits plotted by Nickerson and Newhall (1943) in Munsell renotation space, Munsell value being plotted as distance above the base plane, Munsell chroma as distance from the central black-white axis, and Munsell hue by angle about this axis. The inner construction of this solid summarizes the Munsell information both on lightness and on chromaticness scales. Comparison of present pigment gamuts with the outer boundary indicates in the most direct terms the possibilities of producing colors by nonfluorescent pigments; that is, it shows for each Munsell hue what improvements are possible, and it shows in terms of the uniform Munsell value and chroma scales how significant such improvements would be as judged by a customer.

The Munsell color-spacing data are the most important single body of data that has been developed so far on lightness and chromaticness scales. Several attempts have been made to find a coordinate system for the 1931 CIE standard observer that would yield essentially the same chromaticity spacing. Adams (1942) developed a nonlinear transformation known as the "chromatic-value diagram" to fit these data; this diagram will be considered in detail in connection with specification of color tolerances. It has been used for assessing the importance of color changes. Farnsworth (1943) developed a graphical

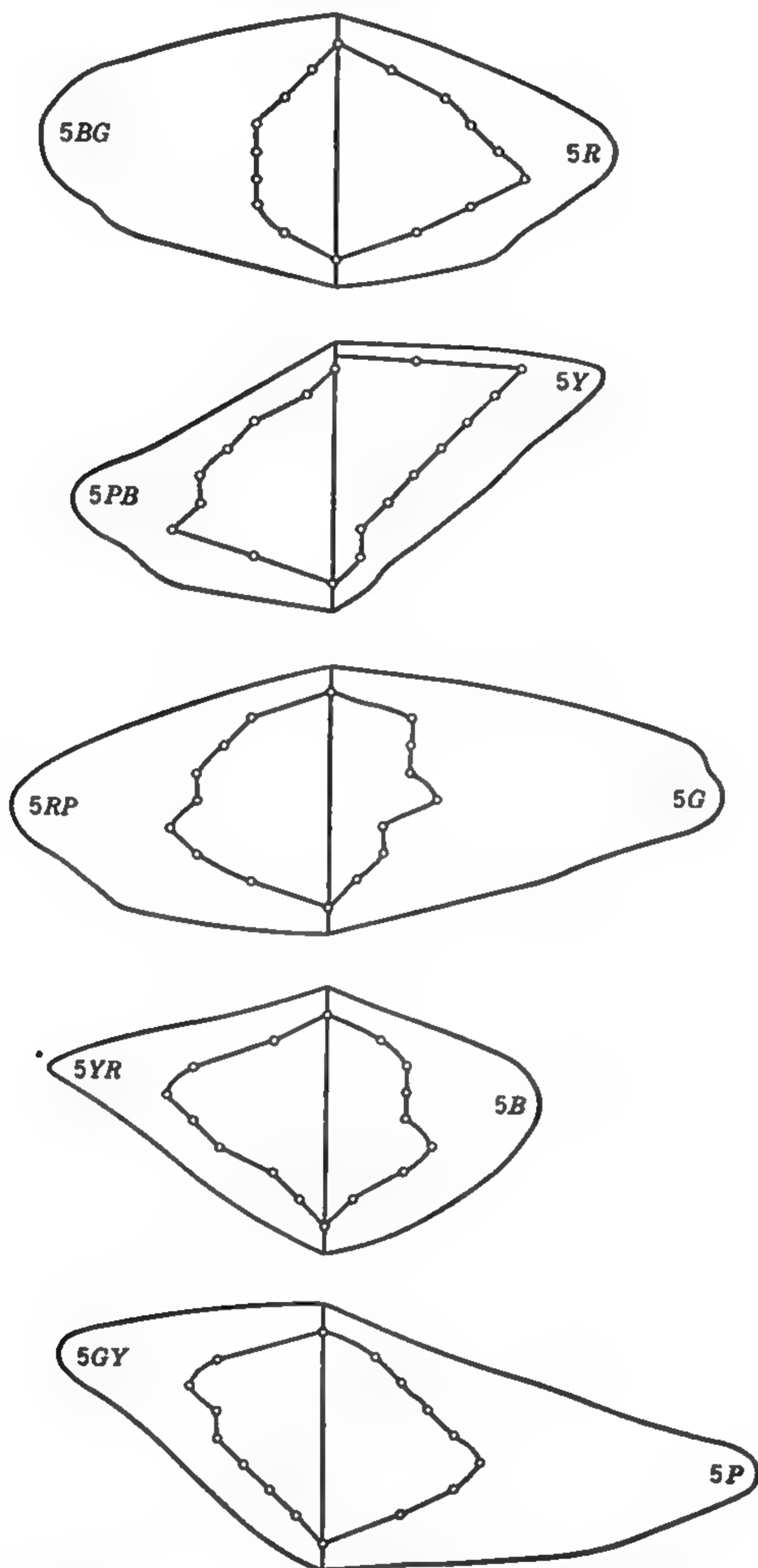


FIG. 71. Vertical sections of the solid figure defined by the MacAdam (1935) limits plotted in ideal Munsell space (after Nickerson and Newhall, 1943). The inner irregular boundaries correspond to the colors in the *Munsell Book of Color*.

nonlinear transformation of the (x, y) diagram for this purpose and later developed a projective transformation (Nickerson, 1947) for the same purpose that has been used extensively in color studies by the U. S. Naval Submarine Base in New London. Moon and Spencer (1943, b) developed a nonlinear transformation known as "omega" space intended to represent Munsell color spacing but found by Saunderson and Milner (1944) to be importantly nonrepresentative. Saunderson and Milner later (1946) developed a nonlinear transformation known as "zeta" space based on the Adams chromatic-value diagram and giving a much improved agreement with the Munsell spacing. "Zeta" space has been used by the Dow Chemical Company for color specification and for the expression of color tolerances. The advantage of being able to tell from the Munsell data how much different any two color samples will appear has been avidly seized by some segments of American industry.

Uniform-Scale Chromaticity Diagrams. Another way to develop a uniform two-dimensional chromaticness scale is to place reliance on the more or less scattered data on chromaticity spacing published prior to the Munsell data. For example, the chromaticity spacing of the spectrum colors has been determined many times and is known to good approximation; see Fig. 72. There have also been several good determinations of the chromaticities just noticeably different from some central nearly neutral chromaticity (Priest and Brickwedde, 1938; Purdy, 1931); see Fig. 73. Other determinations of just noticeably different chromaticities refer to those at constant dominant wavelength (Martin, Warburton, and Morgan, 1933; Donath, 1926), at constant colorimetric purity less than unity (Tyndall, 1933), along the locus of chromaticity (Judd, 1933) defined by Planck's radiation law (see equation 18), along the Lovibond yellow scale (Judd, 1933), and along the Lovibond 35-yellow plus red scale (Judd, 1935). It was found by Judd (1935) that these data are reasonably concordant (that is concordant within a factor of 2) with representation in a plane diagram, and a projective transformation (r, g) of the 1931 CIE (x, y) coordinate system for colorimetry was derived to represent them. If the chromaticity coordinates of a color are x, y in the standard CIE system, this color would have chromaticity coordinates r, g in this projective transformation defined by:

$$\left. \begin{aligned} r &\equiv \frac{2.7760x + 2.1543y - 0.1192}{-1.0000x + 6.3553y + 1.5405} \\ g &\equiv \frac{-2.9446x + 5.0323y + 0.8283}{-1.0000x + 6.3553y + 1.5405} \end{aligned} \right\} \quad (24)$$

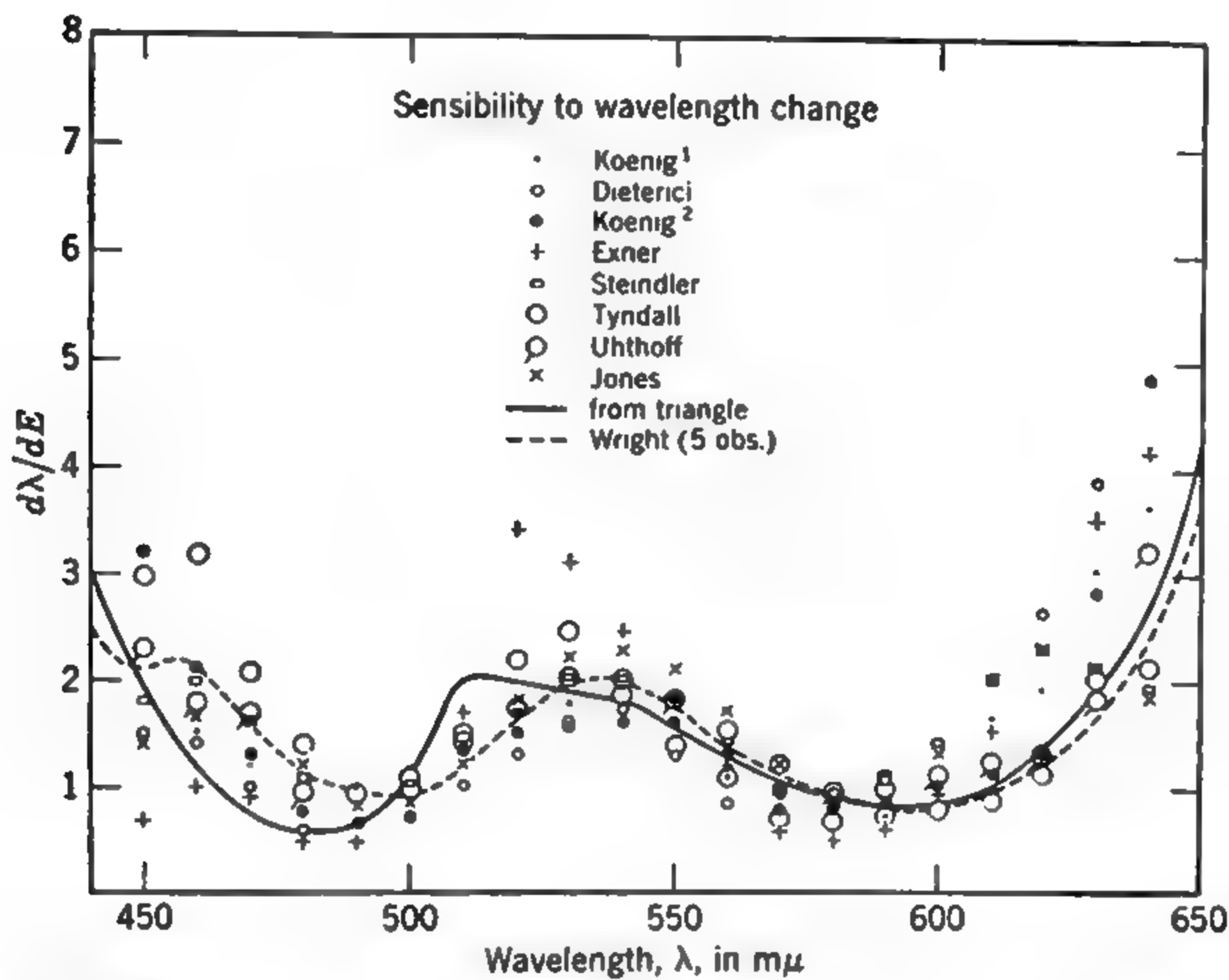


FIG. 72. Wavelength intervals corresponding to just noticeable chromaticness differences throughout the spectrum (after Judd, 1935).

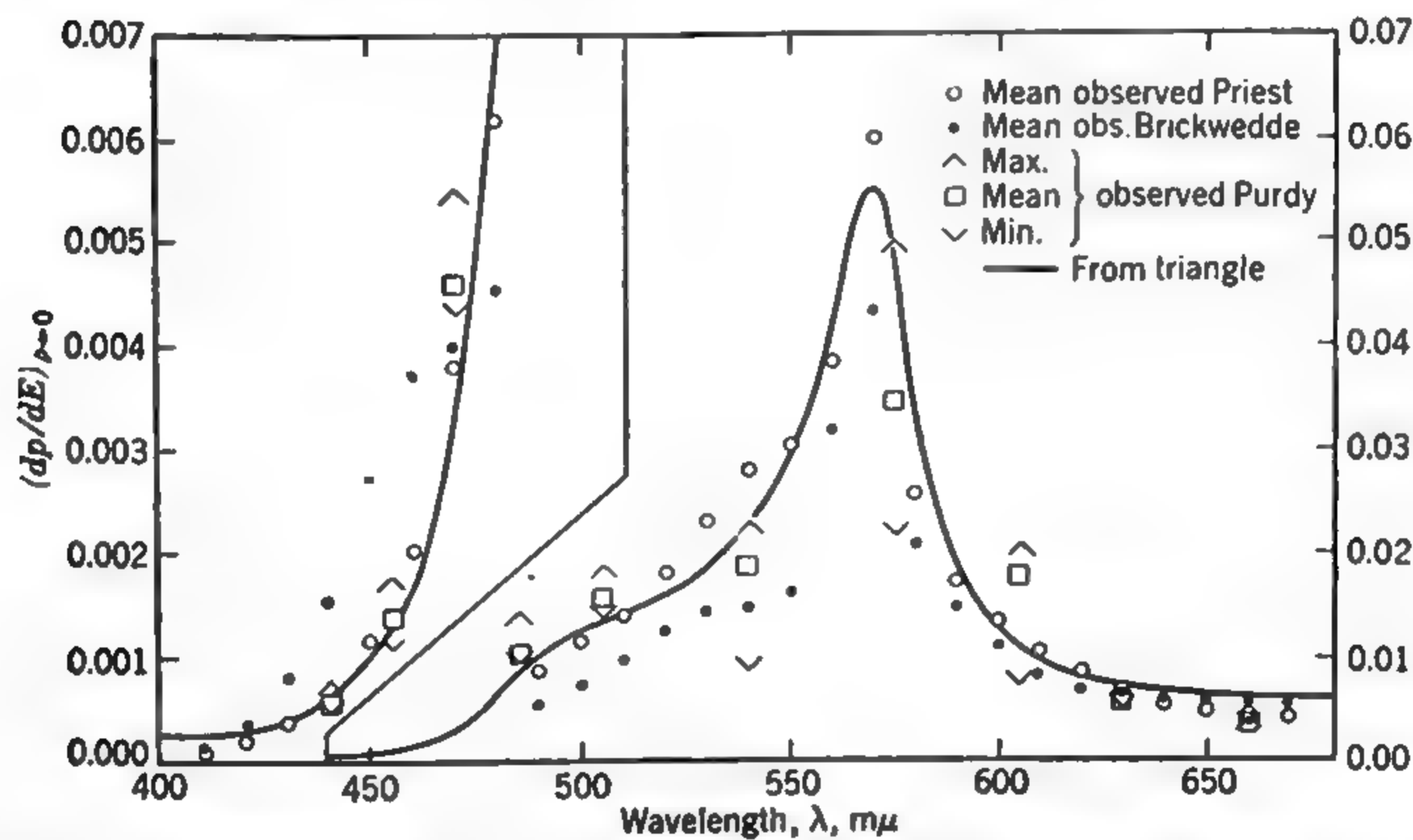
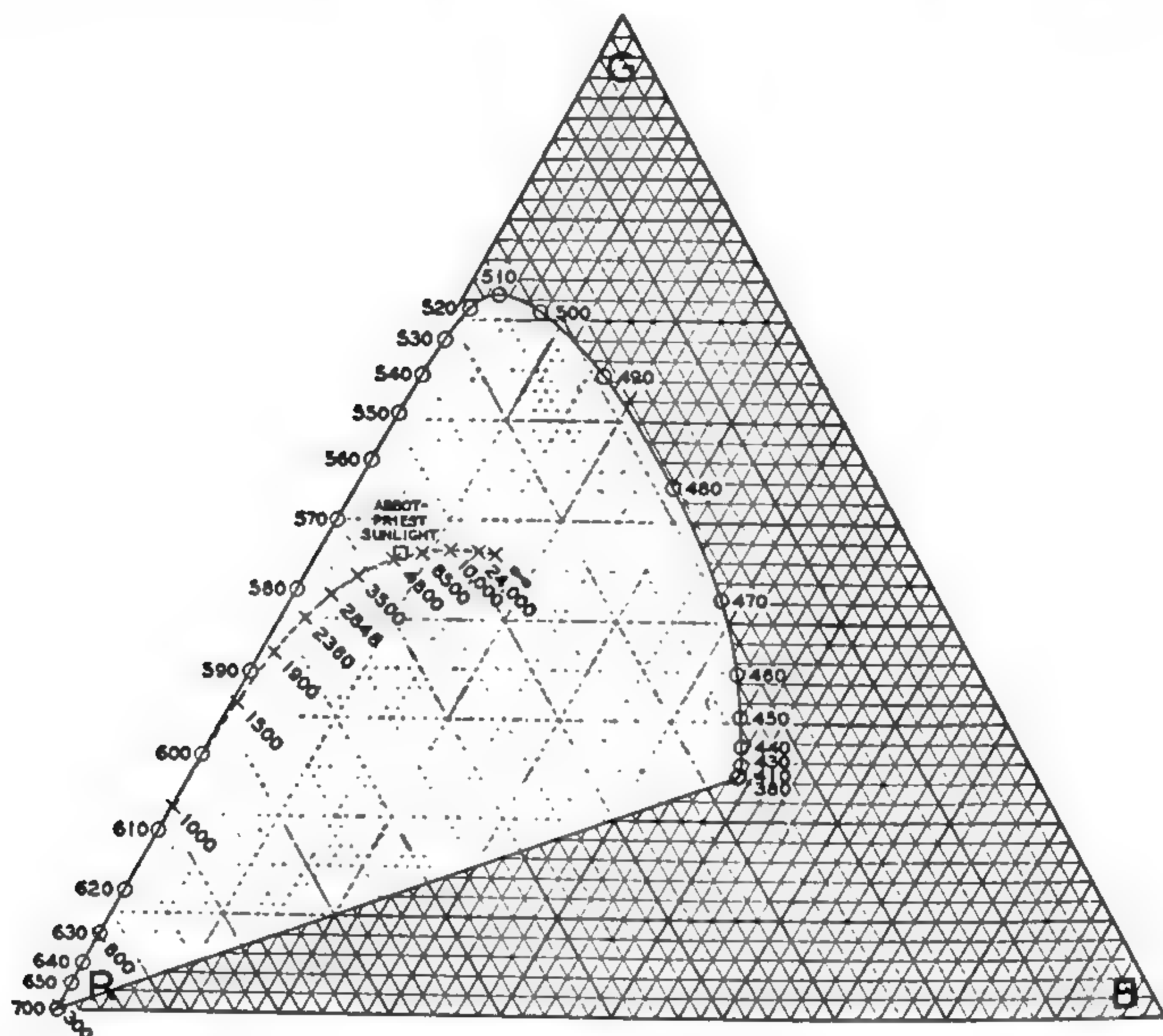


FIG. 73. Colorimetric purities of chromaticities just noticeably different from neutral (after Judd, 1935).

By plotting r against g in triangular coordinates (axes intersecting at 60°), Fig. 74 is produced. This diagram is called a uniform-chromaticity-scale (UCS) diagram because it yields, according to the spacing data on which it is based, approximately constant distance between pairs of points corresponding to chromaticities found experimentally



nates (r, g) would provide a two-dimensional approximately uniform chromaticness scale in accord with the data summarized by the UCS triangle.

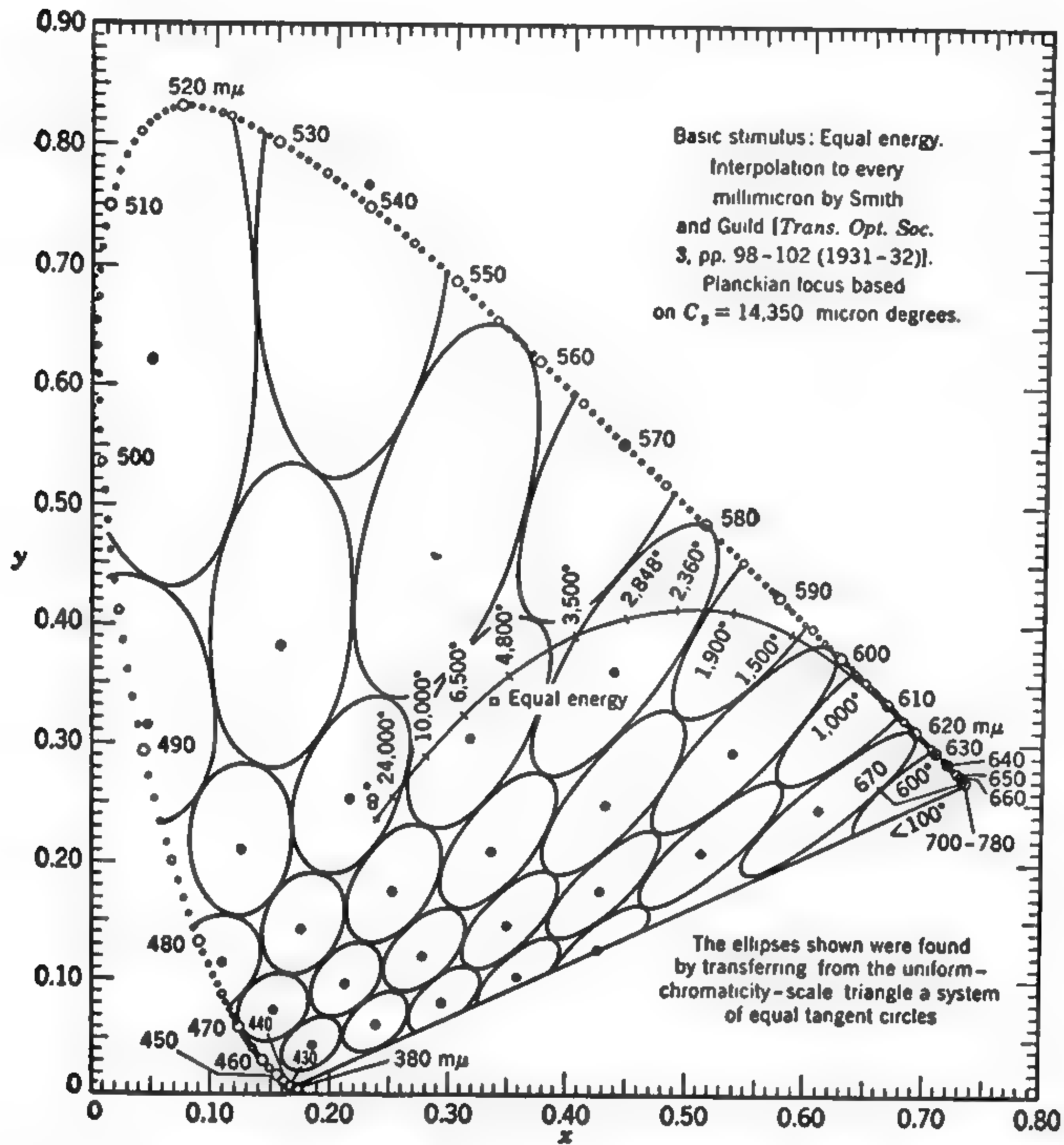


FIG. 75. Perceptibility scales for the (x, y) -chromaticity diagram (after Judd, 1936). All the distances from points on boundary of each ellipse to the indicated point within it correspond approximately to one hundred times the chromaticity difference just perceptible with certainty under moderately good observing conditions. The ellipses shown correspond to equal tangent circles on the uniform-chromaticity-scale diagram shown as Fig. 74.

Figure 75 shows a series of ellipses on the (x, y) -chromaticity diagram corresponding to a set of equal tangent circles on the UCS diagram. The chromaticity interval from the center of any of these ellipses to any chromaticity point on its edge corresponds to about

100 just noticeable chromaticity steps. It is seen that the standard (x, y) diagram spaces out the green too much and greatly compresses the bluish purples.

The UCS triangle (Fig. 74) has been used as a guide in the chromaticity spacing of glass standards for the colors of rosin (Brice, 1940) and of petroleum products (Judd, Plaza, and Belknap, 1950). It has also been used in the interpretation of one-dimensional color scales applied to colors not falling exactly on the one-dimensional locus defined by the members of the scale. Thus, the isothermperature lines on Fig. 61 would be lines of shortest distance between the point representing the non-Planckian chromaticity and the Planckian locus on Fig. 74. These isothermperature lines are used in setting chromaticity tolerances for the purchase of fluorescent lamps by government agencies (General Services Administration, 1950). The UCS triangle was used by McNicholas (Fig. 63) to assign designations on the 35-yellow plus N -red scale to the colors of vegetable oils not falling exactly on that scale. It was used (Judd, Plaza, and Belknap, 1950) to assign Union Color Numbers to petroleum products whose colors do not fall exactly on that one-dimensional color scale (see Fig. 65). The UCS triangle was used by Tintometer, Ltd., for the form of graphs prepared and sold by them to convert Lovibond readings (number of red, yellow, and blue units on the Lovibond scales) to chromaticity coordinates x, y on the standard CIE system. It was used by Bouma (1937) to assess the importance of color differences caused by changing from one light source to another. The method of specifying the color-rendering properties of a light source derived in this way by Bouma has been provisionally adopted by the International Commission on Illumination (1948). The UCS triangle was used by McNicholas to derive a color designation for the signals used in his study of selection of signal colors for certainty of identification (1936). It was used (Judd, 1935) to extend to all dominant wavelengths the method derived empirically by MacAdam (1934) for one dominant wavelength (575 $m\mu$) for defining an index of whiteness for laundered textiles. The extended method has been applied to whiteness of paper (Judd, 1935, 1936). The UCS triangle has thus found its way into commercial transactions of widely different kinds.

The chromaticity spacing of the UCS triangle has been incorporated into two other projective transformations of the CIE system so chosen that the chromaticity diagram can be plotted in rectangular coordinates. One of these, the (u, v) plot developed by MacAdam (1937), has been included in the report of the OSA Colorimetry Committee

(OSA, 1944); see Fig. 76. The chromaticity coordinates, u , v , are related to those, x , y , in the CIE system by the defining equations:

$$\left. \begin{aligned} u &\equiv \frac{0.4661x + 0.1593y}{1.0000y - 0.15735x + 0.2424} \\ v &\equiv \frac{0.6581y}{1.0000y - 0.15735x + 0.2424} \end{aligned} \right\} \quad (25)$$

The other projective transformation is the (x'', y'') plot developed by Breckenridge and Schaub (1939), which has become known as the RUCS system (rectangular-uniform-chromaticity-scale system). The connection of the RUCS coordinates, x'' , y'' , with the CIE standard chromaticity coordinates, x , y , is given in two steps by way of intermediate coordinates x' , y' :

$$\left. \begin{aligned} x' &\equiv \frac{0.82303x + 0.82303y - 0.82303}{1.00000x - 7.05336y - 1.64023} \\ y' &\equiv \frac{3.69700x - 5.07713y - 1.36896}{1.00000x - 7.05336y - 1.64023} \\ x'' &\equiv 0.0750 - x' \\ y'' &\equiv y' - 0.5000 \end{aligned} \right\} \quad (26)$$

Tables have been prepared for carrying out this transformation by interpolation with uncertainties less than 1 in the third decimal (Breckenridge, 1948). The RUCS system places the chromaticity of the equal-energy source at the origin of the coordinate system. Since this source appears under the usual observing conditions to have nearly an achromatic or neutral color, the coordinate axes of the RUCS system may be said to divide the chromaticity diagram approximately into four ranges of hues. The first quadrant corresponds to the hue range, yellow to green; the second quadrant, green to bluish purple; third quadrant, bluish purple to red; and the fourth quadrant, red to yellow. The RUCS system has the further convenience that the long-wave branch of the spectrum locus is parallel to one of the coordinate axes. It has been used by Holmes (1941) to plot his results on the recognition of colored light signals, and these results go far to corroborate the correctness of the chromaticity spacing for the colors of small spots viewed against a dark background.

The chromaticity spacing of the UCS triangle has been subject to two further exhaustive tests. The first of these is by Wright (1941)

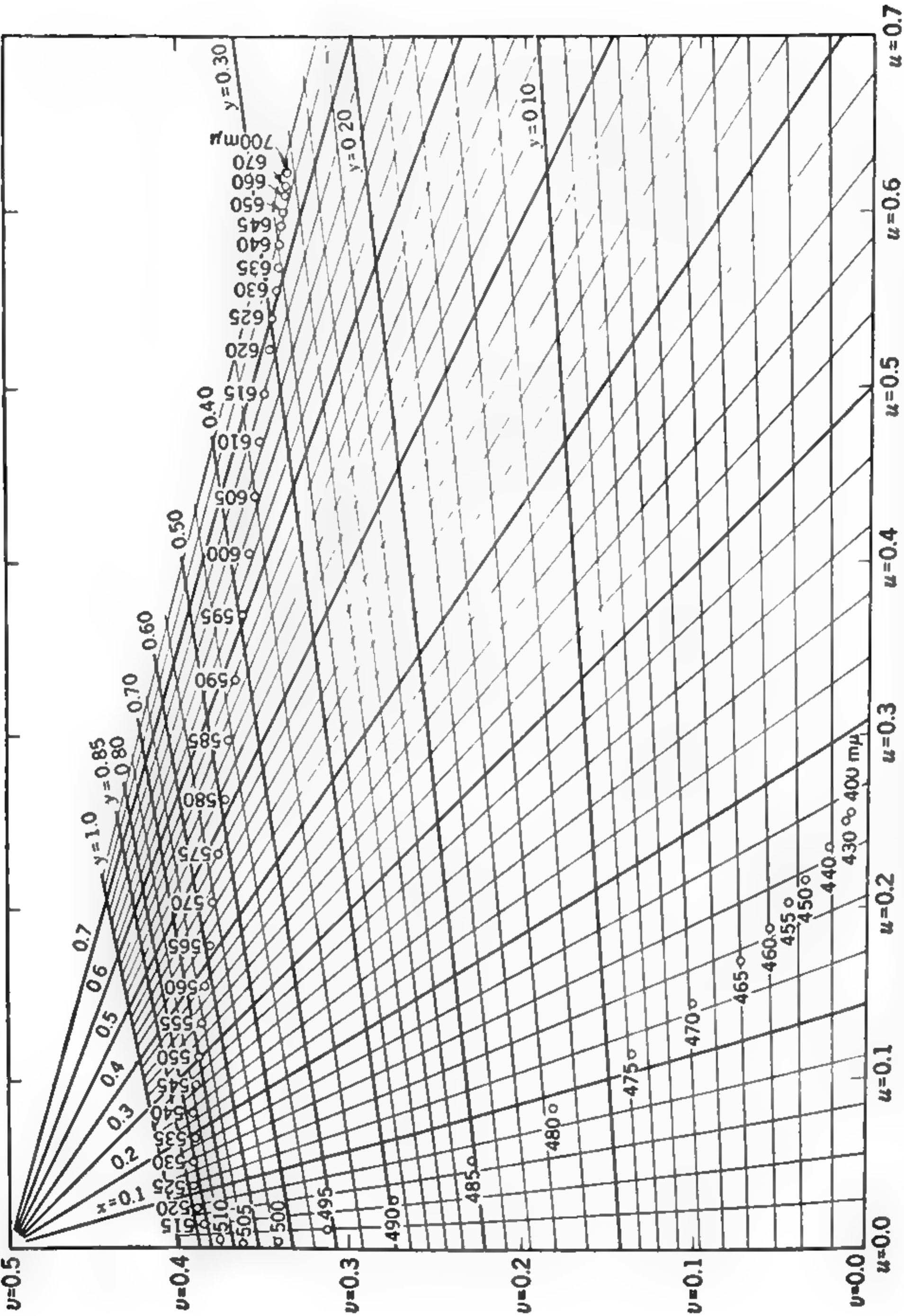


FIG. 76. Rectangular uniform-chromaticity-scale diagram (after MacAdam, 1937).

who studied the perceptibility of chromaticity differences along the spectrum locus and along a large number of paths produced by addi-

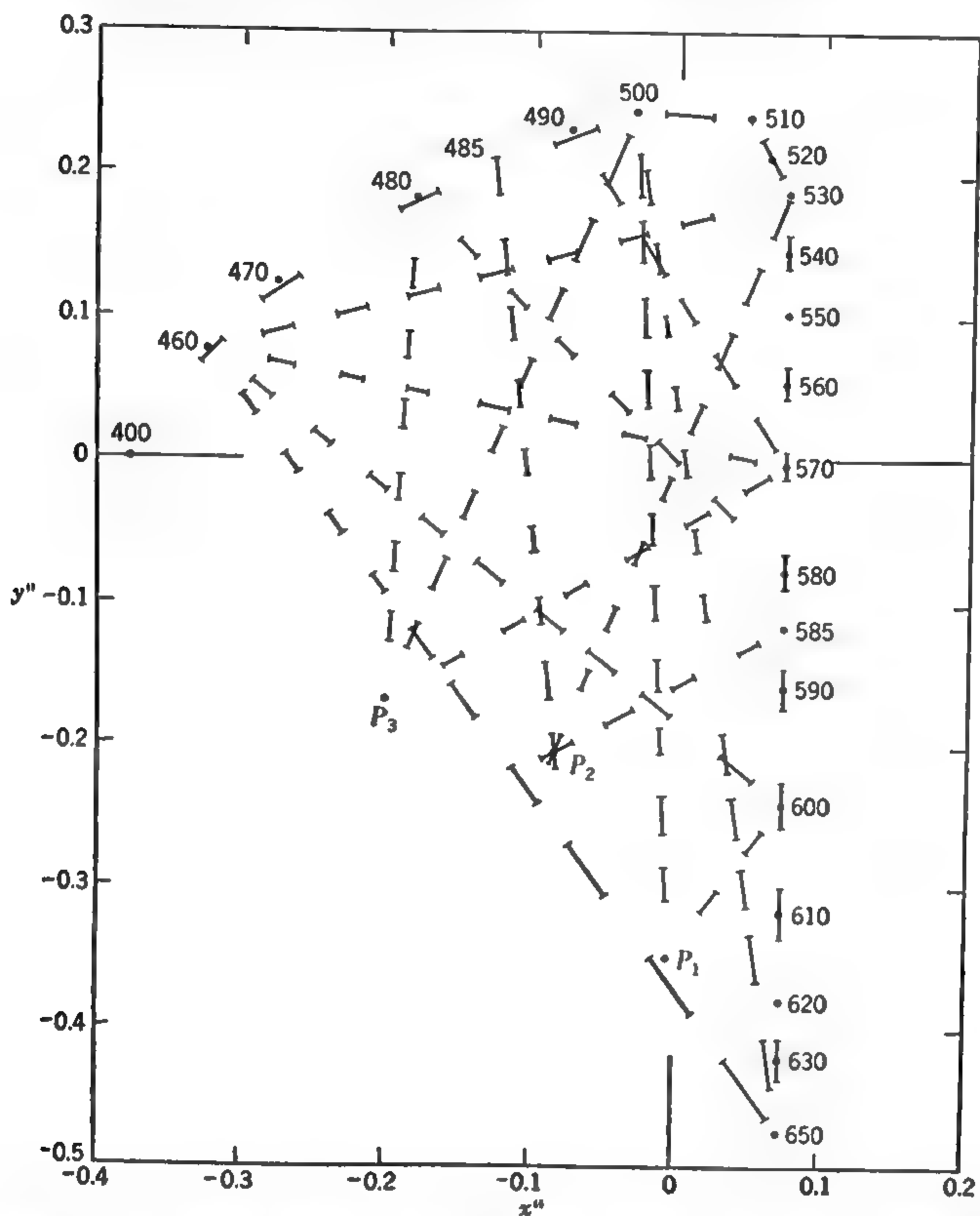


FIG. 77. Rectangular uniform-chromaticity-scale diagram developed by Breckenridge and Schaub (1939). The dashed lines indicate chromaticity intervals, all of which were found by Wright to correspond to the same degree of perceptibility (after Wright, 1943).

tive mixture of pairs of lights. Figure 77 shows on the RUCS triangle a few of the results selected by Wright (1943) as typical. The points P_1 , P_2 , and P_3 represent the chromaticities of three purple lights mixed with various parts of the spectrum as indicated. The chromaticity

interval defined by each dash in the dashed lines was found by Wright to correspond to a chromaticity difference judged to be the same as that for each other dash. If the chromaticity spacing of the RUCS

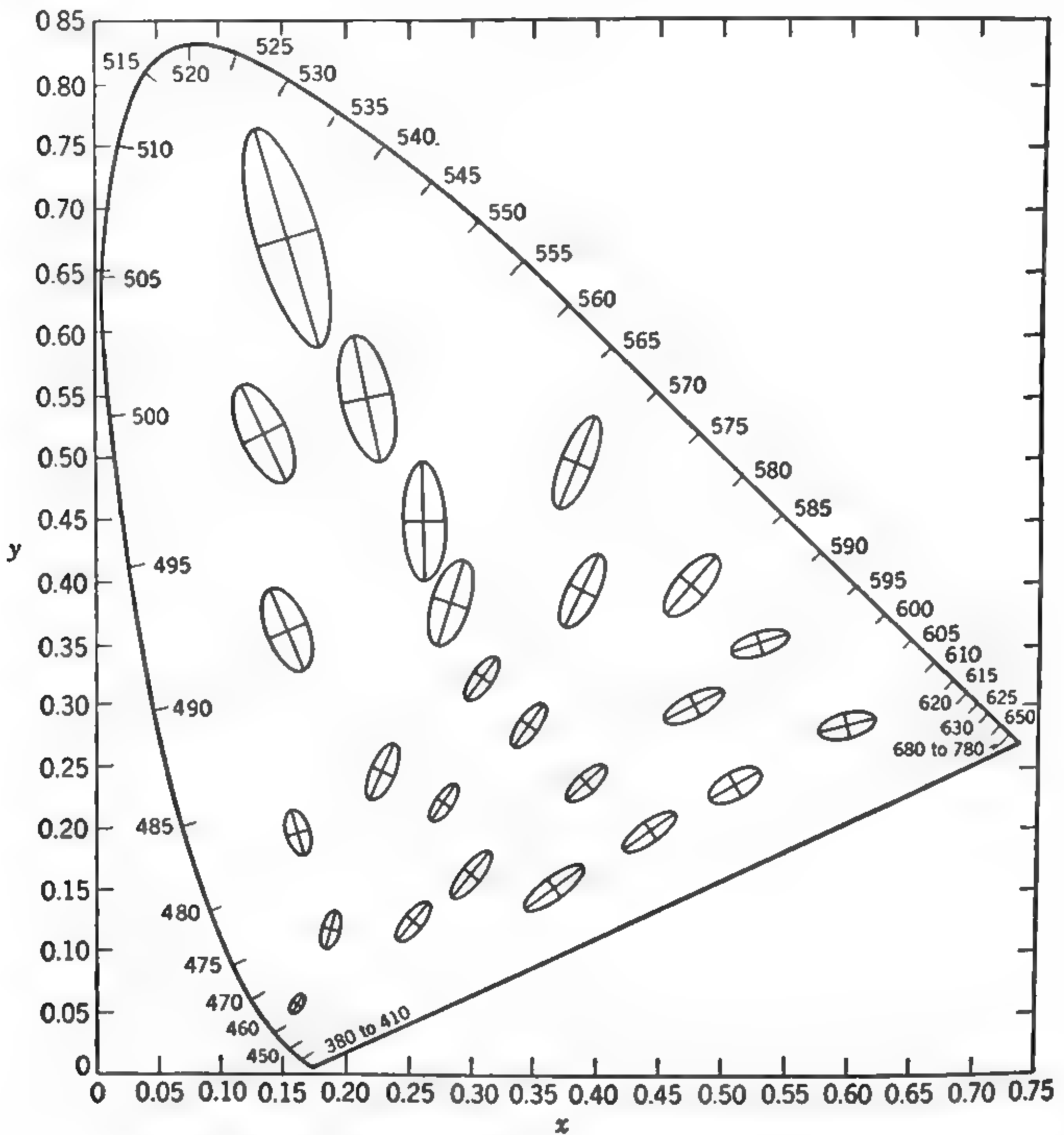


FIG. 78. Nutting's data on perceptibility of chromaticity differences shown on the (x, y) -chromaticity diagram (after MacAdam, 1942). The elliptical loci correspond to chromaticities separated from the central point by ten times the standard deviation of settings for chromaticity match.

diagram were uniform as intended, each dash would be exactly the same length. Figure 77 shows the lengths of dashes in general about equal, but there are some consistent deviations from equality. For example, the dashes along the line representing mixtures of the spectrum at 460 $m\mu$ with that at 650 $m\mu$ become progressively larger as 650 is approached, suggesting that the red end of the spectrum is pulled out too far on the UCS triangle.

The second extensive test of the spacing of the UCS triangle was made by MacAdam (1942), the bulk of the observations referring to one observer, P. G. Nutting, Jr. The criterion of the size of a chromaticity difference was the extent of errors in that direction in settings for a chromaticity match. The field observed was 2° in diameter, had a luminance of 15 millilamberts, and was surrounded by a 21° field of daylight quality (standard source C) and luminance of 7.5 millilamberts. The results are shown on Fig. 78, the noticeabilities of chromaticity differences being represented by loci of ten times the standard deviation on the (x, y) -chromaticity diagram surrounding each chromaticity point studied. Comparison with Fig. 75 shows that the main features of the UCS spacing have been corroborated; that is, the green part of the (x, y) diagram is shown to be considerably overspaced, and the bluish purple portion overcompressed. MacAdam (1943) made a careful analysis of his data, however, and proved that not only did the UCS triangle fail significantly to accord with them but also no linear transformation of the (x, y) -chromaticity diagram could account for them, nor even could any non-linear transformation such as proposed by Moon and Spencer (1943, a) do it. In short, no plane chromaticity diagram can be plotted by any means whatever that will represent by equal distances chromaticities judged by the one observer (Nutting) to be equal. MacAdam constructed the curved surface required to do it, and the photograph of it (MacAdam, 1949, b) shows that this surface is far from being a plane. It looks something like a very much rumpled felt hat. There are a bulge in the middle and ruffles around the edge, and it is otherwise irregular and skewed. The kind of difficulty run into in trying to represent Nutting's observations on a plane diagram may be seen from MacAdam's plot of these data on the UCS triangle; see Fig. 79. Chromaticity spacing in accord with that of the UCS triangle would, of course, be represented by circles of equal size; and the fact that the ellipses are of nearly the same size and something like circles shows the approximate corroboration of the UCS triangle already noted from Figs. 75 and 78. However, note that the ellipses in the center of the diagram tend to be smaller and more like circles than those around the border. To represent these chromaticity differences, judged by Nutting to be equally noticeable, by lines of equal length in a two-dimensional surface requires us to make the surface dome shaped; that is, the moderately peripheral parts of the diagram have to be made smaller than shown on the plane diagram of Fig. 79, and, if a circuit around the central part of the diagram ($x = y = 0.3$) is squeezed together, the central part will have to bulge. This is known as a surface of positive curvature. Further inspection of Fig. 79 will

disclose another regularity. There is a tendency for the major axes of the ellipses in the extreme periphery to point in the general direction of the center ($x = y = 0.3$) of the diagram. To represent these equally noticeable chromaticity differences by lines of equal length in a two-dimensional diagram requires that the radial distances be contracted and the peripheral distance be expanded. This again forces us to leave the plane, but the dome shape given to the center of the surface expands

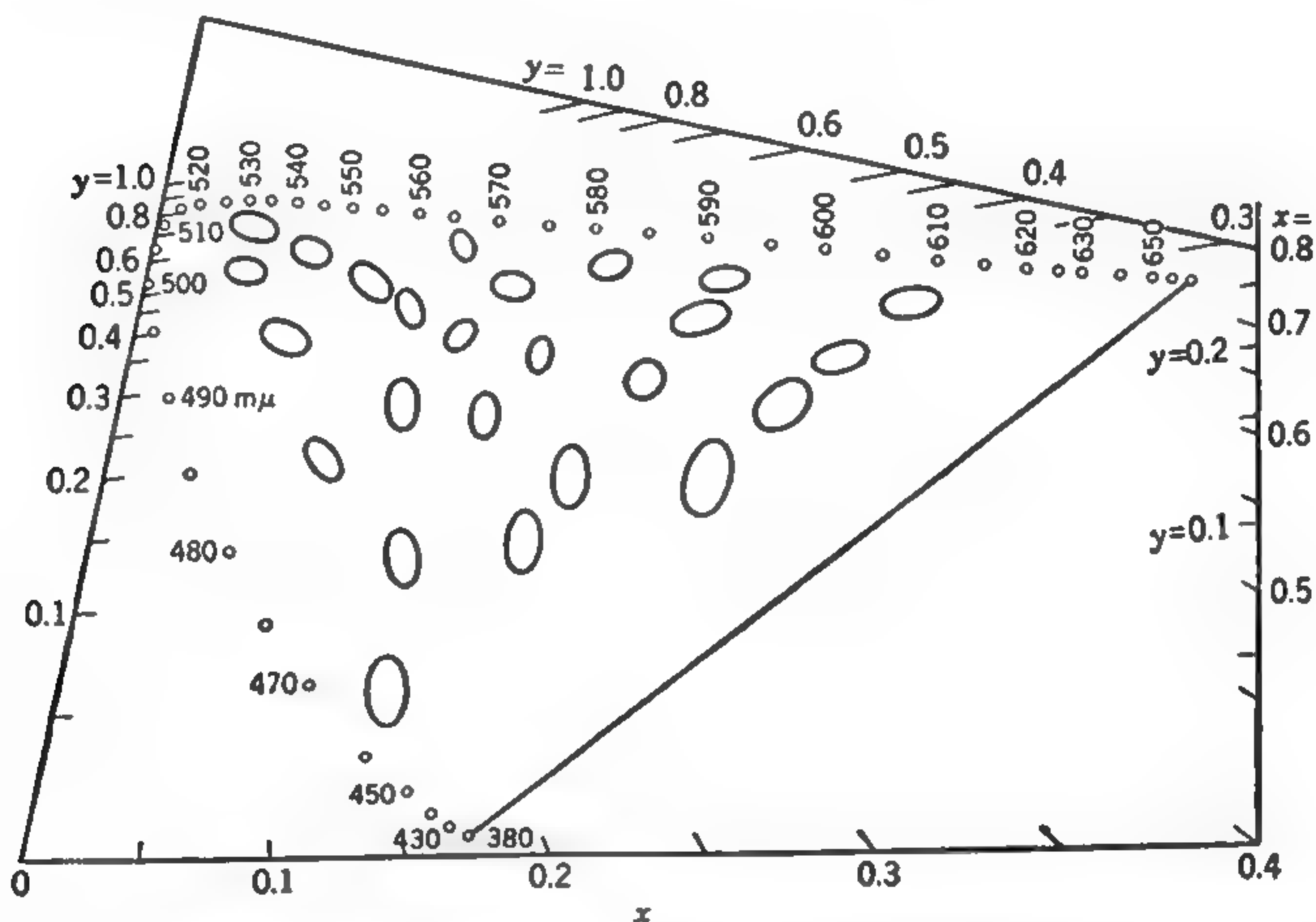


FIG. 79. Nutting's data on perceptibility of chromaticity differences shown on the UCS triangle of Fig. 76 (after MacAdam, 1943).

radial distances relative to peripheral and so is the exact opposite of the kind of surface required. What is required is a surface of negative rather than positive curvature, such as a saddle-shaped surface. This accounts for the ruffles found by MacAdam to be required around the edges of the two-dimensional surface representing Nutting's determinations of chromaticity spacing.

MacAdam (1943) has carefully worked out a special differential geometry applying to this complicated surface, which has partly negative and partly positive curvature, so that industrial color differences can be evaluated in accord with Nutting's data on chromaticity spacing if anyone cares to do it. It should be pointed out, however, that both the bulge and the ruffles of this surface refer to properties of the eye well known in other terms. This is the same property that was found to exert a controlling influence on the character of the lightness

scale, namely, adaptation. The most favorable adaptive condition for detecting chromaticity differences (Schönfelder, 1933) is to have the eye adapted to a chromaticity as closely like the two being compared as possible. In MacAdam's study his observer's adaptation was largely determined by the surrounding field of daylight quality. The loci of constant-chromaticity difference near the center of Fig. 79 ($x = y = 0.3$) ought, therefore, to be smaller than those in the periphery. This accounts for the bulge (positive curvature) in the center of the two-dimensional surface. Now, as to deviations from chromaticities far different from daylight quality, it should be noted that the observer, thus adapted to daylight, will perceive radial deviations as saturation differences and will perceive peripheral deviations as hue differences. When the observer fixates one of two chromaticities yielding the perception of a saturation difference, the perceived saturation immediately commences by local adaptation to decrease and will continue to decrease until a blink forces a change of fixation. Newhall (1932) succeeded in measuring the time course of such local color adaptation between blinks. The amount of this decline in saturation is very large, easily more than 10 times the just noticeable saturation difference. As the observer looks from one half-field to the other to decide whether or not they are different, first one half-field and then the other is viewed by a parafoveal part of the retina adapted to the daylight surrounding field. Each half-field is thus seen to be much more saturated than the other in turn, and the saturation difference corresponding to comparison of the two half-fields by equally adapted retinal areas is largely obscured. This is an analysis and explanation of why the surrounding field should be as much like the colors to be compared as possible. If the two chromaticities differ in the peripheral rather than the radial sense, the disturbing influence of the daylight surrounding field is less. The fixated half-field still appears less saturated than the other, but as the fovea and parafovea together become more and more adapted to the average of the chromaticities being compared, the hue difference becomes more and more apparent, until for complete local adaptation the hues become actually complementary. The result to be expected is exactly that shown on most of the peripheral ellipses of Fig. 79. Radial differences cannot be reported on as certainly as differences represented by vectors perpendicular to the radius. The loci of equally perceptible chromaticity differences from any fixed chromaticity near the periphery should be expected to be ellipses with the major axis pointing toward the chromaticity to which the observer is adapted. This accounts for the ruffles around the edge

of the two-dimensional surface constructed by MacAdam. It also accounts for the difficulty pointed out in step *g* of Fig. 60: the number of just noticeable hue steps experimentally found in the hue circuit at constant saturation is usually greater than 2π (6.28...) times the number of just noticeable saturation steps between the neutral chromaticity and any one of the chromatic colors. This analysis supports the hope of constructing a two-dimensional chromaticness scale that is uniform in the sense that, if the observer becomes completely adapted to the particular part of the array of chips that he is inspecting, the spacing will appear uniform. Note the good approach to circles for the loci near the center of Fig. 79. This analysis also supports MacAdam's contention that no two-dimensional chromaticness scale can be constructed that will appear uniformly spaced if the entire array is inspected quickly by an observer with any fixed chromatic adaptation. Just as in lightness scales, so for chromaticness scales also, the key to uniformity of spacing is the adaptation of the observer.

Stiles (1946) has developed a body of chromaticity-spacing data from experiments on the noticeability of the color change produced by adding a light of one chromaticity to that of another. The fields being compared thus differ both in chromaticity and in luminance (Stiles, 1939). The reduction of the data to take account of the increase in luminance caused by adding the one light to the other involves a rather high-powered but sensible application of the Helmholtz three-components theory of vision. This elegant method of reduction seems to take adequate account of the troublesome question of chromatic adaptation, and the results (shown in Fig. 80 on the *x, y* diagram) are free from the local irregularities found by MacAdam for his one observer (Nutting), as comparison with Fig. 78 will show. It is not possible, however, to plot a plane diagram showing uniform chromaticness spacing by Stiles' data. A somewhat ruffled surface is required, that is, one having a negative curvature throughout (Stiles, 1946). Comparison of Fig. 80 with Fig. 78 shows that Stiles' data corroborate the chief trends of MacAdam's data apart from those ascribable to the special adaptive conditions used by MacAdam and suggests strongly that Stiles' results are closer to the truth than any previous generalized statement of chromaticity spacing for small self-luminous areas.

Comparison of the Munsell Chromaticity Spacing with That Obtained from Self-Luminous Areas. Two extensive bodies of data on uniform chromaticness scales are available. One of these, the body of Munsell data, was obtained by observation of somewhat separated painted chips in which a considerable fraction of the field of view was

filled with colors as light or lighter than those of the particular chips being compared. The other was obtained by comparing the colors of two small, juxtaposed photometric half-fields (about 2° subtense) surrounded either by a completely dark field or by a field considerably

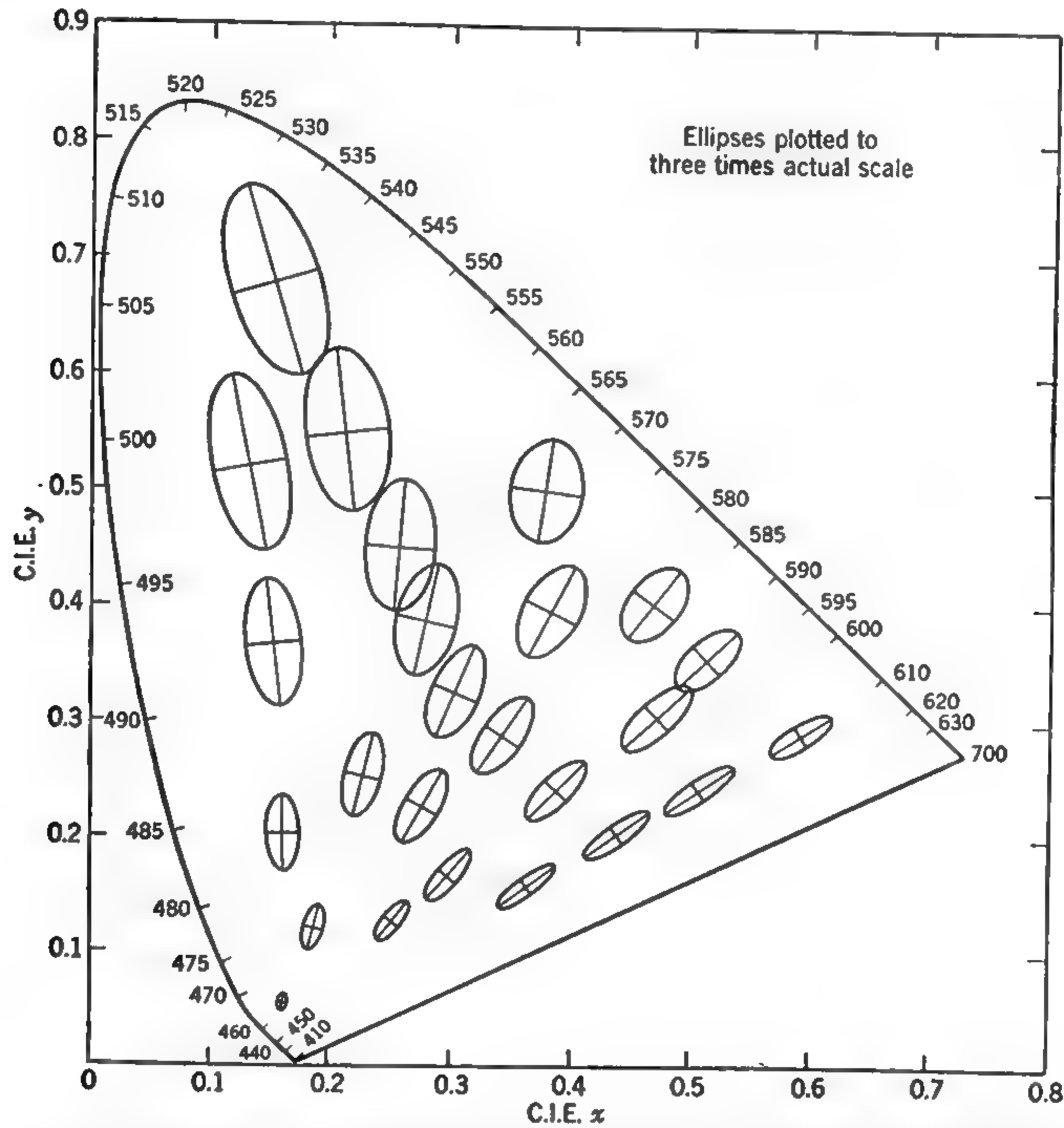


FIG. 80. Perceptibility of chromaticity differences derived from the three-components theory and shown on the (x, y) -chromaticity diagram (after Stiles, 1946).

darker than the two half-fields being compared. The first body of data refers to chromaticness spacing of objects viewed in an object-color environment. The second body of data refers to self-luminous areas viewed in more or less dark surroundings. It is of interest to inquire whether uniform chromaticness spacing of lights is found to be the same as uniform chromaticness spacing of objects. If they were substantially the same, a single colorimetric coordinate system might

serve to show the relative importance of chromaticity differences for both. The answer is somewhat complicated. On the one hand, the two bodies of data agree so well in general that one is indeed tempted to take some kind of average to apply both to lights and to objects (Burnham, 1949). On the other hand, in certain particulars the two bodies of data show disagreements of such size and regularity as to suggest that it would be useful to have two coordinate systems, one adapted to uniform chromaticness spacing for small lights, the other for large objects.

The most evident disagreement between the Munsell data on chromaticness spacing of object colors and those derived from comparisons in a photometric field may be seen by reading, first from Fig. 69, then from Fig. 74, the wavelength of the part of the spectrum most confusable in chromaticity with daylight. From Fig. 74 this is fairly easy. The point on the Planckian locus for a color temperature of $6,500^{\circ}\text{K}$ is a close representative of daylight quality. By dropping a perpendicular from this point to the long-wave branch of the spectrum locus, it may be seen that the required wavelength is about $568\text{ m}\mu$ (greenish yellow). From Fig. 69, however, the indication is less decisive. The region between 572 and $576\text{ m}\mu$ (yellow) corresponds closely to Munsell chroma $/12$ and so would seem to be about equally confusable with daylight. We know that the first value ($568\text{ m}\mu$) is correct if the light is either very weak (Abney, 1910; Judd, 1949, *a*) or very small in area, because under such conditions the normal eye makes tritanopic confusions (König, 1894; Willmer and Wright, 1945; Hartridge, 1945, 1947; Middleton and Holmes, 1949). Note from Fig. 29c that the wavelength of the spectrum confused by the typical tritanope with daylight (standard source C) is $568\text{ m}\mu$. Why a normal eye stimulated either by insufficient luminance or size makes the same chromaticity confusion as one kind of partially color-blind observer has attracted considerable recent attention. Some peculiarity of our retinas must explain this, and several explanations have been offered. By the Young-Helmholtz, three-components theory of vision the confusion would be explained if the violet-responding receptors were considerably less sensitive than those for red and green, or were much fewer in number. Be that as it may, among lights seen as point sources of low intensity the normal eye can detect only red-green and bright-dim differences and confuses daylight with the spectrum at $568\text{ m}\mu$. For small nonpoint fields it is reasonable to suppose that daylight quality would resemble $568\text{ m}\mu$ more than any other part of the spectrum, as is indicated by the UCS triangle (Fig. 74); and indeed, the direct

experiments of Guild (1929) and Holmes (1941) show that this is true. The Munsell spacing leads to a wrong conclusion in this regard and cannot be extended safely to apply to these experimental conditions. Conversely, the chromaticness-spacing data derived from observation of small photometric fields with dark surroundings probably apply only to conditions in which the normal eye approaches to a degree the form of partial color-blindness known as tritanopia.

Hunter (1941) developed a projective transformation of the (x, y) diagram yielding chromaticity spacing more or less in agreement with both sets of data. His chromaticity spacing agrees fairly well with both, and not exactly with either. This transformation is known as the "alpha-beta" diagram. The chromaticity coordinates, α , β , of any color may be found from those, x , y , in the standard CIE coordinate system by the defining equations:

$$\left. \begin{aligned} \alpha &\equiv \frac{2.4266x - 1.3631y - 0.3214}{1.0000x + 2.2633y + 1.1054} \\ \beta &\equiv \frac{0.5710x + 1.2447y - 0.5708}{1.0000x + 2.2633y + 1.1054} \end{aligned} \right\} \quad (27)$$

and these chromaticity coordinates may be found with good approximation from the readings (A, G, B) of a photoelectric colorimeter equipped with the amber, green, and blue tristimulus filters developed by Hunter (see Fig. 43) by the simple equations:

$$\left. \begin{aligned} \alpha &\doteq (A - G)/(A + 2G + B) \\ \beta &\doteq 0.4(G - B)/(A + 2G + B) \end{aligned} \right\} \quad (27a)$$

Figure 81 shows the spectrum locus on the "alpha-beta" diagram, and it also shows solid dots representing the chromaticities of some of the samples of the *Munsell Book of Color* for Munsell value 4/ and Munsell chroma /6 (the 4/6 locus). The chromaticity of standard source C is represented by the origin ($\alpha = 0$, $\beta = 0$) of this system of coordinates. The part of the spectrum closest to the origin is at wavelength about 571 m μ ; this is not far from the part (568 m μ) most confusable with daylight in a small-field comparison. Also the conformity of the spacing to the Munsell data, though far from perfect, is not too bad.

Since most of the commercial interest in uniform chromaticness scales has to do with objects and materials (plastics, textiles, papers, painted surfaces, vitreous enamel on metal, and so on), the present trend is away from such compromises and toward the Munsell data and coordinate systems intended to conform to it.

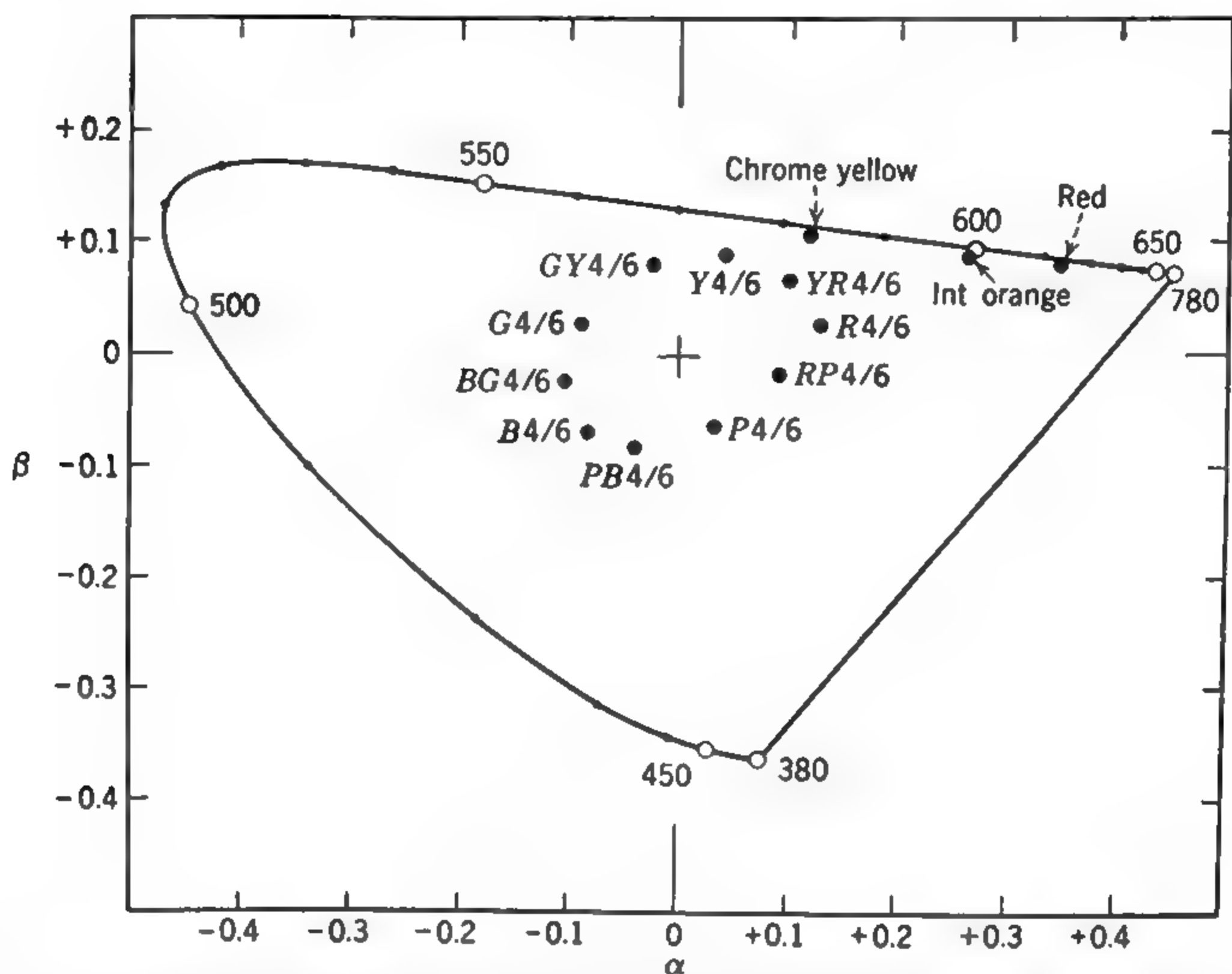


FIG. 81. The alpha-beta chromaticity diagram derived to adjust the spacing of the UCS diagram of Fig. 74 to conform more closely to the Munsell spacing (after Hunter, 1942).

Combined Lightness and Chromaticness Scales. The Munsell spacing data provide a reliable basis for telling how easily perceptible the difference between two color chips is, provided that they have identical luminous reflectances. Furthermore, the coordinate systems developed by Adams (1942) and by Saunderson and Milner (1946) provide convenient ways to calculate this perceptibility. The bulk of the color differences encountered in industry are, however, not of this special sort. Most of the color differences of commercial importance are of the sort involving a combination of chromaticity and reflectance differences. Thus, if a textile is guaranteed fast to a certain number of hours of sunlight, what is meant is that it will not change color enough to make the customer complain. Such fading usually causes the textile to become both lighter and more grayish (so-called "on-tone" fading). What is required to assess the importance of a color difference of this combined sort is a uniform tridimensional color scale. By this is meant a collection of color chips making a tridimensional array such that each color differs from each of its nearest neighbors by perceptually equal amounts. We know how to arrange dots in space meeting an

analogous requirement. This is the three-layer close-packing arrangement indicated in Fig. 60. But we do not have a set of color chips making a uniform tridimensional color scale, we do not know how to construct such a set, and, indeed, we have already considered the distressing influences of chromatic adaptation sufficiently to recognize that no set of colors covering the entire gamut will appear equally spaced except under very special conditions of chromatic adaptation, such as adaptation to the particular group of nearly identical colors being inspected for uniformity of spacing. The number of careful determinations required to establish tridimensional uniformity of color spacing over the entire gamut of colors is staggering. There are about 10 million definitely distinguishable colors. Spacing in all directions in the color solid for a reasonably close sampling of the entire color solid must be checked by such methods as those used by Brown and MacAdam (1949). The influence of many variables related to color adaptation must be separately determined. Some of these are field size, field luminance, luminance ratio of field to surround, chromaticity of surround, individual observer variation, mode of appearance, and so on. The complexity of the problem is such as to discourage even the most energetic research worker. The resources of several organizations have been pooled to study this problem under the direction of a Committee on Uniform Color Scales, Optical Society of America, and studies are in progress. But, even if they go forward without interruption or mistake, no definitive answer for all possible color differences can be expected for many years.

In the meantime questions such as, "Is this textile (or plastic, or paper) more fast to light than that other one taken as standard?" or, "Is the delivered plastic a commercial match to the standard?" keep coming up. Most practice has been to avoid giving any definite general answer. The inspector simply has to remember what size of color difference was accepted in this application before, and he applies this mental standard. It is hardly surprising that two inspectors should obtain different judgments even if they are both trying to be fair.

A standard difference with which to refresh the memory of the inspector is a step in the right direction. Thus, in tests of fastness to light, to weathering, to salt water, to laundering, and the like, a standard piece is agreed on, the sample and standard are both separated into two parts, and one part of each is run through identical weathering or laundering tests. The inspector then looks at the four swatches, standard and sample originals and standard and sample tested. He has to decide whether the color difference between the two standards, original and tested, is greater or less than that between the

two samples, original and tested. This is easier than comparison with a purely mental standard, but there is still room for an honest difference of opinion. The standard does not match the sample in color; so the inspector has to decide perhaps whether two blues differ in color by as much as two reds. Furthermore, the sample and standard may have faded by different kinds of color change. Thus the red standard may have become chiefly whiter (lighter and less saturated) corresponding to the disappearance of some of the dye causing the red color ("on-tone" fading); but the blue sample may have turned redder as well as more grayish. To obtain a really valid answer as to which color difference is more easily perceptible is a little research problem in its own right; several unbiased observers must be used and careful attention paid to compliance with standardized conditions of observation previously agreed on. It is obvious that a set of chips covering all colors with steps in all directions previously found to be equally perceptible would provide immediate ready-made answers to these commercially important questions. Even if we had to put up with only an approximation to the ideal tridimensional color scale formed by such a set of chips, this approximation would be very useful because it would provide a basis that could be agreed to in advance by buyer and seller for writing the color requirement in a contract. To formulate such an approximation in our present state of ignorance requires an obvious technic. We select some chromaticity diagram with approximately uniform chromaticness spacing and some function of luminous reflectance giving approximately uniform lightness spacing; then we make some kind of guess regarding how the two should be combined to define a unit of color difference. Any such guess can form the basis for an agreement as to color tolerance in a contract.

The first such guess was made by Nickerson (1936) in her definition of an index of fading, I . She adopted the Munsell spacing as it was known at that time, evaluated the relative importance of one step of Munsell value, one step of Munsell chroma, and one step of Munsell hue at chroma /5, and made the assumption, or guess, that the perceptibility of the color difference is sufficiently well evaluated by the weighted sum of its components (ΔH , ΔV , ΔC) along these parameters:

$$I \equiv (C/5)(2\Delta H) + 6\Delta V + 3\Delta C \quad (28)$$

It is seen that by this definition 1 step of Munsell value is placed equal in perceptual importance to 2 steps of Munsell chroma and to 3 steps of Munsell hue at chroma /5. The chief advantage of this formula for evaluating the perceptual size of a color difference is its simplicity. If

the Munsell notations of the two colors are known, the differences in Munsell hue, value, and chroma are found by simple subtraction, and the calculation of I is very direct. This formula, however, defines a complicated non-Euclidian geometry. For equiluminous colors ($\Delta V = 0$), the Munsell 100-step hue circuit ($\Delta H = 100$) at chroma $\frac{1}{3}$ ($\Delta C = 0$) corresponds to $(2 \times 100)/(3 \times 5) = 40/3 = 13.3$ steps of fading. Since this hue circuit is but 1 fading step ($\Delta C = \frac{1}{3}$) away from neutral, the number of hue steps is more than twice the number ($2\pi = 6.28 \dots$) representable by uniform distances on a plane. It would require for its representation a two-dimensional ruffled surface something like that found by MacAdam (1949) from his experiments on chromaticity spacing (see Fig. 79).

Balinkin (1939, 1941) proposed an index of fading, I' , likewise based on the Munsell color spacing, that conforms for infinitesimal hue differences ($\Delta H \rightarrow 0$) to Euclidian spacing:

$$I' \equiv [(2C \Delta H/5)^2 + (6\Delta V)^2 + (20\Delta C/\pi)^2]^{1/2} \quad (29)$$

This type of formula was used by Forrest, Kreidl, and Pett (1948) to assess the importance of color differences among different melts of glass.

Note that formulas 28 and 29 are not applicable, however, to color differences involving differences in Munsell hue covering an appreciable fraction of the hue circuit. Thus, if the two colors whose difference is to be evaluated by I' are equiluminous ($\Delta V = 0$) and of the same chroma ($\Delta C = 0$), but nearly complementary in hue ($\Delta H \approx 50$), equation 29 says that $I' = 20C$. But it is obvious that this difference really corresponds to $\Delta C = 2C$, the diameter of the hue circuit, instead of to half the circumference. From equation 29 it may be seen that for $\Delta V = \Delta H = 0$, and $\Delta C = 2C$, the color difference, I' , comes out to be $40C/\pi$. The value, $20C$, obtained from unthinking mechanical application of equation 29 is thus too high by the ratio, $\pi/2$, of the length of the semicircle to that of its diameter. Equations 28 and 29 should not be used for ΔH much exceeding 10 Munsell hue steps. The rigorous statement of the Euclidian distance element expressed in cylindrical coordinates, though rather complicated, has been used by Godlove (1951) in a study of 350 dyeings.

Other definitions of the approximate perceptibility of color differences avoid this trouble by steering clear of cylindrical coordinates. Judd (1939) based a definition on the chromaticity spacing of the UCS triangle (see Fig. 74) combined with the reflectance spacing of the square-root law (equation 20). By this definition the size of the color

difference, ΔE , between two colors (Y_1, r_1, g_1, b_1) and (Y_2, r_2, g_2, b_2) defined in terms of the chromaticity coordinates, r, g, b , ($r + g + b = 1$), on the UCS triangle (equation 24) and luminous reflectance Y , is taken as:

$$\Delta E = 600 \{ Y^{1/2} \overline{\Delta S}^2 + (k/600)^2 [\Delta(Y^{1/2})]^2 \}^{1/2} \quad (30)$$

where Y is the average of Y_1 and Y_2 , $\overline{\Delta S}^2$ is the square of the distance on the UCS triangle between the two points (r_1, g_1) and (r_2, g_2) equal to $(r_1 - r_2)^2 + (g_1 - g_2)^2 + (b_1 - b_2)^2$, $\Delta(Y^{1/2})$ is an abbreviation for $Y_1^{1/2} - Y_2^{1/2}$, and k is a constant relating in importance the lightness and the chromaticness scales. If the comparison is made across a very narrow dividing line (as in comparing paint blobs squeezed together under a cover glass, or in comparing overlapping sheets of paper), the lightness constant k should be taken as about 120. This corresponds to 120 lightness steps from black to white. If there is a narrow dividing area of contrasting color (such as in the comparison of two plaques of vitreous enamel on metal), better agreement with visual estimate is found from equation 30 if k is set at 90 or 100. Separation of the compared areas by a conspicuous broad design requires k to be as small as 30 or 40 for good agreement. The size of color difference, ΔE , is given in units known as NBS units. The constant, 600, in equation 30 is such as to make the NBS unit about 5 times the smallest difference perceptible under the best experimental conditions. Differences of one NBS unit or smaller are usually disregarded in commercial transactions. Measurements by Nutting (1935) indicate (Judd, 1939) that a commercial match for wool dyeings corresponds to 2 NBS units or less, and for cotton dyeings to 4 NBS units or less. This definition of the NBS unit was used at the National Bureau of Standards until both the Wright (1941) check of the UCS triangle and the Munsell spacing data indicated that the UCS triangle somewhat overspaced the saturated reds.

The redefinition of the NBS unit (Hunter, 1942) is based on the "alpha-beta" chromaticity diagram (see Fig. 81 and equation 27). The present definition is:

$$\Delta E = f_g \{ [700 Y^{1/2} (\overline{\Delta \alpha}^2 + \overline{\Delta \beta}^2)^{1/2}]^2 + [k \Delta(Y^{1/2})]^2 \}^{1/2} \quad (31)$$

where f_g is a factor to take account of the masking influence of a glossy surface on the detection of color differences, and k is as given for equation 30. If the determinations of directional reflectance Y and chromaticity coordinates α, β are made with 45° illumination and perpendicular viewing, but the visual estimation of the size of color difference is

carried out as usual in a room near a window, the inclusion of the factor f_g yields improved correlation by taking account of the light sent to the eye of the observer from both samples by mirror reflection from the ceiling or upper walls of the room. These mirror reflections tend to mask the color difference between the samples and make smaller the visual appreciation of it. The gloss factor f_g is defined as $Y/(Y + K)$, where, for the usual inspection room, K should be set at about 0.025. One NBS unit is equivalent to about one-tenth Munsell value step, to 0.15 Munsell chroma step, or to about 2.5 Munsell hue steps at chroma /1.

Equation 31 has also been used to derive a formula for whiteness, the degree of approach of the sample color to the color of the standard white. Whiteness, always of great interest in the laundering of textiles, use of chemical bleaches, and bluing, has since 1940 taken on even greater importance because of the development of blue fluorescent dyes, the so-called optical bleaches. The formula for whiteness, W , is based on the definition:

$$W \equiv 1 - \frac{\Delta E_{\text{MgO to specimen}}}{\Delta E_{\text{MgO to black}}} \quad (31a)$$

where magnesium oxide (MgO) is taken as the whiteness standard. This relation gives a scale of whiteness from 1.00 for the whiteness standard to zero for black ($Y = 0$) and has been found (Judd, 1941) to correlate well with visual estimates of whiteness of near-white specimens made without visual reference to any standard when the color difference, ΔE , is computed from equation 31 with the lightness constant k set equal to 20. Since the index of whiteness, W , is applicable only to near-white specimens, it can be computed from equation 31b:

$$W = 1 - \{[30(\alpha^2 + \beta^2)^{1/2}]^2 + [(1.00 - Y)/2]^2\}^{1/2} \quad (31b)$$

It has been found that the chief cause for disagreements among visual estimates of whiteness of near-white papers arises from discrepancies from one grader to another as to what shall be the standard white (Judd, 1935, 1936). If some whiteness standard other than magnesium oxide is adopted (such as the color of paper made from good-grade, well-bleached, undyed pulp, called "natural paper white") equation 31a should be rewritten relative to that standard and used to derive a substitute for equation 31b. If the whiteness judgments are made by direct comparisons over a fine dividing line between sample and standard, departures of the sample color from the standard color toward gray count for more and the lightness constant k to produce good correlation should be set much higher than 20. Selling and Friele (1950) found that by setting $k = 70$, equation 31 gave good correlation with

the visual whiteness estimates made by their observers by direct comparison with a fine dividing line.

Neither equation 30 nor equation 31 conform to Euclidian space even though ΔE is evaluated as the square root of the sum of squares. This is because of the term, $Y^{1/4}$, by which the length of the distance on the chromaticity triangle is weighted to evaluate the chromatic component of the color difference. It approaches a Euclidian form if the colors to be compared are nearly equiluminous ($\Delta Y \rightarrow 0$).

Scofield (1943) proposed a modification of equation 31 for estimating the size of color differences:

$$\Delta E = [(L_1 - L_2)^2 + (a_1 - a_2)^2 + (b_1 - b_2)^2]^{1/2} \quad (32)$$

where

$$\begin{aligned} L_1 &= 100Y_1^{1/2} & L_2 &= 100Y_2^{1/2} \\ a_1 &= 7L_1\alpha_1 & a_2 &= 7L_2\alpha_2 \\ b_1 &= 7L_1\beta_1 & b_2 &= 7L_2\beta_2 \end{aligned}$$

This formula is simpler than equation 31, and for $L_1 - L_2$ small it differs only by assessing chromaticity differences among dark colors less by a factor of $Y^{1/4}$. For colors differing considerably in luminous reflectance Y , however, equations 31 and 32 differ radically. The Scofield formula 32 agrees much better with formula 29 than formula 31 does.

Hunter (1948) has developed an improved photoelectric reflectometer in which the calculations are largely done by the electric circuits attached to the barrier-layer cell serving as light receptor. The illumination is at 45° , and the direction of view is perpendicular. The instrument is direct reading in terms corresponding closely to $L_1 - L_2$, $a_1 - a_2$, and $b_1 - b_2$ of equation 32. One type of circuit yields the quantities R , a_R , b_R , defined as:

$$\left. \begin{aligned} R &= 100Y \\ a_R &= 175f_Y(1.02X - Y) \\ b_R &= 70f_Y(Y - 0.847Z) \end{aligned} \right\} \quad (32a)$$

where $f_Y = 0.50(21 + 20Y)/(1 + 20Y)$, and X , Y , and Z are the tri-stimulus values of the color on the standard CIE system. Another type of circuit yields the quantities L , a_L , b_L , defined as:

$$\left. \begin{aligned} L &= 100Y^{1/2} \\ a_L &= 175(1.02X - Y)/Y^{1/2} \\ b_L &= 70(Y - 0.847Z)/Y^{1/2} \end{aligned} \right\} \quad (32b)$$

Figure 82 shows the Munsell renotations for Munsell value 5/ plotted on the (a_R, b_R) diagram taken from a paper by Nickerson (1950). Sets of these graphs for all 9 Munsell values may be purchased from the Munsell Color Company for use as work sheets. The units are equivalent to the NBS unit for differences between light colors involving small values of $L_1 - L_2$. The instrument is called the "Color-Difference Meter," and it has been favorably received by industry, particularly

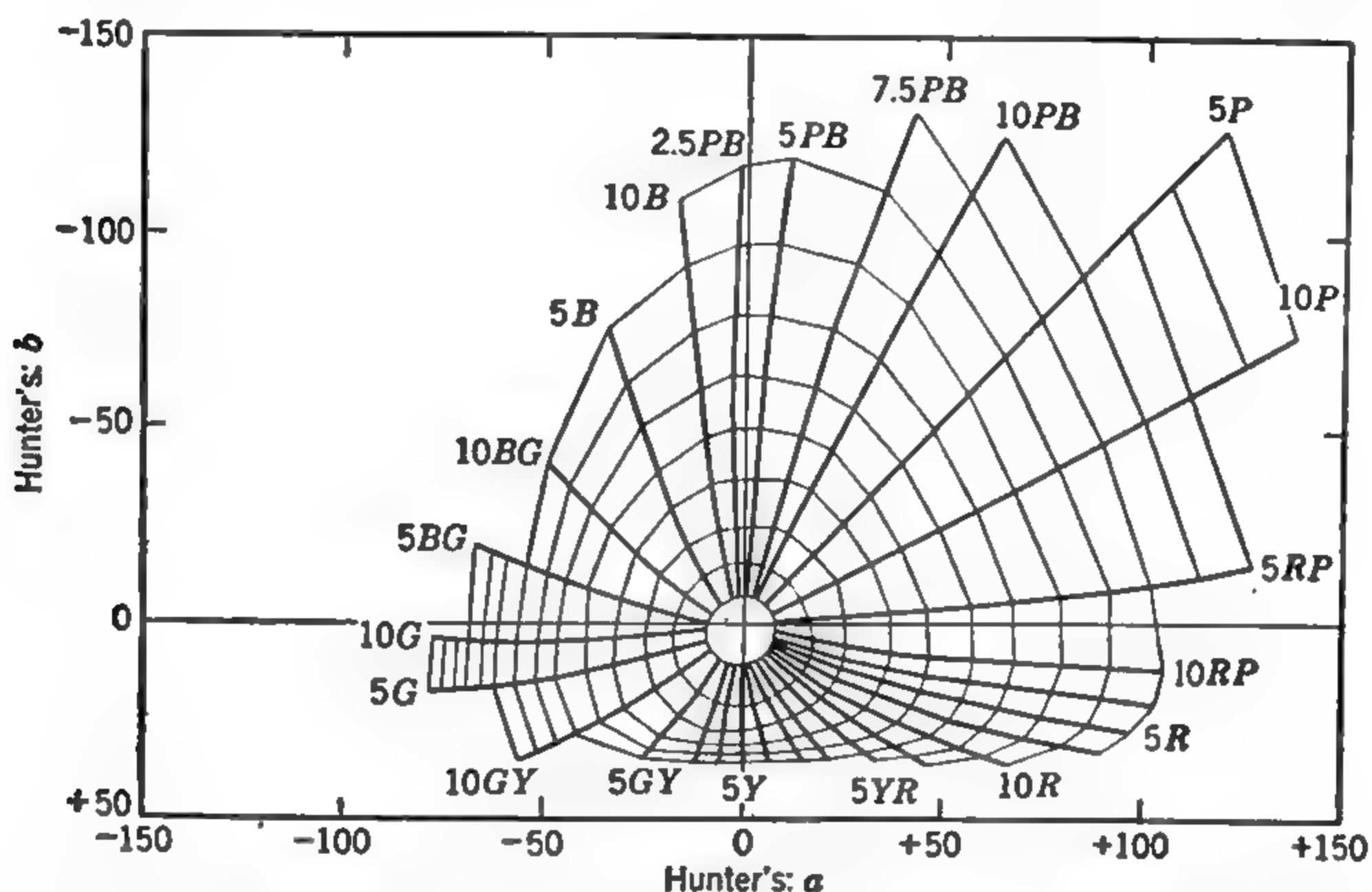


FIG. 82. Chromaticities of ideal Munsell colors, value 5/, shown on the (a_R, b_R) -chromaticity diagram (after Nickerson, 1950).

for paints, ceramics, and agricultural products (Kramer, Guyer, and Smith, 1948; Kramer, 1950; Younkin, 1950). The use of equation 32 in industry has been chiefly to reduce data taken on the Color-Difference Meter. Figure 83 shows a photograph of the instrument. Figure 84 shows a view of an automatic colorimeter for raw cotton (Nickerson, Hunter, and Powell, 1950) based on the Color-Difference Meter.

The definition of the size of color differences given by equation 32 has been adopted as test method 623.1 in Federal Specification TT-P-141b (Paint, Varnish, Lacquer, and Related Materials; Methods of Inspection, Sampling, and Testing, 15 January 1949).

Although all the thinking that guided the guesses leading to equations 28, 29, 30, 31, and 32 has been based on the idea of a Euclidian color space, it is noteworthy that equation 32 is the first one that corresponds strictly to Euclidian geometry. In equation 32 the distance element, ΔE , obviously corresponds to the distance between two points

in the rectangular coordinates defined by the three variables, L , a , and b . All other formulas for color difference yet to be considered here are also of this form, differing only in the definitions of the three variables. All such formulas, or distance elements, describe a form of color spac-

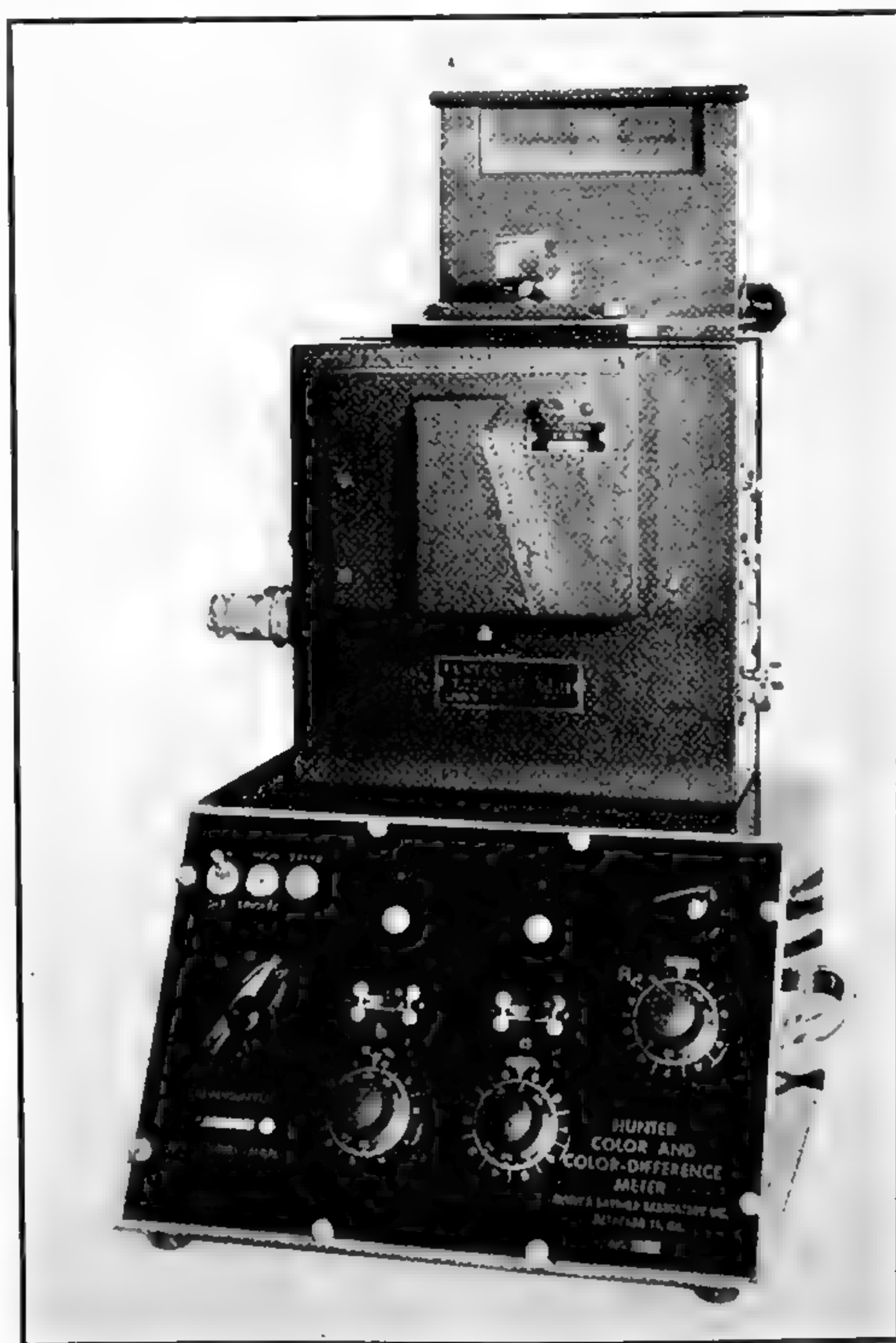


FIG. 83. General view of the Hunter Color-Difference Meter. (Courtesy of Henry A. Gardner Laboratory.)

ing that could be embodied in a set of color chips making a tridimensional color scale uniform to the extent that the spacing data on which they are based is correct. The set of chips would be displayed in a tridimensional array organized on the three-layer hexagonal basis (see Fig. 60) giving a uniform close-packed sampling of the space. Figure 85 shows one central point, O , with the 12 nearest neighbors (A to L) all equally distant; that is, if this kind of space lattice were inserted,

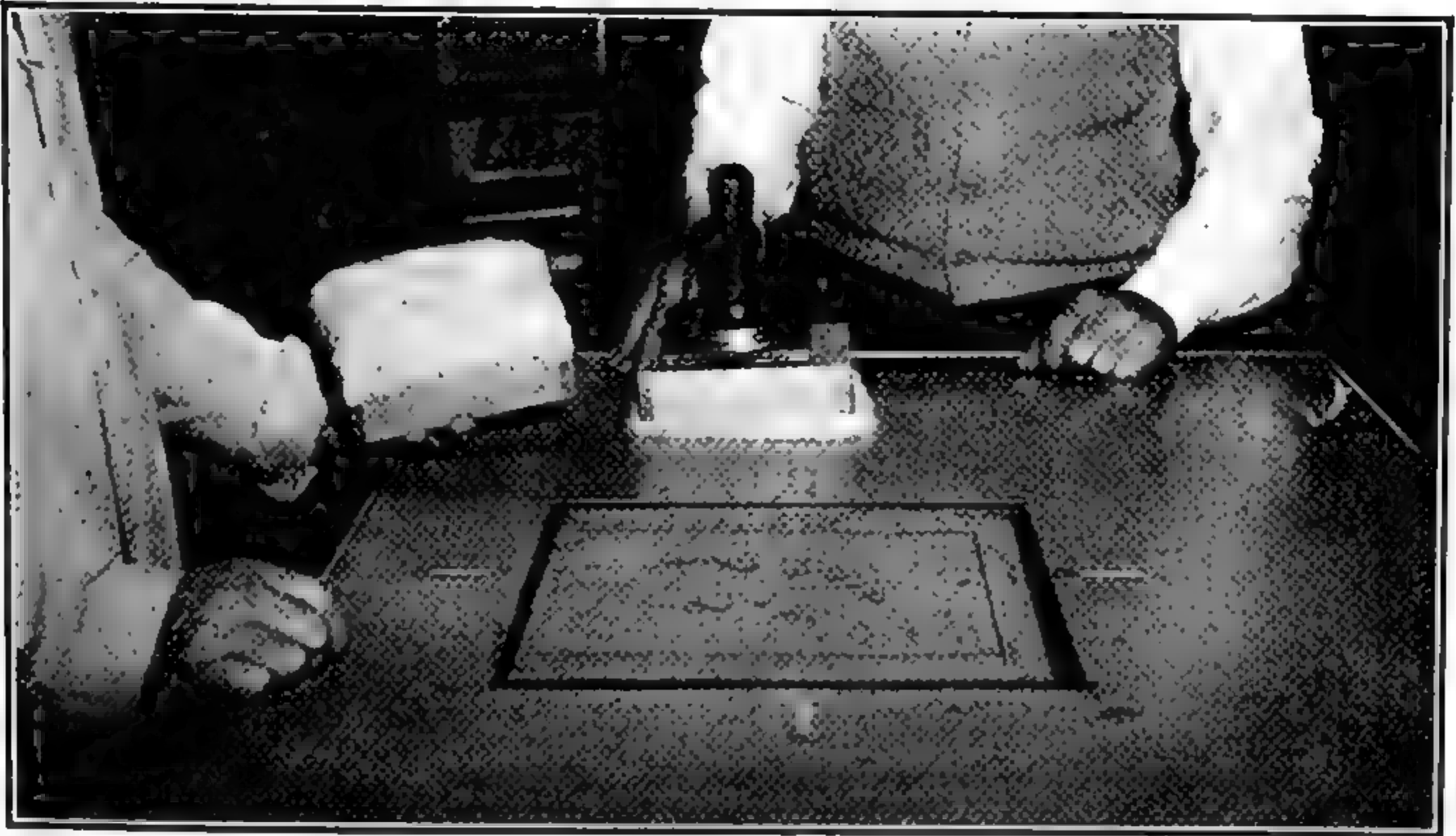


FIG. 84. General view of the Nickerson-Hunter-Powell (1950) cotton colorimeter.

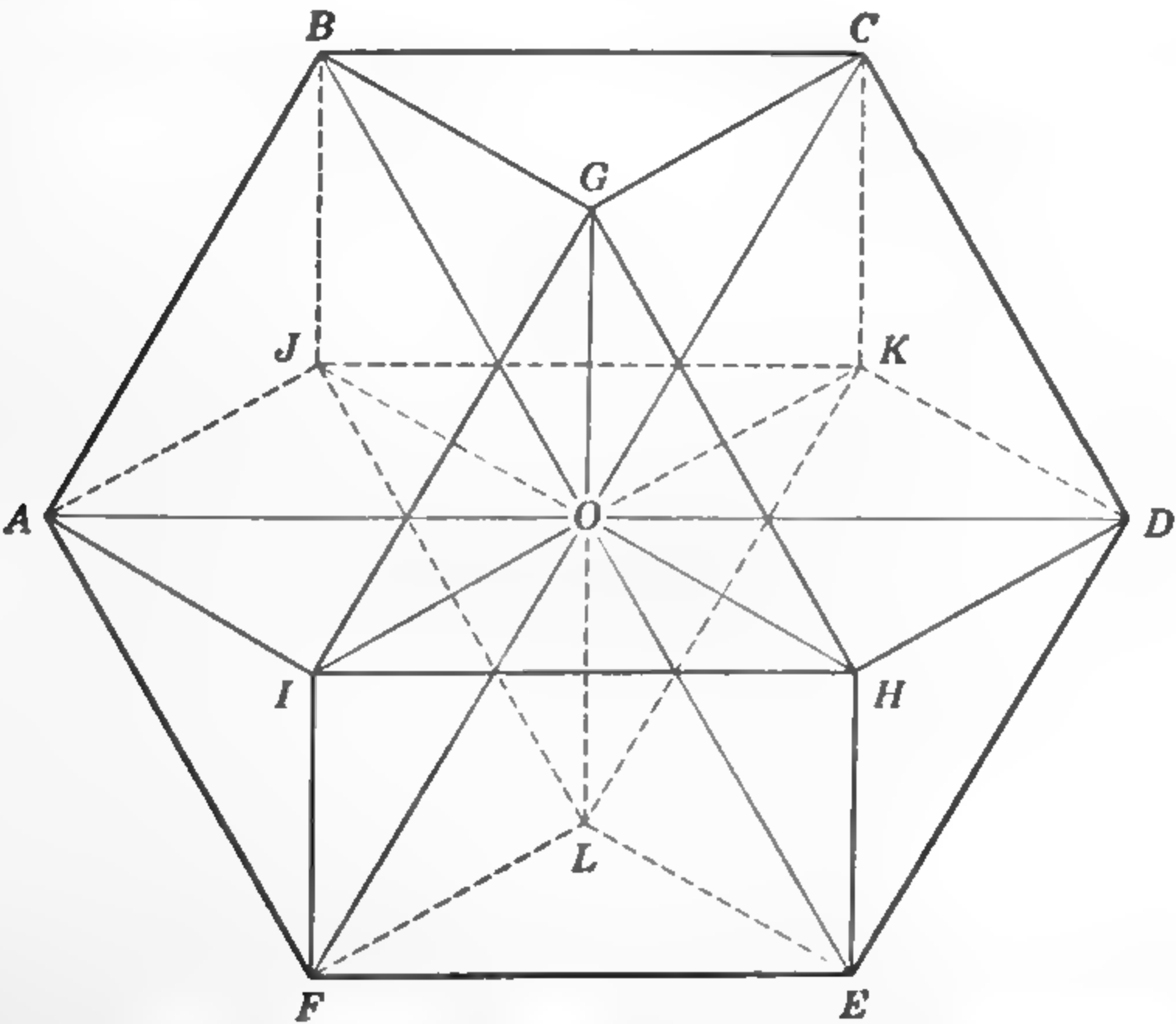


FIG. 85. The 12 nearest neighbors, equally distant from, and surrounding a central point, O, in three-layer close packing.

for example, into Munsell color space and the Munsell notation corresponding to each point (not too close to the neutral axis) were read by interpolation, then the index I' computed from equation 29 would be closely the same for the color intervals OA , OB , $\dots OL$. If the equiluminous plane through color O is taken as the plane of the paper containing the regular hexagon $ABCDEF$, the colors I , G , H would represent an equiluminous triad lighter than colors $OABCDEF$, so that the triangle IGH may be thought of as above the paper. Similarly the colors J , K , L would represent an equiluminous triad darker than colors $OABCDEF$, so that the triangle JKL should be thought of as below the paper. It will be seen that each point in the array is at the center of a 14-face solid figure, 6 faces being squares (such as $ABGI$ and $CDGH$), and 8 being equilateral triangles (such as AFI and BCG). The tetrahedrons and octahedrons whose edges define these 14-face solids pack tightly into space, exhausting it completely. If the array of points is split along the plane of any of the triangles, a two-dimensional triangular coordinate system is revealed; but, if it is split along the plane of any of the squares, a two-dimensional rectangular coordinate system is revealed. The Munsell spacing has been utilized by Foss (1949) in conjunction with this kind of close packing to derive the Munsell notations for a uniform tridimensional color scale, and a start has been made toward preparation of the corresponding tri-dimensional array of color chips.

Nickerson and Stultz (1944) tested several formulas for size of color differences. One of these was based on the Adams (1942) chromatic-valence space. By this formula the size of color difference depends on differences in three functions, V_v , W_x , W_z , of the tristimulus values, X , Y , Z , thus:

$$\Delta E = [(0.5\Delta V_v)^2 + (\Delta W_x)^2 + (0.4\Delta W_z)^2]^{1/2} \quad (33)$$

where V_v is the Munsell value function given in equation 20b, $W_x = V_v[(X_c/Y) - 1]$, $W_z = V_v[(Z_c/Y) - 1]$, X_c being the ratio $X/X_{MGO} = X/0.9804$, and Z_c being the ratio $Z/Z_{MGO} = Z/1.1810$. For source C, $X_c = Y = Z_c$; see sums for source C in Table 10. The geometry described by this formula is obviously Euclidean, and the lightness scale corresponds exactly to Munsell spacing because the Munsell value function is included explicitly. The special functions W_x and W_z and the constants 0.5 and 0.4 were also chosen because they give good agreement with the Munsell spacing. This formula has been used to assess the importance of distortions in object-color rendi-

tion due to the wearing of selectively absorbing goggles (Air Force-Navy Aeronautical Specification AN-G-22a, Glasses; Flying Sun, 28 April 1948).

Another formula tested by Nickerson and Stultz (1944) was based on the Adams chromatic-value diagram (Adams, 1942) combined with the Munsell value scale (equation 20b). It is:

$$\Delta E = \{ (0.23\Delta V_v)^2 + [\Delta(V_x - V_v)]^2 + [0.4\Delta(V_z - V_v)]^2 \}^{1/2} \quad (34)$$

where V_v is the Munsell value function, V_x is the Munsell value found from the same function (equation 20b) by setting $Y = X_c$, and V_z is the Munsell value for $Y = Z_c$, X_c and Z_c being the same as for equation 33. The Adams chromatic-value diagram is a nonlinear transformation of the (x, y) -chromaticity diagram of the standard CIE system. The transformation is based on the Adams theory of vision (Adams, 1923, 1942). It is doubtful whether American industry has been impressed by the theory, though it is among the most promising of the current guesses as to why we see as we do. What has impressed industrial users of the Adams chromatic-value diagram is the simplicity of its definition and the remarkable agreement between its chromaticity spacing and the Munsell spacing. Figure 86 shows the loci of Munsell renotation hue and Munsell renotation chroma (Newhall, 1943) at Munsell value 5/ plotted on the Adams chromatic-value diagram by Nickerson (1947, 1950). The approximation of the loci of constant chroma to circles is evident.

Saunderson and Milner (1946) proposed a formulation for the size of color differences based on the Saunderson-Milner "zeta" space, a virtually perfect adjustment of the chromatic-value diagram to the Munsell spacing. The formula is:

$$\Delta E = [(\Delta \zeta_1)^2 + (\Delta \zeta_2)^2 + (\Delta \zeta_3)^2]^{1/2} \quad (35)$$

where $\zeta_1 = (V_x - V_v)(9.37 + 0.79 \cos \theta)$

$$\zeta_2 = kV_v$$

$$\zeta_3 = (V_z - V_v)(3.33 + 0.87 \sin \theta)$$

θ being the angle whose tangent is $0.4(V_z - V_v)/(V_x - V_v)$, and V_x , V_v , and V_z are as in equation 34. Here k is a constant whose value may range between 1 and 5, depending on the observing conditions such as fineness of the dividing line.

Equation 35 is more complicated than equation 34, but Saunderson and Milner (1946) have developed a slide rule and graph to simplify

the simplest; if the data are in the form of readings A, G, B on a photo-electric, tristimulus colorimeter equipped with Hunter filters (1942), equation 31 or 32 should be used; and, if the data are in the form of tristimulus values X, Y, Z , such as result from reduction of spectrophotometric data by means of the 1931 CIE standard observer, equation 33 or 34 should be used.

Each of the formulas (28 to 35) evaluates the sizes of color differences between specimens approximately in accord with the perceptibility of the differences by an observer adapted to working in a daylight-illuminated room, but each has a somewhat different set of scale constants. Table 19 gives the factor, f , by which the results of the

Table 19. Factor f by Which the Results of the Various Formulas May Be Multiplied to Convert Them on the Average to NBS Units and So Make Them Comparable

Equa- tion	Identification of Color Space Used	Factor f					Recom- mended
		Hue differ- ences	Value differ- ences	Chroma differ- ences	All equal	Random (100 differ- ences, Nickerson)	
28	Munsell non-Euclidian (fading index)	1.0	1.7	2.2	0.84	1.3	1.0
29	Munsell Euclidian	1.0	1.7	1.1	1.2	1.4	1.2
31	Alpha-beta- $Y^{1/2}$ (NBS unit)	1.0	1.0	1.0	1.0	1.0	1.0
32	$a-b-Y^{1/2}$ (Scofield-Hunter)	1.5	1.0	1.5	1.3	1.2	1.2
33	Adams chromatic-valence	25	20	27	23	21	21
34	Adams chromatic-value	56	43	51	51	46	50
35	Saunderson zeta ($k = 2$)	5.9	5.0	6.1	5.6	■

various formulas should be multiplied to express them in NBS units so as to be comparable. Since no two formulas give exactly the same color spacing throughout the color solid, the factor f depends on what part of the color solid contains the two points representing the two colors to be discriminated and on the direction of the line connecting the two points. The factors corresponding to directions tangential to the hue circles (value and chroma constant) are given in the column headed "hue differences." Those corresponding to vertical directions (hue and chroma constant) are headed "value differences"; and those corresponding to radial directions (hue and value constant) are headed "chroma differences." Those corresponding to diagonal directions for which the hue, value, and chroma differences are equally perceptible are headed "all equal." Note that for the non-Euclidian form (equation 28) the factor, f , for the diagonal direction is not an average of the factors for hue, value, and chroma differences but is lower than any

of them. Color differences chosen at random will, in general, require a factor, f , intermediate to those given as representative of the four directions. For comparison the factors, f , found by Nickerson and Stultz (1944) for the 100 color differences studied by them are given. The factors recommended to reduce the results by the various color-difference formulas to approximate NBS units are based partly on those found by Nickerson and Stultz, but it is obvious from the other values tabulated that the conversion to NBS units is only approximate for any given color difference.

Because of the extensive industrial use and study of formulas (such as equations 34 and 35) based on the Adams chromatic-value diagram, there are included 1,000-entry tables in the Appendix, showing the dependence of the Munsell value functions V_x , V_y , and V_z on the tristimulus values X , Y , Z . Table A shows the interdependence of X and V_x ; Table B connects Y with V_y ; and Table C, Z with V_z . All three tables are taken from a paper by Nickerson (1950) and refer to standard source C. The use of these tables may be shown by an example. Suppose that it is required to determine the size of the color difference in standard daylight (CIE source C) between the two color chips $X_1 = 0.2251$, $Y_1 = 0.1200$, $Z_1 = 0.0475$ (Munsell renotation 5R 4/14) and $X_2 = 0.3080$, $Y_2 = 0.1977$, $Z_2 = 0.0776$ (Munsell renotation 7.5R 5/12), by equations 34 and 35, where, in equation 35, $k = 2$. The computation is as follows:

	5R 4/14	7.5R 5/12
From Table A, Appendix:	$V_x = 5.34$	6.12
B,	$V_y = 4.00$	5.00
C,	$V_z = 2.32$	3.00
	$V_x - V_y = 1.34$	1.12
	$V_z - V_y = -1.68$	-2.00
	$\Delta(V_x - V_y) =$	0.22
	$\Delta(V_z - V_y) =$	0.32
	$\Delta V_y =$	1.00
	$0.23\Delta V_y =$	0.23
	$\Delta(V_x - V_y) =$	0.22
	$0.4\Delta(V_z - V_y) =$	0.13
From equation 34	$\Delta E =$	0.34

Use of equation 35, more precisely in agreement with the Munsell renotations, involves the following additional steps:

	5R 4/14	7.5R 5/12
$0.4(V_z - V_y)/(V_x - V_y) =$	-0.50	-0.71
$\theta =$	335.5°	324.6°
$\sin \theta =$	-0.45	-0.58
$\cos \theta =$	0.89	0.82
$(9.37 + 0.79 \cos \theta)(V_x - V_y) = \zeta_1 =$	13.5	11.2
$2V_y = \zeta_2 =$	8.0	10.0
$(3.33 + 0.87 \sin \theta)(V_z - V_y) = \zeta_3 =$	-4.9	-5.7
$\Delta \zeta_1 =$	2.3	
$\Delta \zeta_2 =$	2.0	
$\Delta \zeta_3 =$	0.8	
From equation 35 $\Delta E =$	3.1	

The last step in each of these computations is to square three terms, add them, and take the square root. This is usually done graphically by laying off diagonals on rectangular coordinate paper.

To express these two results approximately in NBS units so that they may be compared, we must read the factor f from Table 19, and compute $f \Delta E$, thus:

$$\text{From equation 34, } f \Delta E = 50 \times 0.34 = 17$$

$$\text{From equation 35, } f \Delta E = 6 \times 3.1 = 19$$

Equation 34 is probably the most widely used method of assessing the color difference between two samples whose tristimulus values, X , Y , Z , are known by spectrophotometric measurement. Godlove (1950, 1951, a) and Vickerstaff and Tough (1949) used it in studies of light-fastness of textiles; Boyd (Nickerson, 1950) used it in his study of color tolerances for paint at the Industrial Test Laboratory of the Philadelphia Naval Shipyard; and Stearns (1951, b) has recommended it for seven applications in the textile industry.

Of course, if the two colors are already known in Munsell notation, as in the example chosen, equations 28 or 29 give about the same result with much less work, thus:

By equation 28:

$$I = (1\frac{3}{5})(2 \times 2.5) + 6 \times 1 + 3 \times 2$$

$$= 13 + 6 + 6 = 25$$

$$fI = 1.0 \times 25 = 25$$

By equation 29:

$$\begin{aligned} I' &= [(13)^2 + (6)^2 + (12.7)^2]^{\frac{1}{2}} \\ &= (169 + 36 + 161)^{\frac{1}{2}} = 19.1 \end{aligned}$$

$$fI' = 1.2 \times 19.1 = 23$$

Color Tolerances. The failure to set tolerances on the color requirement in a purchase order has been the cause of much grief. Sometimes the setting of tolerances causes grief, too. On small amounts of goods it does not pay. Reliance on the integrity of the supplier or brand name backed up by a liberal policy of making good on legitimate complaints is better business for retail or small wholesale dealings. Large corporations and government agencies, however, often make large single purchases on the open market and demand in their bid requests to know in advance exactly what they are buying, including exactly how closely the color requirement will be met. Government purchases during World War II introduced many suppliers to their first quantitative statements of color tolerances.

Whether a color tolerance should be set also depends on the economic situation. In a seller's market the buyer is glad to get the goods even if badly off-color; he will not complain and a color tolerance is not necessary. But in a buyer's market the insertion of a tolerance on the color requirement is a protection to the supplier against arbitrary rejection of the goods as off-color by an unscrupulous buyer who has found another supplier offering the goods at a lower price. In normal times, it is a protection to both buyer and seller. A color tolerance, like other tolerances in a purchase order, represents a compromise between what the purchaser really would like to get and what the manufacturer can supply at a reasonable price.

The actual decision as to what color range is to be written into the contract as acceptable is not easy. Several hundreds, or even thousands, of dollars may have to be spent to make sure that this choice is a wise one. There is a great temptation to make a guess as to the correct color, tell the supplier what the color has to do with the intended use of the article or material, and trust to his competence to deliver goods suited to the purpose. If the operation involves mass production, it is dangerous to yield to this temptation. If an automobile manufacturer were to order from one supplier engine blocks with 4-inch cylinders, and from another pistons to go into 4-inch cylinders, he might have to waste time sorting through the delivered pistons trying to find one to fit each separate cylinder. Similar risks are run on off-color goods. The color serves some purpose. It must coordinate

with some other color to some degree. If the production is to run smoothly, no attention can be diverted to finding uses for materials or parts having a color importantly different from that planned. If the color tolerance is large, money can be saved by adopting simple inspection procedures; if it is small, as in the case of upholstery fabrics, the most careful selection of color tolerances coupled with the most reliable color measurement and control possible will pay high dividends.

Color tolerances can be stated in terms of special color standards made up for one order, or they may be stated in terms of a collection of color chips possessed both by purchaser and by supplier, or they may be based upon measurements, either by direct colorimetry or by spectrophotometry.

Special Color Standards. One of the most common methods for manufactured items in regular production is to give the prospective purchaser samples illustrating the color variation normal to the production process, and to guarantee that all goods delivered except a certain small percentage will be within the limits defined by those samples. Alternatively, the purchaser will supply samples illustrating the color range acceptable for his intended use of the goods. A method used by the late Arthur S. Allen, well-known package and advertisement designer, will serve as an example. Tolerances on colors used for trade-marks have to be set quite small. Mr. Allen first selected the ink to be used. Then he coated a piece of paper with five different film thicknesses. Alternatively he simulated the colors produced in this way by introduction of the correct proportion of transparent extenders and printed the inks on actual stock at normal film thickness. The central color represented the desired trade-mark color. The two colors adjacent to this central color represented acceptable variations. The two end colors represented nonacceptable variations. The printer thus knew in advance exactly what degree of control he would have to exert on the composition of the ink and on the thickness of the ink film laid down on the paper. The print jobs handled by Mr. Allen involved a minimum of argument or rejections due to off-color work. The method is simple and practicable.

A logical extension of this method is to show 12 limit samples deviating from the standard in the directions shown in Fig. 85. The tolerances should not be uniform, but instead the variation corresponding to film thickness (usually, light to dark) should carry less strict tolerances. This method has been used in a number of commercial applications by Carl Foss.

Collections of Color Standards. Another way to set a color tolerance is by reference to a collection of color standards, copies of which are

in the possession of both buyer and seller. Systematic collections, like the *Maerz and Paul Dictionary*, the *Munsell Book of Color*, or the *Color Harmony Manual*, are best suited for the specification of color tolerances. For example, if Yale Blue (44-D-12) in the *Maerz and Paul Dictionary* were the desired color, the tolerance might be stated by saying that colors 44-C-12 and 44-E-12 and intermediates would be acceptable. This gives a hue tolerance, and this one-dimensional tolerance is far better than no tolerance at all. A saturation tolerance indicating how much grayer than standard the delivered goods are permitted to be might be stated by including 44-D-11 as an acceptable variation. These two stipulations, taken together, make a two-dimensional tolerance, but they fail to specify how much lighter or darker the delivered goods may be and still be acceptable. The light and dark limits might be specified by stating that colors 43-D-12 and 45-D-12 and their intermediates would be acceptable. The three-dimensional tolerances set up by mentioning these 5 colors in the *Maerz and Paul Dictionary* would serve for most practical purposes. These tolerances still leave some room for argument, first because no upper saturation limit can be set in this collection of color chips, Yale Blue being at the gamut limit for this collection, and second, because no precise way has been specified for passing on delivered goods that are, for example, both darker and grayer than chip 44-D-12. A slightly more precise way to state how the inspector is to make his decision would be to designate color 44-C-12 as the green limit, 44-E-12 as the red limit, and 45-D-11 as the gray limit, 43-D-12 as the light limit, and 45-D-12 as the dark limit. The inspector must then be able to judge which of two blues is grayer, darker, lighter, redder, or greener.

The *Munsell Book of Color* is well adapted to the setting of hue, lightness, and saturation limits of this sort because the color scales shown in the book exemplify those variations for usual conditions of observation. Adjacent color chips on these scales differ too much, however, to permit actual citation of particular chips as limit colors for most purposes. Direct use of the Munsell scales by visual interpolation must be made. Yale Blue corresponds in Munsell notation (Reimann, Judd, and Keegan, 1946) to 7.2PB 2.9/9.1. The limits might be expressed as two Munsell hue steps ($\Delta H = 2.0$), one Munsell chroma step ($\Delta C = 1.0$), and one-half Munsell value step ($\Delta V = 0.50$). Inspection of the delivered goods would be made by taking an adequate sample and determining the Munsell book notation of its color by visual tridimensional interpolation along the color scales of the *Munsell Book of Color*. The hue, value, and chroma differences between sample and the specified standard notation (7.2PB 2.9/9.1) would be found by

subtraction and compared to the stated tolerances. Note that reading the Munsell book notation of a color sample also requires that the inspector be able to judge which of two nearly identical colors is the lighter, grayer, redder, or greener, and, in addition, he must be able to make visual interpolation along the Munsell color scales much as a length scale graduated in millimeters may be read by interpolation to tenths of millimeters. For example, a color intermediate between 7.5PB 3/10 and 5PB 3/10 might be read by hue interpolation as 7.2PB 3/10. An inspector can be trained to make such judgments with considerable reliability. A trained inspector judging whether a sample is a "commercial color match" to the standard will ordinarily make allowances based on his knowledge of the difficulty of controlling the color of the particular sample being judged. If the combination of colorants by which the sample is colored is hard to control, he may pass an easily visible color difference because in his judgment another attempt to match it would probably turn out just as poorly. But for a well-behaved colorant combination he will throw out samples that an untrained observer would call a perfect color match. The use of actual chips showing the limit colors previously agreed on goes far toward making the inspection objective and independent of the training of the inspector in the coloring process whose results he must judge.

Fundamentally Specified Standards. If the color to be chosen has a permanent meaning, it is a wise safeguard to measure in fundamental terms any special color standards developed for its control. This permits duplicate color standards to be selected at any future time and also permits a statement of the intended color range in fundamental terms. This range may be stated in terms of the CIE coordinate system (chromaticity coordinates, x , y , z , daylight reflectance or daylight transmittance, Y), or in terms of any coordinate system derived from the standard system, such as that based on dominant wavelength and purity, or those defined by equations 24, 25, and 26, or those implied in the distance elements defined in equations 32, 33, 34, and 35, or that defined graphically by the OSA Subcommittee on the Spacing of the Munsell Colors (Newhall, Nickerson, and Judd, 1943). Or the color range might even be specified in terms of the color difference, ΔE , from any of equations 28 to 35. It should be noted, however, that the perceptibility of the difference between sample and standard is usually only a part of the consideration on which the tolerance is based. Slight deviations from the standard in one sense may be intolerable; but comparatively large deviations in another may make little difference to the intended use of the material or article.

set these color tolerances; but difficulties of color control in manufacturing signal glassware were carefully taken into account. Inspection of signal glassware is carried out by visual comparison with standard limit filters exemplifying the chromaticity limits. Thus, for green signal glassware the inspector is supplied with a blue limit filter and a filter giving both the pale limit and the yellow limit, and he is required to be able to judge whether the glass roundel under test is bluer than the blue limit, or less saturated than the pale limit. The AAR specification includes strict tolerances on the chromaticities of these limit filters. These tolerances are stated in terms of chromaticity coordinates x, y . The master standards for the limit filters are on file at the National Bureau of Standards, which, for a fee, will compare any submitted filter with the corresponding master standard and report whether it complies with those tolerances. Similar methods are used to control the colors of signal lights for airplane traffic (Colors, Aeronautical Lighting, Fed. Std. No. 3, 21 March 1951, obtainable from Superintendent of Documents, Washington 25, D. C., price 15 cents).

Colorimetry and Spectrophotometry. If the inspection of the delivered goods is to be made still more independent of the personal traits of the inspector, resort must be had to measurement of the color of the delivered goods by visual colorimetry, photoelectric colorimetry, or by a fundamental measurement by spectrophotometry plus computation by means of the standard observer. If the measurement is to be made by direct colorimetry, the statement of the color tolerance is often given directly in terms of the instrument readings. For example, if a subtractive colorimeter using Lovibond glasses is specified, the standard color and the limits might well be stated directly in terms of Lovibond red, yellow, and blue numeral. Since 1931, however, it has become more common to express color tolerances in terms of the CIE standard observer and coordinate system, or in some terms with a known relation to them.

If the color tolerances are based on a study of the probable variations in color arising from the manufacturing process, the tolerances for each new color have to be individually studied, and it does not matter much what coordinate system is used to express them, so long as it is defined in terms of the standard (Y, x, y) coordinate system. The coordinate system used for the study is a logical choice.

But, if the color tolerances are set purely on the basis of objectionableness of the variations from the standard, or on the basis of the perceptibility of such variations, a coordinate system yielding nearly uniform color scales, such as the Munsell renotation system (Newhall, Nickerson, and Judd, 1943), or those implied in equations 28 to 35

have an advantage. If perceptibility of the variations is the sole basis, the advantage is obvious. The color tolerances may be stated directly in terms of size of color difference by means of those equations. The following set of color tolerances has been proposed by the Munsell Color Foundation for three different degrees of closeness of match:

Designation of the Closeness of Color Match	Munsell Hue	Munsell Value	Munsell Chroma	Index of Fading, I (Equation 28)
AAA	2/C	0.05/	0.2	1.4
AA	3/C	0.10/	0.4	2.4
A	4/C	0.20/	0.6	3.7

AAA tolerances are recommended to the Munsell Color Company for use in determining whether a repaint of any of the chips of the *Munsell Book of Color* shall be accepted for issuance. These rigid tolerances (AAA) correspond to about one NBS unit. Tests for conformity to AAA tolerances can be made only by test methods that involve direct comparison to material standards. The less rigid AA tolerances correspond to about 2 NBS units. Tests for conformity to AA tolerances may be made by careful application of indirect instrumental measurements based on the magnesium oxide reflectance standard (Keegan, 1939) alone. For example, studies of the measurements of the Munsell samples by the spectrophotometer at the Massachusetts Institute of Technology (Glenn and Killian, 1940), the National Bureau of Standards (Kelly, Gibson, and Nickerson, 1943), and the Interchemical Corporation (Granville, Nickerson, and Foss, 1943) indicate that the determination can be carried out so as to meet the AA tolerances. The more lenient A tolerances correspond to about 3 NBS units.

If conformity to color tolerances set on the sole basis of perceptibility of the variations from the standard is to be decided by spectrophotometric measurement, the Adams chromatic-value coordinate system is a good choice. The tolerance may then be set conveniently in terms of size, ΔE , of color difference by equation 34. In such a case no special study of tolerances for each new color need be made. All that is required is a specification of the standard color and the number of units of color difference that is to be tolerated.

If the color tolerance is to be set on the basis of the objectionableness of the color variations from standard, it is still advantageous to use a coordinate system yielding nearly uniform color scales. Noticeability of the variations is a very helpful place to start an analysis of the objectionableness of color variations. In writing color tolerances for textiles, it has usually been found that a close hue match is imperative,

but larger tolerances may be allowed for variations in lightness and saturation. Not only are hue variations more easily detectable because of lesser dependence on chromatic adaptation of the observer and on precise uniformity in amount of light falling on the samples but also, when detected, hue variations carry the meaning that a wrong dye has been used. A considerable lightness difference even though easily seen is not objected to by the ultimate consumer because she sees similar differences all the time caused by uneven illumination. A saturation difference even though detectable is not as objectionable as a hue difference because it indicates only that too much or too little dye has been used. This is not so bad as using the wrong dye; it fails to give the impression that a gross mistake has been made. Also two colors of the same hue that are perceptibly different in lightness and saturation still harmonize, but a perceptible though small hue difference produces an unpleasant effect.

To the degree, and it is considerable, that it is valid to analyze the difference between objectionableness and noticeability of color difference in terms of the relative importance of hue, lightness, and saturation differences, the radial organizations of color space have an advantage (Godlove, 1951, c). Thus equations 28 or 29 may be modified by insertion of different multiplying factors for ΔH , ΔV , and ΔC to correspond to the differing objectionablenesses of the corresponding variations.

COLOR LANGUAGES

The explanation of color phenomena involves several branches of science (chemistry, physics, physiology, psychophysics, psychology), and color is an incidental concern of others (botany, zoology, geology, archaeology, anthropology). The production of colored materials involves many industries and technologies (paper, textiles, ceramics, paints, plastics). The various approaches to color have given rise to various color languages. Few people are thoroughly conversant with any one of these languages, and probably no one with all. Yet everyone dealing with color should have a good knowledge of one color language and a smattering of the rest. Correct use of color language is a valuable tool. It permits those working on different aspects of color to understand each other and to cooperate successfully. Many hours have been wasted in arguments between people trying to say the same thing in different color languages. Dollars have been wasted, too, because a purchaser thought he understood what the supplier meant about the colors to be supplied but did not. This book has used chiefly the psychophysical language of color developed by the OSA

Colorimetry Committee (OSA, 1944), together with a few words from physics, physiology, psychology, and colorant technology. As the reader no doubt has noticed, the psychophysical language of color is rather elaborate; it is to be hoped that he has also noticed that it is well adapted to the description and solution of many of the color problems arising in industry and commerce. It is a mistake to think of this terminology as static, because it is really in an embarrassingly rapid state of development. It is also a mistake to think that any language of color developed from any one point of view can serve to express conveniently all that has to be said about color. An example may serve to illustrate this point. This is the story of Jack and Jill.

It seems that Jack, a young psychophysicist, had been forced, because of war-time shortages, to try oleomargarine as a substitute for butter and had finally had his taste so perverted that he preferred it to butter. His bride, Jill, objected to oleomargarine not so much from its taste but because she had to spend time mixing in the capsule of yellow dye supplied in those unenlightened times with the oleomargarine which, to avoid a federal tax of 10 cents a pound, was sold white.

JACK. Well, why don't you serve it white, then? It makes no difference to me.

JILL. Why, I wouldn't have a white spread for bread on my table. It reminds me of lard and takes away my appetite.

JACK. Ah! this is right in my line. Color is defined by dominant wavelength, purity, and reflectance. I know a way to color oleo without dye. All you have to do is to use a yellow light over the dining-room table. This will give the uncolored oleo the same dominant wavelength, purity, and reflectance as butter. You won't be able to tell them apart.

JILL (doubtful). I don't know anything about dominant wavelength, but go ahead and put in a yellow bulb.

So Jack brought home the yellow bulb and proudly displayed a pat of butter and a pat of uncolored oleo side by side on the same dish.

JACK. See, just as I told you. Under the yellow light you can't tell them apart.

JILL. I see all right. What you have done is make them both look like lard! You may as well throw that yellow bulb away.

JACK (somewhat taken down). H'm. They do both look white, don't they? Oh! I understand now. To make them look yellow you have to use a black background.

JILL. You never would admit being wrong. If you think I am going to use black dishes and a black tablecloth, you have another think coming.

So Jill won the argument, as she usually did.

The psychophysical terms dominant wavelength, purity, and reflectance refer to the light reflected by the specimen. White and yellow are psychological terms describing the color perceived to belong to the specimen. As quoted before from Dr. Adelbert Ames of the Dartmouth Eye Institute, "What we see is our best guess as to what is out front." With a dark background, the best guess is that both spreads are yellow. With a white background illuminated by yellow light, the yellow character of the light is unmistakably revealed, and the guess that both spreads are yellow immediately gets demoted in favor of the guess that both spreads are white. This phenomenon is often erroneously ascribed to chromatic adaptation. It is true that after looking at a white tablecloth illuminated by yellow light for five minutes or more, the eye becomes yellow adapted so that the color perceived to belong to the illumination changes from strong yellow to faint yellow. But the lights reflected from the spreads, the dishes, and the tablecloth change color together, and the objects themselves are perceived as white throughout the course of the adaptation. The phenomenon has been thoroughly studied by a German psychologist (Jaensch, 1921) under the name color transformation.

There is a moral to this story. It is that dominant wavelength by itself never makes anyone lose his appetite. But the perceived aspect of the color of foods may, if it is different from previous pleasant experiences with that food. These aspects must be described in psychological (that is, personal) terms. Business must interest itself in what the customer sees. The color of the light leaving the object is only a part of the story.

A good start on a dictionary of color terms has been made by the Inter-Society Color Council. This is an organization of 21 societies of national scope, each having an interest in color.* Lists of color terms with definitions were supplied by many of these societies and were compiled by a subcommittee of the Council to form a comparative list

* American Artists Professional League, American Association of Textile Chemists and Colorists, American Ceramic Society, American Institute of Architects, American Institute of Decorators, American Oil Chemists Society, American Pharmaceutical Association, National Formulary, American Psychological Association, American Society for Testing Materials, Federation of Paint and Varnish Production Clubs, Gravure Technical Association, Illuminating Engineering Society, Industrial Designers' Institute, National Association of Printing Ink Makers, Optical Society of America, Society of Industrial Designers, Society of Motion Picture and Television Engineers, Technical Association for the Graphic Arts, Technical Association of the Pulp & Paper Industry, Textile Color Card Association of the United States, United States Pharmacopoeial Convention.

of 1,550 color terms alphabetically arranged.* This list reveals hundreds of minor discrepancies in the meanings of color terms by different groups, and several scores of major discrepancies. It also includes many terms dealing with one aspect of color or another that are quite unfamiliar to one who, like the author, is accustomed chiefly to the terms used in the Optical Society of America. As examples may be cited such terms as: barre, isopaque curve, bloom, blushing, cast, high, ghosting, amaurosis, episcotister, Horner's law, phosphene, chatters, crock, duochroism, duotonic, hungry, level, mealy, nicks, picking-off, resist, snaps, throwster colors, timbre, distemper, gouache, juicy color, reticence, rutilant, scumbling, shot color, tender, truculent, upper register, valuable discord, and skitteriness. No doubt some of those who use these terms are equally at a loss to know the meaning of such optical terms as: minus color, tricolor ratio, alychne, yellow spot, and dichroism. To attempt to resolve discrepancies in the various definitions used, or to define all unfamiliar color terms found in print, is quite beyond the scope of this book. It would involve many diverse sciences and technologies. The reader is referred to the ISCC list as a unique compilation of this kind of information. It does seem useful, however, to collect in one place the terms and definitions for the most important color concepts used by American industry. This has already been done for British industry by the Colour Group of the Physical Society (1948).

Basic or Psychological Color Terms. Basic or psychological color terms apply to color concepts evaluated personally by each observer to describe his color perceptions. There is no way in which a manufacturer can be sure what color will be perceived to belong to his product. Each customer will see it somewhat differently, depending on the type of vision that he or she has, or what the customer has been looking at before. These terms apply also to the judgments of an inspector checking delivered goods for conformity to a color requirement.

Hue (of a color perception). The attribute denoted by blue, green, yellow, red, purple, and so on.

Saturation (of a color perception). The attribute determining the degree of its difference from the achromatic color perception most resembling it; opposite of grayness.

Achromatic color perception. One not possessing a hue.

Chromatic color perception. One possessing a hue.

Brightness (of an area perceived as self-luminous). The attribute permitting the color perception to be classed as equivalent to some member of the series of achromatic color perceptions ranging from very dim to very bright or dazzling.

* Available at \$2.00 a copy from the Secretary, Inter-Society Color Council, P.O. Box 155, Benjamin Franklin Station, Washington 4, D. C.

Lightness (of the color perception of a nonself-luminous object). The attribute permitting the perception to be classed as equivalent to some member of the series of achromatic object-color perceptions ranging for light-diffusing objects from black to white, and ranging for regularly transmitting objects from black to perfectly clear and colorless.

Psychophysical Color Terms. Psychophysical color terms are terms derived from the operation of color-matching one photometric half-field to another, or from judgments of similarities and degree of difference between two such half-fields. They are terms used in the specification of color and in the solution of the optical parts of color problems. General terms will be considered first; then the special terms arising from the Ostwald theory, and finally the special terms arising from the Munsell theory.

General Terms. General terms arise from the operation of setting up a color match between two photometric half-fields.

Light. The aspect of radiant energy of which a human observer is aware through the visual sensations that arise from the stimulation of the retina of the eye.

Color. The characteristics of light apart from variations in space and time.

Dominant wavelength (of a light). Wavelength of the spectrum light that, when combined in suitable proportions with the specified achromatic light, yields a match with the light considered.

Spectrum light. That corresponding to radiant energy of one frequency.

Achromatic light. Light chosen because it usually yields an achromatic color perception under the desired observing conditions.

Complementary wavelength (of a light). Wavelength of the spectrum light that, when combined in suitable proportions with the light considered, yields a match with the specified achromatic light.

Colorimetric purity (of a light). Ratio of luminance of the spectrum light, in a mixture with the specified achromatic light required to match the light considered, to the luminance of the mixture.

Luminance. Luminous intensity of a surface (real or virtual) in a given direction per unit orthogonally projected area.

Luminous intensity (of a point source in any direction). Luminous flux per unit solid angle emitted by that source in that direction. Note: Luminous intensity of an extended surface is measured by placing the receptor so far from the surface that it approximates a point source.

Luminous energy. Evaluation of radiant energy according to its stimulation of the brightness attribute of visual perception.

Luminous reflectance (of a body). Ratio of reflected to incident luminous flux.

Luminous flux. Time rate of flow of luminous energy.

Luminous transmittance (of a body). Ratio of transmitted to incident luminous flux.

Luminous directional reflectance (of a body). The luminous reflectance that a diffusing surface of constant luminance regardless of angle of view would need to possess in order to yield the same luminance as the specimen considered under the same illuminating and viewing conditions.

Tristimulus values (of a light). The amounts of the three reference or matching stimuli (primary lights or primary colors) required to give by additive combination a match with the light considered.

Chromaticity coordinates (of a light). The ratio of each tristimulus value of the light to their sum.

Chromaticity (of a light). The color quality of a light definable by its chromaticity coordinates.

Chromaticity diagram. Diagram produced by plotting any one of the three chromaticity coordinates against any other.

Excitation purity (of a light). The ratio of two lengths on a chromaticity diagram, the first length being the distance between the point representing the specified achromatic light and that representing the light considered, the second length being the distance along the same direction from the first point to that of the border of the chromaticity diagram (spectrum locus or the straight line connecting its extremes).

Primary colors. Colors of three lights by whose additive combination nearly all other colors may be produced. Note: These colors are usually said to be either red, green, and blue, or red, green, and violet.

Metametric pair. Two lights of identical tristimulus values but different spectral compositions.

Ostwald Terms. By the Ostwald theory the color of a light-diffusing object is specified by designating which of a family of idealized objects of a particular kind yields a color match. The Ostwald ideal reflecting object is conceived as having spectral reflectance, R_λ , constant at one value between two complementary wavelengths and constant at some value (different from the first except for whites, grays, or blacks) elsewhere in the visible spectrum (Ostwald, 1931; Foss, 1944). The lower of these values of spectral reflectance is called the white content. The difference between unity and the higher of these two values of spectral reflectance is called the black content. The difference between the two values of spectral reflectance is called the full color content; see Fig. 88. The ideal Ostwald object color defined in Fig. 88 would be a purple. The complementary wavelengths chosen are 480 and 580 $m\mu$. It will be noted that the sum of the black content, the white content, and the full color content of a color is necessarily unity. Any two of these terms serve to specify two attributes of color, the two usually chosen being white content and black content. The remaining attribute is called Ostwald hue and corresponds to a fairly uniform spacing of the circuit. Ostwald hue number held constant corresponds either to dominant wavelength constant at some value or to complementary wavelength constant at some value. It is doubtful whether any measurements of color in Ostwald terms have ever been carried out in this country, certainly none of any commercial importance. Perhaps this is because not all commercially important colors can be matched by an ideal Ostwald color. Consequently, an extended dis-

cussion of the Ostwald theory is not warranted here. However, color charts, such as the *Color Harmony Manual*, arranged more or less according to Ostwald principles, have proved to be quite valuable, and a number of the color terms springing from the Ostwald theory have

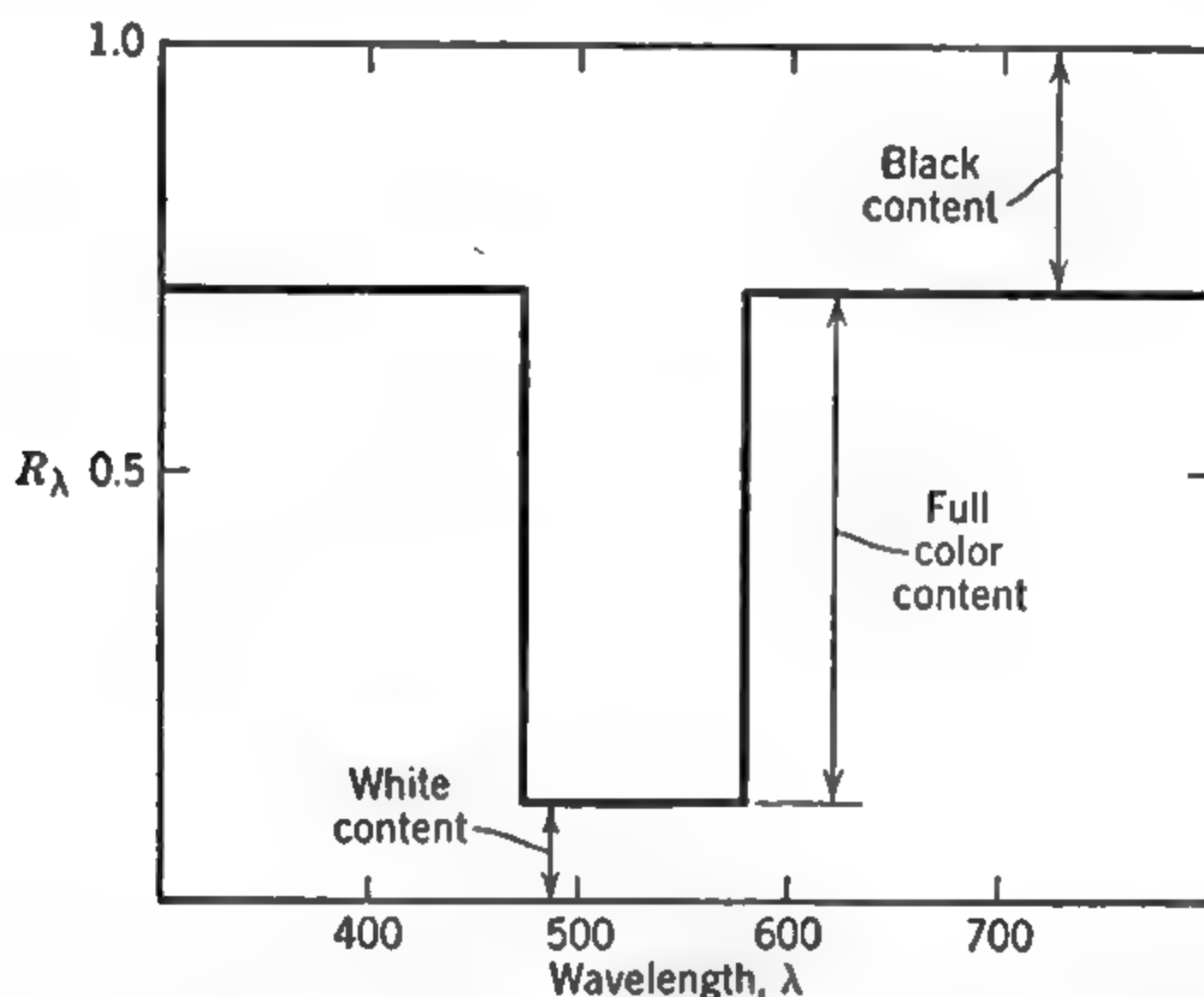


FIG. 88. Spectral reflectance of an opaque specimen illustrating what is meant by an ideal Ostwald color. The lower of the two reflectances is the white content of the color, the difference between unity and the higher of the two reflectances is the black content, and the difference between the two reflectances is the full color content.

been found convenient for expressing color ideas of importance in industry and commerce.

Semichrome. Ideal Ostwald object color for which black content and white content are equal to zero; same as full color.

White content. The lower of the two values of spectral reflectance characterizing an ideal Ostwald object color.

Black content. The difference between unity and the higher of the two values of spectral reflectance characterizing an ideal Ostwald object color.

Full color content. The difference between the two values of spectral reflectance characterizing an ideal Ostwald object color.

Shadow series. Series of colors having the same Ostwald hue and the same ratio of full color content to white content; same as series of constant chromaticity.

Ostwald hue. Designation of dominant wavelength or complementary wavelength by arbitrary numbers usually ranging from 1 to 24.

Isovalent colors. Colors of differing Ostwald hue but having the same black content and the same white content.

Ostwald purity. Ratio of full color content to white content. Note: For any one Ostwald hue, colors of constant Ostwald purity also have constant colorimetric purity and constant excitation purity.

Those who work with mixtures of a chromatic pigment with black pigment and white pigment as in the formulation of paint, or in the mixture of paint on a palette, often find that their color thoughts can be expressed conveniently in these Ostwald terms. Such people find a collection of chips arranged more or less on Ostwald principles very suggestive and helpful. They see immediately what to do in paint mixture to carry out their ideas. A collection of chips organized more or less on the Ostwald basis is sufficiently close to a paint-mixture system so that such suggestions are not importantly misleading. These terms and the color charts to illustrate them have been used successfully in teaching art students (Hiler, 1942) and in package design (Jacobson, 1948). Those who work with printing by screened plates involving combinations of a chromatic pigment and a black pigment with the white of the paper showing between the printed dots will find much the same advantage in the Ostwald terms and in the use of color charts arranged more or less on the Ostwald basis. They are on somewhat firmer ground because color mixture by juxtaposed dots corresponds exactly to the principles of color mixture by averaging on which the Ostwald system is based. Furthermore, the variations of popular color preferences can be expressed conveniently in terms related to the Ostwald theory. Thus, pastels or spring colors are colors having high white content and little or no black content (Birren, 1944). Fall colors are those having little or no white content. Muted colors are those having considerable black content and considerable white content, but very little full color content. Peasant colors are those having very little black or white content and which therefore approach a semichrome. The selection of a combination of colors for an industrial purpose is likely to involve some thinking along these lines, and the conclusions reached are already expressed in terms showing approximately what to do in pigment or ink mixture to implement them.

Munsell Terms. Munsell terms have arisen from judgments of similarities and degrees of difference between colors. The organization of the Munsell color system has been described as an introduction to the consideration of uniform lightness and chromaticness scales, and the Munsell notation has been given in connection with the *Munsell Book of Color*. The Munsell color terms correlate closely (Munsell, 1941) with the attributes of the color perception itself.

Munsell hue. Expression of one aspect of an object color on arbitrary scales defined (Newhall, Nickerson, and Judd, 1943) in terms of its luminous reflectance (Y) or luminous transmittance (Y) and its chromaticity coordinates (x, y). Note: The Munsell hue scales have approximately uniform perceptual steps; under ordi-

nary observing conditions Munsell hue of a specimen correlates well with the hue of the color perceived to belong to it.

Munsell value. Expression of the luminous reflectance (Y) or luminous transmittance (Y) of an object color on a scale (equation 20b) giving approximately uniform perceptual steps under usual observing conditions. Note: Under these conditions the Munsell value of a specimen correlates closely with the lightness of the color perceived to belong to it. Table B, Appendix, gives values of Y as a function of Munsell value, V .

Munsell chroma. Expression of the degree of departure of an object color from the nearest achromatic color on arbitrary scales defined (Newhall, Nickerson, and Judd, 1943) in terms of its luminous reflectance (Y) or its luminous transmittance (Y) and its chromaticity coordinates (x, y). Note: The Munsell chroma scales have approximately uniform perceptual steps; under usual observing conditions Munsell chroma of a specimen correlates well with the saturation of the color perceived to belong to it.

Munsell power (of a color). Product of Munsell value by Munsell chroma. Note: Under usual observing conditions (observer adapted to daylight) Munsell power of a color correlates well with its power to attract attention. In rotary mixture of two complementary colors on a Maxwell disk, the sector areas required to produce a match for a mixture of black and white are approximately in inverse proportion to their Munsell powers.

Munsell hue is designated in four different ways: (a) by word (the basic hue terms, red, yellow, green, blue, purple, and their combinations

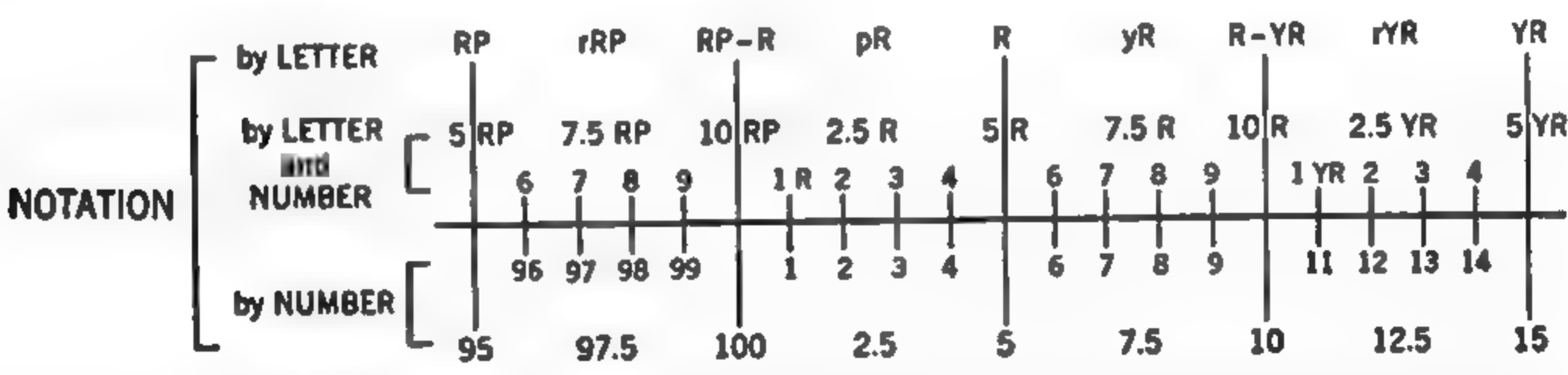


FIG. 89. Detailed relation between the various ways of designating Munsell hue. The relation between the letter, the number, and the combination letter and number designations of Munsell hue are shown here for a portion of the hue circuit. By means of the combination letter-number method, more widely used than the other designations, the third intermediate set of hues is designated: 7.5RP, 2.5R, 7.5R, 2.5YR, and so on. (Courtesy of Munsell Color Company.)

such as yellow-red, green-yellow, and so on), (b) by letter (R, Y, G, B, P , and their combinations such as YR, GY, BG , and so on), (c) by letter and number ($5R, 7.5R, 10R, 2.5YR, 5YR$, and so on), and (d) by number ($5, 7.5, 10, 12.5, 15, 17.5, 20$, and so on, where $R = 5R = 5, yR = 7.5R = 7.5$, and so on). The letter and number method of designating Munsell hue is the most used. Figure 89 shows the various methods of designating Munsell hue in detail.

Munsell value ranges from 0/ for ideal black ($Y = 0$) to 10/ for ideal white ($Y = 1$, or $Y/Y_{MgO} = 1.0257$).

Munsell chroma expresses departure of the color from the equivalent gray (gray of the same Munsell value) in arbitrary steps and is written /1, /2, /3, and so on.

The complete Munsell notation is written in the form: Hue value/chroma, such as 5R 4/14, which means a color of red hue, value slightly darker than middle value, and chroma 14 arbitrary steps from the equivalent gray (N 4/). If it is obtained approximately by visual interpolation along the scales of the *Munsell Book of Color*, the notation is called Munsell book notation. If it is obtained precisely from the tristimulus values (X , Y , Z) of the color in accord with the definitions it is called Munsell renotation.

The Munsell color terms, hue, value, and chroma, since they relate closely to the hue, lightness, and saturation of the color perception, have a wide application. Difference in Munsell value between figure and ground is a direct indication of how well the figure will stand out from the ground. The Munsell terms have been used in design of packages and advertising where legibility is important. They are well adapted to the analysis of color harmony (Moon and Spencer, 1944, 1950) and have been used successfully in teaching art students (Graves, 1941). Their preeminent use, however, is in color specification. A Munsell renotation can have nearly the precision of chromaticity coordinates and luminous reflectance and at the same time indicate in an immediately understandable way what color is meant. Munsell notations were widely used during World War II in contracting and subcontracting under American War Standard Z44-1942 (American Standards Association, 1942), and they are recommended in the current American Standard Z58.7.3-1951.

Experienced decorators, industrial designers, and artists make use of many organizations of color space, the Munsell, the Ostwald, and others invented to order (Birren, 1945, 1950). They, so to speak, serve their apprenticeship by learning the Munsell and Ostwald color languages, then graduate perhaps to the complicated language of drawing and painting (Pope, 1949).

Physical Terms for the Stimulus to a Color Perception. The stimulus for a color perception is a pattern formed by radiant energy on the retina of the eye. The radiometric terms given here are those used to evaluate this pattern.

Radiant energy. Energy traveling through space in the form of electromagnetic waves of various lengths.

Wavelength. Distance traversed by the spectrally homogeneous radiant energy during a single period. Note: Wavelength varies inversely with the frequency

of vibration, or number of waves passing any given point per unit period of time.

Radiant flux. Time rate of flow of radiant energy.

Radiant emittance (of a source). Radiant flux emitted per unit area.

Spectral transmittance (of a body). Ratio of transmitted to incident radiant flux of narrow wavelength range.

Spectral internal transmittance (of a body). Ratio of radiant flux of narrow wavelength range incident internally on one face of a body to that penetrating the opposite face.

Spectral reflectance (of a body). Ratio of reflected to incident radiant flux of narrow wavelength range.

Spectral reflectivity (of a material). Spectral reflectance of a layer of the material so thick that further increase in thickness, however great, produces no change in the value of spectral reflectance.

Irradiance (of a surface). Radiant flux incident per unit area.

Spectral distribution. Amounts of a radiant quantity for the various parts of the spectrum. Note: The radiant quantity may be the radiant flux of a beam, the radiant energy obtained from a beam over a given time interval, the irradiance of a surface by one or more beams, and so on.

The psychophysical term and psychological term corresponding to several of these physical terms are given in Table 2.

Chemical Terms Relating to Spectrally Selective Absorption of Radiant Energy. The articles and objects imaged on the retina of the eye determine by their chemical and physical properties the spectral distribution of energy in the retinal image. Those working with the relation between chemical constitution of the substance (pigment or dye) and its ability to modify by absorption the incident radiant energy have their own color language, a few words of which are defined here.

Transmittancy. Ratio of the transmittance of a nonturbid solution to that of the solvent in equivalent thickness.

Absorbancy. Logarithm to the base 10 of the reciprocal of transmittancy.

Absorption curve. Curve showing the variation of absorbancy with wavelength; same as absorption spectrum.

Absorption band. A maximum in the absorption curve of a substance.

Bathochromic effect. An effect by which the absorption band or curve of a substance is shifted to a longer-wave part of the spectrum with little change in shape of the absorption curve.

Hypsochromic effect. Effect by which the absorption band or curve of a substance is shifted to a shorter-wave part of the spectrum with little change in shape of the absorption curve.

Hyperchromic effect. Effect by which, with no change in concentration, absorption is increased with little change in shape of the absorption curve.

Hypochromic effect. Effect by which, with no change in concentration, absorption is decreased with little change in shape of the absorption curve.

A bathochromic effect is usually associated chiefly with a change in dominant wavelength and is usually appreciated visually as a change in hue in the direction from yellow to orange, to red, to purple, to blue, to green. A hypsochromic effect is also usually associated chiefly with a change in dominant wavelength and usually appreciated visually as a hue change in the reverse direction.

A hyperchromic effect corresponds to increasing the strength of a dye, or to increasing the tinting strength of a pigment. Changes in chemical structure to produce these kinds of effects are well known for most dyestuffs.

Colorant Terms. A large fraction of the color problems of industry have to do with the compounding and application of colorants (pigments and dyes) to produce a material or an article of the desired color. Those who compound and apply colorants describe and think about colors in terms of the colorants required to produce them.

Dyeing. Dyed fabric.

Strength (of a dyeing). Amount of dye introduced in the dye bath to dye a given amount of material, usually expressed as a percentage of the weight of the material.

Strength (of a dye). Reciprocal of the strength of the dyeing required with that dye to produce a desired color; same as tinctorial power.

Strength (of a color). Perceptual quality associated with the amount of dye used to produce the color.

Strong color. Color having the appearance of having been produced by the application of a relatively large amount of dyestuff; same as heavy shade.

Weak color. Color having the appearance of having been produced by the application of a relatively small amount of dyestuff; same as thin color, or hungry color, or stingy color.

In the gray (cloth in general), *in the greige* (silk), *in the grease* (wool). Not yet subjected to wet processing.

Deep color (ceramic industry). Color having the appearance of having been produced by a high proportion of chromatic colorant with relatively little white; same as strong color, or intense color.

Pastel color (ceramic industry). Color having the appearance of having been produced by a high proportion of white with relatively little chromatic colorant; same as tint.

Depth (ceramic industry). Perceptual quality of color associated with the proportion of chromatic colorant with white required to produce it; same as intensity, or strength.

Shade. (1) An expression of color difference from the one under consideration in any way not attributable to variation in strength; that is, differing in such a way that another dye must be added to produce a match. Note: By this meaning a shade variation is either a hue variation, or a (dyer's) brightness variation, or a combination of the two.

Shade (as a verb). (2) To add other colorants (as dyes to a dye bath, or toners to a paint batch) to produce the desired color.

Shade. (3) Color slightly different from the one under consideration.

Shade. (4) Any color produced by a pigment or dye mixture having some black pigment or dye in it.

Shade. (5) Any color. Note: Shade is the most overworked of the colorant terms.

Shade (of a dyeing). Shade of its color.

Shade (of a dye). Shade of the colors of the dyeings made with it.

Full shade. Dyeing such that further additions of the dye will not change the color materially.

Bright color (textile industry). Color perceived as saturated; same as brilliant color, or vivid color, or clean color.

Dull color (textile industry). Color perceived as grayish; opposite of bright color.

Dyer's brightness (of a color). Vividness.

Dyer's brightness (of a dye). Vividness of the colors produced by the dye.

Clean color (textile, paint, and ceramics industry). Color perceived as not at all gray, or dull, or dingy, or dusty, or cloudy; same as clear color.

Muddy color (paint industry). Color perceived as not clean; same as dirty color.

Cleanness (paint and ceramics industry). Freedom of a color from gray or black; same as clarity.

Tint (paint and pigment industries). Any color producible by mixing a chromatic pigment with white pigment, with the latter predominating.

Tint (printing industry). Any color producible by mixing a chromatic pigment with white pigment.

Tint (dyeing industry). Weak dyeing with a bright dye.

Tint (as a verb). Same as to shade (2).

Tone. (1) Hue.

Tone. (2) Color.

Tone. (3) Variation of a color other than in hue.

Tone (as a verb). Same as to shade (2), or to tint.

Tone (decorating trade). Lightness.

Pure color (paint industry). Color produced by undiluted chromatic pigment; same as color of a toner.

Toner. Chromatic pigment added to a batch of paint to produce the desired color.

Masstone (of a chromatic pigment). Color of a thick layer of the pigment incorporated in a vehicle; same as self-tone, or overtone, or mass color.

Undertone (of a chromatic pigment). Color of a thin layer of the pigment incorporated in a vehicle and drawn down on white paper, or color of a tint of the pigment, sometimes as viewed by transmitted light; same as print tone.

Draw-down. Sample used to judge undertone and masstone, prepared by spreading a blob of pigment onto a white backing with the single stroke of a blade.

Tight draw-down. Draw-down prepared with considerable pressure on the blade so as to spread the sample thin; used to judge undertone.

Loose draw-down. Draw-down prepared with only a slight pressure on the blade so as to leave a thick layer of the sample; used to judge masstone.

Let-down. Draw-down of a mixture of white pigment with the pigment to be tested (frequently about 20 to 1 ratio of white to nonwhite pigment).

Primary colors. Colors of three colorants by mixture of which nearly all colors may be produced. Note: These colors are usually said to be magenta, yellow, and cyan, or sometimes simply red, yellow, and blue. They are quite different from primary colors (red, green, violet) in the psychophysical sense.

In the dye-stuff industry, color is specified by two terms: shade and strength, shade being a two-dimensional quantity comprising hue and dyer's brightness. Color differences are estimated (Sundstrom and Stearns, 1950) as weak or strong (referring to strength); red, yellow, green, blue (referring to hue); and bright and dull (referring to dyer's brightness). The redundant terms, light and dark, are also commonly used. Brighter in the dyer's sense usually means lighter and more saturated. Stronger in the dyer's sense usually means darker and more saturated, though for heavy dyeings it may mean simply darker, and exceptionally it may mean darker and less saturated. To explain this apparently ambiguous meaning of strong, it must be noted that as the strength of a dyeing of white fabric is increased the color perception of the dyeing at first darkens and becomes more saturated until maximum saturation is produced. Further increase results in colors perceived to be darker and less saturated (Godlove, 1951, b), until for some dyes an approach to a black color is produced. Figure 90 shows on a plot of lightness against saturation the characteristics (solid lines) of the color perceptions corresponding to dyeings of different strengths of four dyes, one a bright dye, one a dull dye, and two of intermediate dyer's brightness. Each of these solid lines shows in a general and diagrammatic way the changes in color perception caused by variation of strength of dyeings by a single dye. Each of these lines shows the color perceptions corresponding to constant dyer's brightness or constant cleanness. In the lighter colors, an increase in strength corresponds to a decrease in lightness and an increase in saturation. This is the most important range commercially. For an intermediate lightness range, an increase in strength corresponds chiefly to a decrease in lightness; and for the darkest colors it corresponds to a slight further decrease in lightness accompanied by a considerable decrease in saturation. For the lowest lightness range, the expression clean color or bright color is particularly confusing to the uninitiated, and, indeed, to many dyers (Davidson, 1951), because such colors are darker than a color said to have equal strength with less brightness. For very dark colors, dyer's brightness thus corresponds somewhat to what in everyday language is called darkness. This colorant terminology quite logically gives rise to such apparently contradictory expressions as a "bright black" or a "good, clean black." The dotted

lines show approximately the color perceptions corresponding to constant strength with varying dyer's brightness or cleanness. It should be remarked, however, that the dyer will not try to compare in strength dyeings differing very much in brightness.

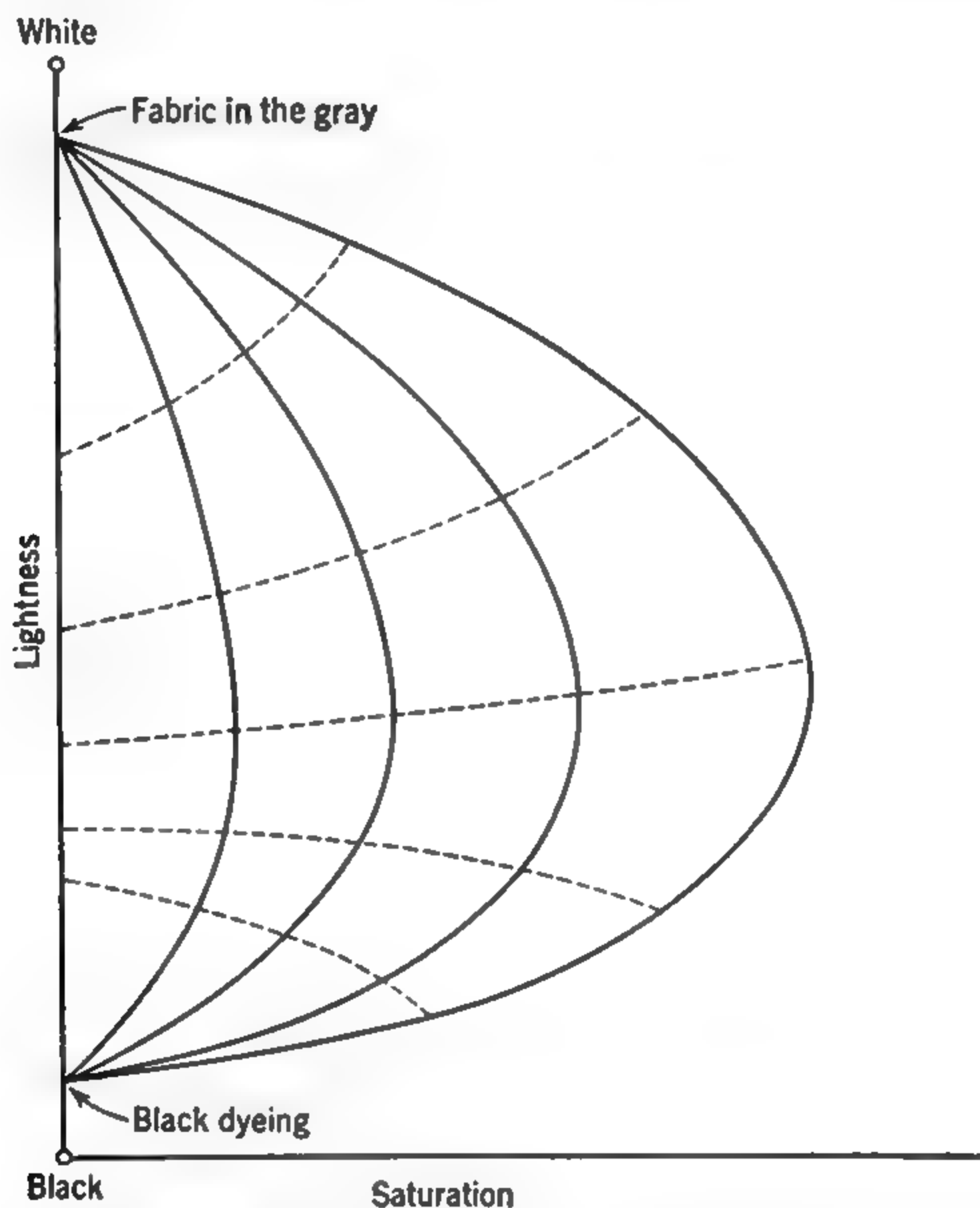


FIG. 90. Loci of constant dyer's brightness (solid lines) on a plot of lightness against saturation of the corresponding color perceptions. Dotted lines show schematically the loci of constant strength.

Figure 90 is a generalized summary of the behavior of dyeings, and it summarizes equally well the general behavior of mixtures of chromatic pigment with white pigment dispersed in a vehicle. It should not be supposed, however, that the loci of constant cleanness for particular colorants are as regular as this. Two colorants of nearly the same cleanness may give rise to loci that cross; that is, for small concentrations one colorant may be cleaner, and for large concentrations dirtier.

The assessment of color differences in terms of hue, dyer's brightness (or cleanness), and strength (or depth) is directly related to what has to be done to bring one dyeing or paint or ceramic to a

color match with another. The ability to make judgments in these terms depends on many years of experience with colorants and is a very valuable result of such experience. If two colors, a sample and a standard, are judged correctly to differ in strength alone, the sample can be brought to a match with the standard simply by changing the concentration of the chromatic colorant. But, if the sample is correctly judged to be brighter or duller or of different hue, it cannot be brought to a color match with the standard by adjustment of concentration. The sample is then said to be "off-shade" and must be corrected to standard by the addition of another chromatic colorant (or toner).

Chromatic colorants (pigments and dyes) are customarily graded as to strength (tinting strength, tinctorial power). Strength of a colorant is a direct measure of how much coloring can be done with it. Other properties (fastness to light, chemical stability in the presence of the other constituents, and so on) being equal, strength of a colorant directly determines the price that can be charged for it. Bright dyes and clean pigments are valuable not only to produce colors that the customer will perceive as desirably light and saturated but also as toners. Grinding a toner with white pigment until a color is produced that is perceived to have about the maximum saturation is called "developing the color." It is generally wasteful to use a toner with any less admixture of white pigment than this, a good approximation to the darker part of its gamut being producible by additions of other less expensive near-black pigments. A method of specifying pigment colors, alternative to the hue-cleanness-depth method, is to specify the composition of the colorant mixture required to produce it: (a) name (trade or chemical) of the chromatic pigment in the mixture, (b) percentage (by weight or volume) of the white pigment in the mixture, (c) percentage of the black pigment. These three terms are roughly equivalent, respectively, to Ostwald hue, white content, and black content.

In the printing industry a color is often matched by a double printing of fine dots, the first being dots of a chromatic ink, the second being dots of black ink. The color is specified by (a) name of the chromatic ink, (b) the percent of coverage of the paper by chromatic dots, and (c) the percent of coverage by the black ink. These terms are rather closely equivalent to Ostwald hue, full color content, and black content. The same terms can be applied to four-color process printing in which a color is matched by four printings of dots; the first, yellow; the second, magenta; the third, cyan; and the fourth, black.

In summary, the chief colorant terms are: (textiles, dyestuffs), strength and shade, shade comprising the two variables, hue and dyer's brightness; (paints, printing, ceramics), hue, cleanness, depth; or, alternatively, hue, percent of white, percent of black.

Color Names. So far we have been dealing with terms for various color concepts. The ultimate purchaser does not deal explicitly with these color concepts. He or she asks for the color by name. American merchants, salesmen, advertising agencies, and fashion experts have shown considerable ingenuity in devising color names for sales promotion. These names may indicate the source of the color or the use of the article and often manage to include a suggestion of romance and glamour. Color preferences undergo a periodic cycle; the keynote of this activity is often: *New Names for Old Colors*. As a sales-promotion technic, this is highly successful. From the standpoint of permanent color specification it has obvious drawbacks. The Textile Color Card Association shows new color names in its Seasonal Color Cards but has made the attempt to have each new name refer to a color at least slightly different from any color that it has named before.

The color names currently used in mass marketing are listed in the *Descriptive Color Names Dictionary* (Taylor, Knoche, and Granville, 1950), issued as a supplement to the third edition of the *Color Harmony Manual*, also available separately from the Container Corporation of America. The color names are defined by reference to the color chips of the Manual.

The color names more or less permanently used in the textile industry are given and defined by silk swatches in the Standard Color Card of America (TCCA, 1940).

The *Dictionary of Color* (Maerz and Paul, 1930, 1950) is the most extensive and authoritative work on color names; it includes color names from many fields. The Plochere color system (Plochere, 1948) gives a color name for each of the 1,248 colors shown. These names are adapted to sales promotion.

The classic *Color Standards and Color Nomenclature* by Ridgway (1912) listed a name for each of about 1,000 colors. These names were intended to be descriptive only. They are not adapted to sales promotion.

The ISCC-NBS Method of Designating Colors. A method, devised at the request of the American Pharmaceutical Association and the United States Pharmacopoeial Convention, was proposed in 1939 for general descriptive purposes. The plan of the method was worked out by the Inter-Society Color Council, and the details were de-

veloped at the National Bureau of Standards; the method has therefore been referred to as the ISCC-NBS method of designating colors

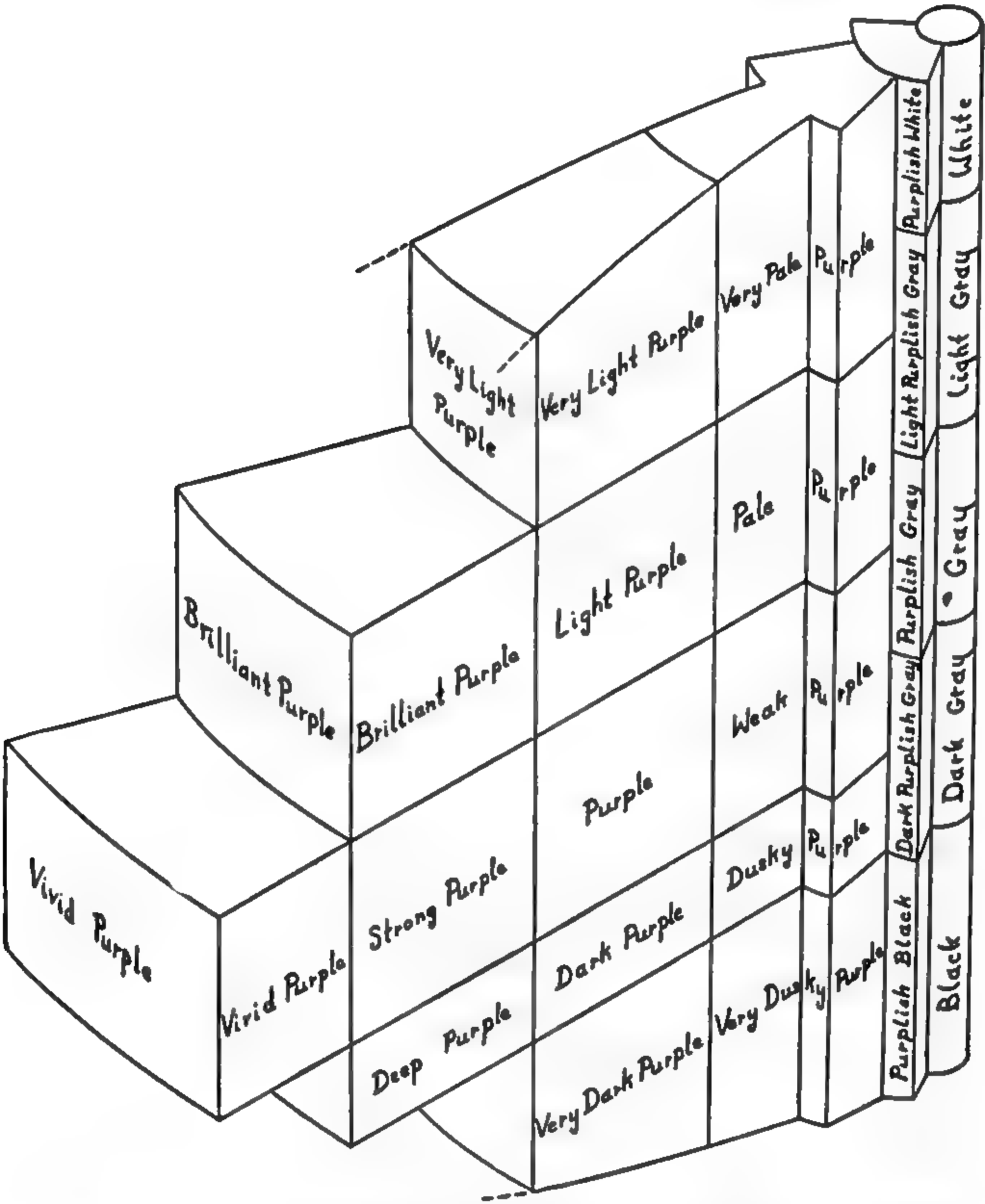


FIG. 91. Illustration of the tridimensional nature of the definitions of the ISCC-NBS color designations.

(Judd and Kelly, 1939; Kelly, 1940). This method provides a designation for every color perceived as belonging to an object (either an opaque surface or a light-transmitting layer), and it has been extended to the colors of self-luminous areas by Kelly (1943). Kelly's extension of the ISCC-NBS method has been used in Figs. 31, 32, and

39. The number of color designations was purposely made small (about 300) for the sake of simplicity. Since about 10 million surface colors can be distinguished by the normal human eye under optimum observing conditions, the ISCC-NBS method falls far short of supplying a different designation for each distinguishable color, or even for all colors (numbering perhaps half a million) considered to be commercially different.

The plan of the method is to divide the surface color solid (see Fig. 12) arbitrarily into about 300 compartments and to assign a designation to each in as good conformity as possible to color nomenclature currently used in art, science, and industry. The compartments embracing the black-white axis are given the following designations: black, dark gray, medium gray, light gray, and white. The compartments adjacent to these are given similar designations formed by adding an adjective indicating the hue, such as yellowish white, dark purplish gray, or greenish black. All other compartments take designations consisting of a hue name (red, orange, yellow, green, blue, purple, pink, brown, olive) preceded by modifiers (light, dark, weak, strong) indicating the lightness and saturation of the perceived color.

The boundaries between the groups of colors known by these designations were adjusted to accord with common usage as closely as that usage was known at the time and have been expressed in terms of Munsell book notation. Figure 91 indicates the shape of the compartments proposed in 1939 for colors of purple hue.

The ISCC-NBS designations are not to be considered a substitute for numerical designations of color resulting from application of a suitable colorimetric method, but they do supply a certain precision to ordinary color designations that had previously been lacking. These color designations are used in the current edition of the *National Formulary*, and in some of the monographs of the *United States Pharmacopoeia*, and in a textbook of qualitative chemical analysis (Pearce, 1946). They have been used for describing the colors of building stone (Kessler, Hockman, and Anderson, 1943) and soils (Rice, Nickerson, O'Neal, and Thorp, 1941), and for a considerable variety of research purposes such as the description of mica colors after heat treatment (Hidnert and Dickson, 1945). Nickerson and Newhall (1941) have found the Munsell book notations for the central color of each compartment assigned an ISCC-NBS color designation and have recommended a system of abbreviations; see Table 20.

Table 20. Abbreviations for ISCC-NBS Color Designations

<i>B</i> = blue	<i>ll</i> = light	<i>pk</i> = pinkish
<i>b</i> = bluish	<i>med</i> = medium	<i>R</i> = red
<i>Bk</i> = black	<i>mod</i> = moderate	<i>r</i> = reddish
<i>Br</i> = brown	<i>O</i> = orange	<i>str</i> = strong
<i>br</i> = brownish	<i>o</i> = orange (adj.)	<i>v</i> = very
<i>d</i> = dusky	<i>Ol</i> = olive	<i>viv</i> = vivid
<i>dk</i> = dark	<i>ol</i> = olive (adj.)	<i>Wh</i> = white
<i>G</i> = green	<i>P</i> = purple	<i>wk</i> = weak
<i>g</i> = greenish	<i>p</i> = purplish	<i>Y</i> = yellow
<i>Gr</i> = gray	<i>Pk</i> = pink	<i>y</i> = yellowish

The ISCC-NBS designations are generally unsuited for sales promotion in which it is important to avoid any suggestion of weakness or adulteration. For example, brick red by this method of designation is *weak red*. No dealer in bricks would care to use such a designation. However, interest in this systematic method of color designation for descriptive purposes only has continued to rise, and a revision of the boundaries defining the color designations has been nearly completed by a subcommittee of the Inter-Society Color Council, so as to bring the names still more closely into accord with general usage, particularly usage in the textile industries. For example, the hue designation, *bluish purple*, is not used in the textile industry; so it has been changed to *violet*. Furthermore, the controversial adjectives, weak and dusky, have been changed to grayish and blackish, respectively. Figure 92 shows the color designations used for the Munsell renotation hue range 4R to 6R in this revision. Figure 93 shows the approximate colors corresponding to these designations; it is a copy by screened-plate printing of the chart for Munsell hue 5R taken from the *Munsell Book of Color*.

The ISCC-NBS method of designating colors is the most ambitious attempt so far to give a precise meaning to color names.

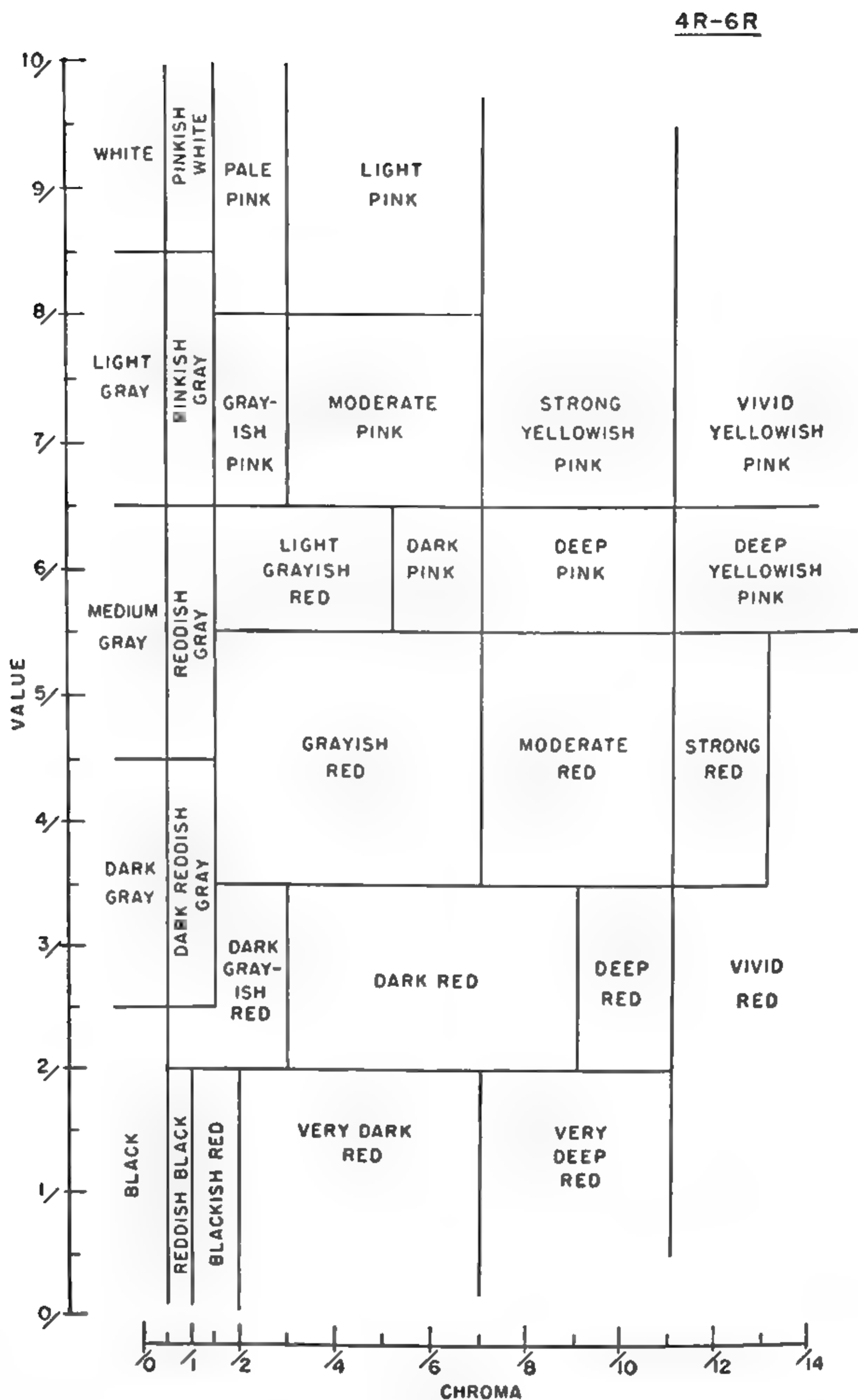


FIG. 92. The Inter-Society Color Council—National Bureau of Standards (ISCC-NBS) method of designation in conjunction with the Munsell notation.

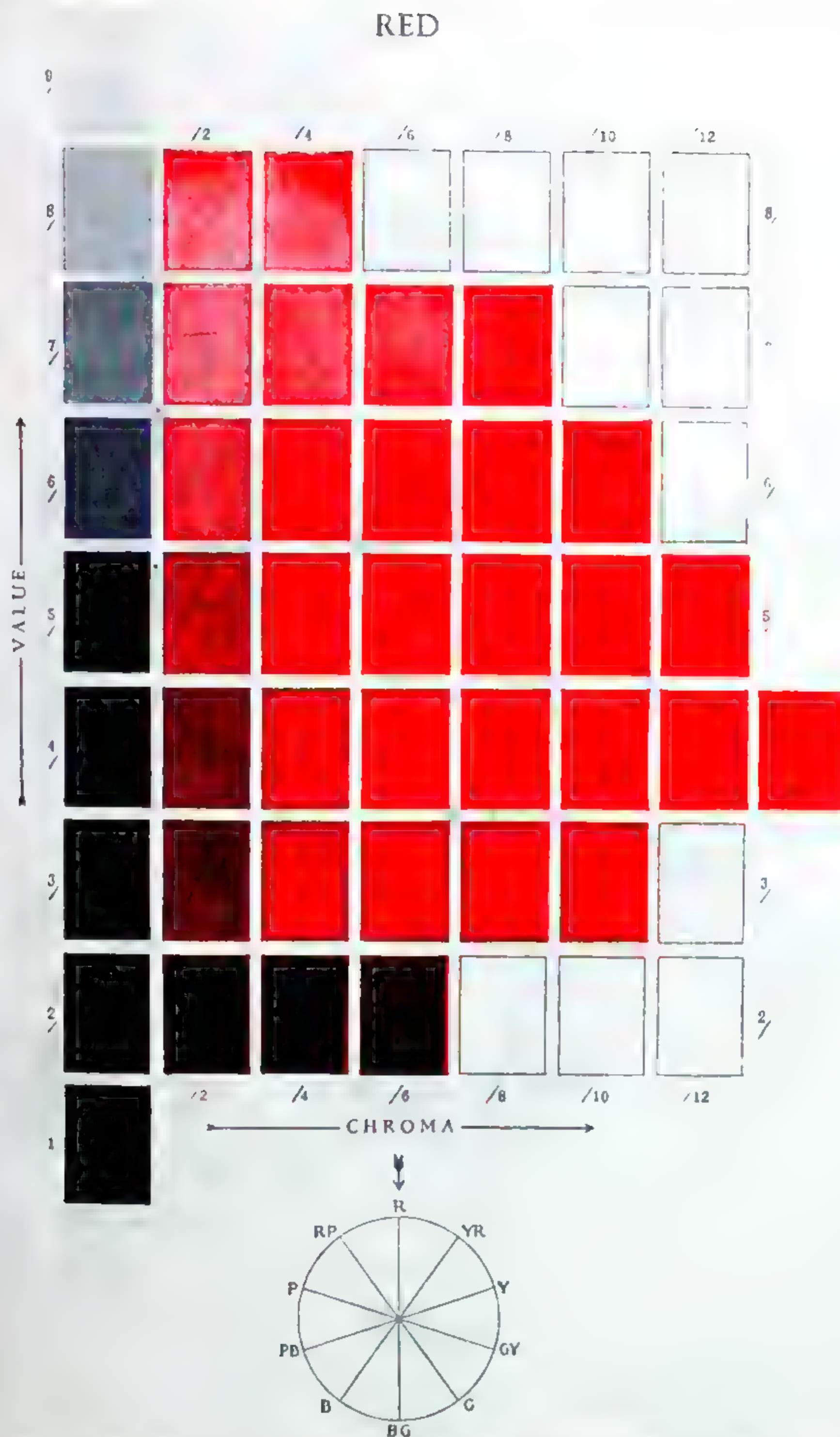


FIG. 93. The RED (Hue 5R) color sheet from the *Munsell Book of Color*. Printed reproduction of a page from the Library Edition. The colors displayed merely serve as an illustration and should not be considered as representing Munsell standard papers. (Courtesy Munsell Color Company, Inc., Baltimore, Maryland.)

PART III

Physics and Psychophysics of Colorant Layers

A large fraction of the color problems in industry has to do with layers of compounded colorants. A varnish film imparts gloss to the surface and reveals the color and texture beneath. A wax film does the same. Paper forms a background permitting type to be legible and hides the type printed on the other side or on the sheet beneath. Cloth is a layer of spun and woven fiber, often dyed. It may be opaque, showing only its own color, or in other uses it may be translucent, as in nylon stockings, both having its own color and revealing something of the underlying color, but hiding the fine detail. A paint layer serves to hide with its own color an unwanted (for example, dirty) color beneath. Vitreous enamel is pigment in a medium of glass supplying its own permanent color as an opaque coat on otherwise impermanent and unattractive metal. Ceramic glazes serve to color and render waterproof clay and ceramic products of a variety of types and uses. Many applications of plastics involve thin pigmented layers of plastics, such as wall tile and wall switches.

A layer of material has a top, a bottom, and an interior whose thickness is small compared to its length and width. Some of the light incident on the top is reflected without penetrating into the body of the film. The various angular distributions of this reflected light flux determine whether the layer appears glossy or mat. Some of the incident light flux penetrates the surface and is absorbed there. The rest leaves either through the top or the bottom of the layer. The power of the layer to absorb and diffuse the light flux penetrating it determines whether the underlying color and pattern are or are not visible. This is known as opacity or hiding power. Gloss and opacity are commercially important properties in their own right, and they have an important bearing on color. How to compound

and apply colored layers for these various purposes is a large part of the "know-how" by means of which many industries serve the public.

GLOSS

A perfect mirror surface is said to have maximum gloss. The ideally perfect mirror surface is a plane surface reflecting all the incident light flux in a perfect image-forming state, whence we see that the surface itself must be invisible. No light leaves a perfect mirror surface in such a way as to permit us to focus on the surface; there is no visible micro-structure, and, in fact, visually there is no surface there at all. Nevertheless the idea of a perfect mirror surface serves very well as a gloss standard. Good approximations are afforded by a plane polished surface of silver, or by the usual mercury-backed glass mirror. The law of mirror reflection is well known. Light incident at an angle, i , from the perpendicular to the surface is reflected in the same perpendicular plane so that the angle, f , of reflection is the same as the angle of incidence, but on the opposite side of the perpendicular, that is: $f = -i$. For an ideally perfect mirror surface the light flux of the reflected beam is the same as that of the incident beam no matter what the angle of incidence. For surfaces of colorant layers the mirror reflectance (ratio of reflected to incident flux) approaches unity only for grazing incidence ($i \rightarrow 90^\circ$). Mirror reflectance of such surfaces depends on the relative index of refraction or ratio of the speed of light in air to that in the part of the layer just beneath the surface. This part of a colorant layer of high gloss is made up of the vehicle, the nearly transparent medium carrying the pigment particles. The indices of refraction of usual vehicles relative to air vary from about 1.45 to 1.60. If the object is immersed in a liquid or solid instead of in air or vacuum, mirror reflectance still depends on the ratio of the speeds of light on the two sides of the boundary. This ratio may be written: n_2/n_1 , where n_2 is the index of refraction of the optically denser medium and n_1 is that of the less dense. For air, n_1 is about 1.0003. It varies with temperature, and this variation permits air rising from a hot surface, and mixing with cold air, to produce a visible shimmer of objects seen through it.

Fresnel Reflection. For perpendicular incidence ($i = 0$) the relation between reflectance, ρ , and the indices of refraction, n_1 and n_2 , is simple:

$$\rho = [(n_2 - n_1)/(n_2 + n_1)]^2 \quad (36)$$

For angles of incidence other than zero, the reflectance, ρ , depends on whether the light has already been reflected by another such plane mirror. When light from an ordinary source (such as the sun or an incandescent lamp) strikes a nonmetallic plane mirror, part of it digs in more easily; the other part bounces off more easily, particularly if the angle of reflection, f , is such that:

$$\tan f = n_2/n_1 \quad (37)$$

In this case the reflected light is said to be completely plane polarized, and it is in condition to bounce off another similarly oriented mirror surface more completely than the original beam. Equation 37 is known as Brewster's law.

The complete statement of the dependence of nonmetallic mirror reflectance ρ on angle of incidence i in the less dense medium, on index of refraction n_2 of the layer, on index of refraction n_1 of the medium in contact with it, and on the state of polarization of the incident beam is known as Fresnel's law. Let ρ_{\parallel} be the reflectance of the mirror surface for plane polarized light so oriented to the surface that its reflection is most facilitated. Let ρ_{\perp} be the reflectance for plane polarized light so oriented that its reflection is most hindered. Fresnel's law states (Moon, 1940) that:

$$\left. \begin{aligned} \rho_{\parallel} &= \left[\frac{\cos i - \sqrt{(n_2/n_1)^2 - \sin^2 i}}{\cos i + \sqrt{(n_2/n_1)^2 - \sin^2 i}} \right]^2 \\ \rho_{\perp} &= \left[\frac{(n_2/n_1)^2 \cos i - \sqrt{(n_2/n_1)^2 - \sin^2 i}}{(n_2/n_1)^2 \cos i + \sqrt{(n_2/n_1)^2 - \sin^2 i}} \right]^2 \end{aligned} \right\} \quad (38)$$

For unpolarized incident light, the reflectance, ρ_T , is simply the average of ρ_{\parallel} and ρ_{\perp} :

$$\rho_T = (\rho_{\parallel} + \rho_{\perp})/2 \quad (38a)$$

By setting $i = 0$ in equation 38 we obtain equation 36 for this special case. By setting $i = 90^\circ$, we find from equation 38 that $\rho_{\parallel} = \rho_{\perp} = 1$; and so, from equation 38a, $\rho_T = 1$. That is, at grazing incidence any mirror surface (glass, varnish, ice, water) has a reflectance approaching unity. Between perpendicular incidence and grazing incidence equations 38 and 38a indicate reflectances ρ_{\parallel} , ρ_{\perp} , and ρ_T as shown in Fig. 94 for $n_2/n_1 = 1.5$. Note that, at the angle whose tangent is 1.5 (about 56°), reflectance for perpendicularly polarized light ρ_{\perp} is zero in accord

with equation 37. All such light striking the mirror surface digs in, and none is reflected. If the beam is unpolarized, reflection at this angle separates out the parallel polarized component; all the perpendicularly polarized light penetrates the surface, leaving only parallel polarized light to form the reflected beam. If this parallel polarized light strikes another similarly oriented mirror surface, just twice as large

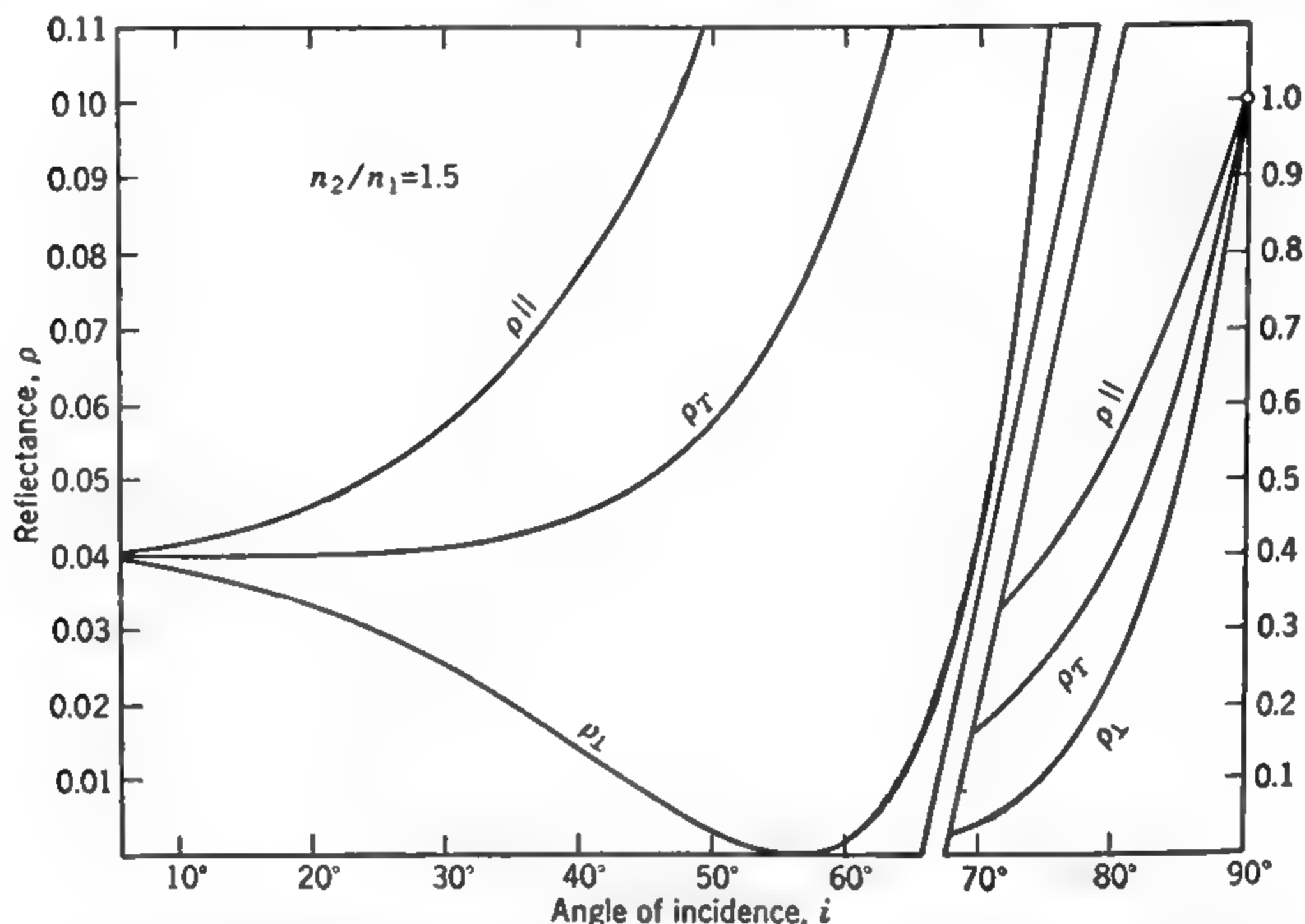


FIG. 94. Fresnel reflectance as a function of angle, i , of incidence on a boundary for which the refractive-index ratio is 1.5 (representative of an air-to-glass boundary). Note that the reflectance, ρ_T , for unpolarized light is the average of the reflectances for light plane polarized in (ρ_{\parallel}) and perpendicular (ρ_{\perp}) to the plane of the illuminating beam.

a fraction of it will be reflected as was reflected from the original unpolarized beam; see equation 38a for $\rho_{\perp} = 0$.

Figure 95 shows the variation of ρ_T with angle of incidence i for n_2/n_1 equal to 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9. By interpolation among these curves, the reflectance, ρ_T , for any angle of incidence and any relative index of refraction, n_2/n_1 , may be read approximately. The values of n_2/n_1 between 1.4 and 1.9 apply to plastics, glass, textile fibers, and paint vehicles. The values between 1.1 and 1.4 refer to these solids immersed in liquids or other solids. More precise values of ρ_T than can be read by interpolation from Fig. 95 can be obtained from published tables (Moon, 1940) or directly by computation from equations 38 and 38a.

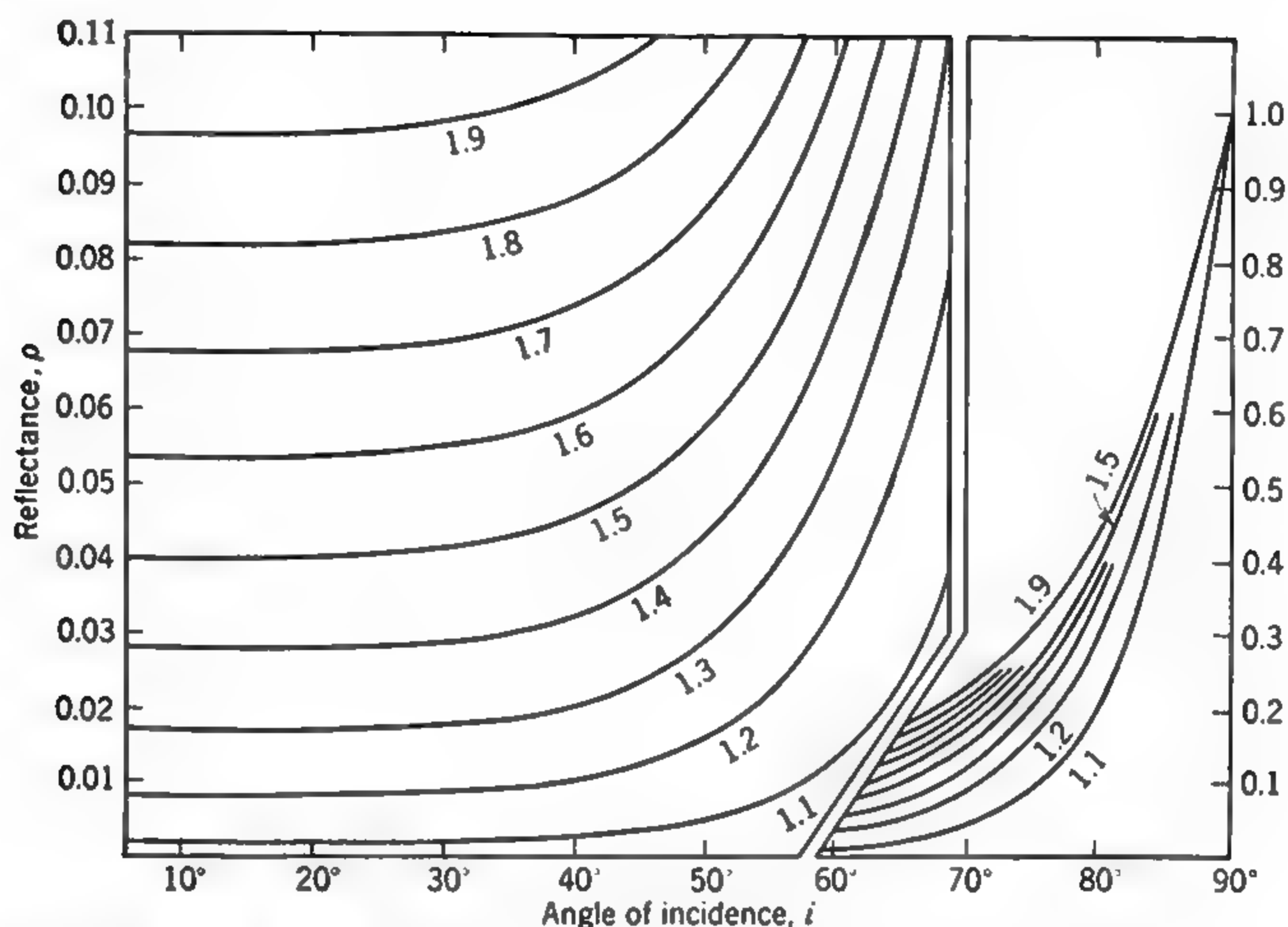


FIG. 95. Fresnel reflectance for unpolarized light as a function of the angle, i , of incidence for various values of refractive-index ratio.

Types of Gloss. The gloss of a surface may be defined as its degree of approach to a mirror surface. The perfectly diffusing surface having luminance constant, regardless of angle of view even with unidirectional illumination, departs from a mirror surface by the maximum amount possible and is said to have zero gloss. Mat surfaces of blotting paper, magnesium oxide, fine-ground glass, and flat paint are practical approximations to this ideal surface of zero gloss. Much of the equipment bought for military use has a finish of this type, lest the enemy detect its presence by mirror reflection of the sun. Such surfaces distribute incident light nearly equally in every direction. A mirror surface sends it away only at the angle of mirror reflection. Surfaces of intermediate character have intermediate light-distributing properties. These properties may be measured by means of a goniophotometer, an instrument that measures luminous directional reflectance as a function of the angle of illumination and angle of view. The goniophotometric curve of a surface is a fundamental determinant of its gloss, much as the spectrophotometric curve determines its color.

Many transition states exist between the perfectly mat and perfectly glossy mirror surface. One type of transition corresponds to small dots of mirror surface spread over an otherwise perfectly mat

surface. If each dot is considered to increase in size until the whole surface is covered, we have one type of transition from mat to glossy. This type corresponds closely to the gloss changes produced by polishing a piece of rough-ground glass by progressive amounts. The reflected light from such a surface could easily be separated by the goniophotometer into two parts, one at the angle of mirror reflection, known as specularly reflected light, the other uniformly distributed regardless of angle of view, known as diffusely reflected light. The same statement applies to lightly sand-blasted glass surfaces. Sometimes reflectance is said correspondingly to be composed of two parts, specular reflectance and diffuse reflectance.

Another type of transition corresponds to a surface made up of small elements each one of which is a perfect mirror but is set at randomly distributed angles to the gross plane of the surface. As the average tilt increases the reflected light flux more and more approximates a uniform angular distribution. This series of surfaces corresponds somewhat to that producible by melting down to various degrees a rough-ground ceramic. Tiles and vitreous enamels on metal show departures from perfect gloss of this general type. The goniophotometric curve of such surfaces varies from the sharp peak of perfect mirror reflectance by the gradual spreading out and reduction of the height of the peak until the reflectance is nearly constant, independent of the angle of view. No division of the reflected flux into specular and diffuse components is possible.

Other more complicated transition states between mat and mirror surfaces are the rule in commercial products. The gloss of a paint layer is customarily controlled by the ratio of pigment to vehicle. If there is a small amount of pigment, as in a paint enamel, all the pigment particles are completely covered by a level film of the vehicle which may give an exceedingly good approximation to a mirror surface. But, if a smaller proportion of vehicle to pigment is used, the dried paint film will not level out perfectly but will follow somewhat the shape of the pigment particles just beneath the surface. This results in a semigloss finish. And, finally, if just barely enough vehicle is used to stick the pigment particles together here and there, as in a cold-water paint, the surface is essentially composed of dry pigment and gives a good approximation to the perfect diffuser. It also rubs off easily. The connection between the gloss of paint and its practical usefulness is clear. A low-gloss paint is desirable for the interior of a room because there are no mirror images of light sources to cause glare, but it is hard to make such paint films so as to resist washing. This is only the beginning.

Brush marks in an otherwise glossy paint film introduce a type of gloss that varies with the orientation of the film in its own plane. Hand-rubbed finishes do the same. Paints formulated to duplicate somewhat the appearance of hand-rubbed finishes without requiring the expense of hand rubbing present their own peculiar gloss characteristics, and so on.

Textiles because of their weave give complicated variations in angular distribution of reflected light with rotation in their own plane. The fiber itself has a gloss; the textile because of its weave introduces characteristic variations, and the processing of the textile both in finishing before sale and in washing and ironing afterward affects its gloss.

The peak of the curve of directional reflectance of a woven textile made of glossy or lustrous fibers, because of the weave, is almost never at the angle of mirror reflectance with respect to the gross plane of the specimen. Even felt will show a blunt maximum at an angle somewhat greater than the angle of mirror reflection. This may be understood in terms of Fresnel's law. The elementary mirrors of which the surface may be thought to be composed have random tilts. There are about as many tilted so as to reflect incident light at an angle smaller than that of mirror reflection as there are tilted so as to reflect light at larger angles. For the latter elements, however, the angle of incidence is greater, and the reflectance is greater (see Fig. 95). On this account the blunt maximum is at an angle greater than that of mirror reflectance. The same argument applies to such near-mat nontextile surfaces as mimeograph paper and mat films of vitreous enamel or paint.

A "surface" having a pronounced tridimensional microstructure, like white velvet, or that of a forest viewed from an airplane, is likely to show the peak of directional reflectance along the angle of incidence instead of near that of mirror reflection. Such a surface is composed of cavities, some of which are lit up by light falling in any one direction onto the surface. Whatever light leaves these cavities by reflection must necessarily leave them more or less in the direction of the source. These surfaces are sometimes said to have "negative gloss." By this idea the retroreflective devices of cube-corner mirrors or clear glass beads embedded in white paint are examples of extreme negative gloss.

Most commercial products have surfaces too complicated to be subject to a useful analysis on the basis of the actual shape or contour of the surface. They may be classified according to appearance

in many cases. The subjective appreciation of gloss is called glossiness. Table 21 lists five different kinds of glossiness (Hunter, 1937;

Table 21. Various Kinds of Glossiness and the Functions of Directional Reflectance with Which They Correlate

Kind of Glossiness	Function of Directional Reflectance	Diagram of the Angular Conditions
Specular	Ratio of $R_{60,-60}$ for the specimen to that of a perfect mirror	
Sheen	Ratio of $R_{85,-85}$ for the specimen to that of a perfect mirror	
Contrast	$R_{60,-60}/R_{60,0}$	
Distinctness of image	Rate of change of $R_{i,-\theta}$ with the angle of incidence i , where the angle of view, $-\theta$, differs by a few minutes of arc from $-i$, that of mirror reflection	
Absence of bloom	Ratio of $R_{i,-i}$ to $R_{i,-\theta}$, where the angle of view, $-\theta$, differs from the angle of mirror reflection, $-i$, by a few degrees	

Hunter and Judd, 1939). Each kind correlates with some aspect of the goniophotometric curve giving directional reflectance as a function of angle of view for unidirectional illumination at some fixed angle. Table 21 also gives the function of directional reflectance with which each kind of glossiness is correlated and shows a diagram of the angles of illumination and view used in the measurement. This list of the kinds of glossiness by no means exhausts the possibilities, but it will at least indicate that gloss of a surface is far from a simple property, and that no one measure of gloss can serve all purposes. It is very hard to tell from the complete goniophotometric curves of two surfaces which of the two will be judged the more glossy. The judgment will depend on the directions of illumination and view, the angular size of the light source, and what the observer is looking for. Surface uniformity will also influence the judgment. Of two varnished surfaces of identically high specular gloss, the one free from bubbles will usually be judged the glossier. The same influence applies to judgments of sheen, contrast glossi-

ness, distinctness-of-image glossiness, and absence-of-bloom glossiness. This influence is particularly marked in the case of high-gloss finishes of a textured material such as a furniture finish. If the surface is so uniform that there are no scratches, bubbles, pits, or other imperfections visible, the observer cannot focus on the surface itself but looks through the surface at the texture of the wood. This is called "depth of finish." It is a special case of what might be called "surface-uniformity gloss."

The National Bureau of Standards (Hammond and Nimeroff, 1950) issues sets of 10 gloss standards each for checking the calibration of instruments measuring specular gloss.

Connection between Gloss and Color. The compounding of colorants so as to yield the desired color is considerably complicated by the fact that there is usually a desired gloss, too. The color match between paint specimen and color standard is usually checked by placing them side by side on a table in front of a north window and viewing them more or less along the perpendicular. The gloss match is then checked by backing away until an image of window-pane frame, or shade cord, silhouetted against the sky is formed by both specimen and standard. In this position the inspector can judge equivalence between the two in specular glossiness (brightness of high light), absence-of-bloom glossiness (haze near the high light), and distinctness-of-image glossiness. If he finds the color match satisfactory but the specular gloss of the specimen too high, he knows that the simple addition of more pigment to the paint will correct the gloss but will also throw the color off. The pigment formulation itself has, therefore, to be changed, too. To get it right the next time, the shader has to have either experience or luck, probably plenty of both.

Leaving aside the question of poorer dispersion of the pigment in the more pigment-rich formulation, we can easily see a reason for this connection between color and gloss. If a piece of polished black glass has part of its surface fine ground, the fine-ground spot will no longer appear black but gray. The incident light collected into the high light by the polished area, and avoided by the inspector during his check of the color, is diffused by the depolished area so that a portion of it reaches the eye of the inspector no matter what the angle of view. This surface-diffused light has closely the same chromaticity as that of the source itself, and it is combined by additive mixture with the light reaching the inspector's eye from the body of the specimen. In the case of black glass, the color change is very striking because there is no body-reflected light at all. In the case of dark chromatic specimens, the addition of more surface-

reflected light may be just as striking. The effect is to raise the directional reflectance, decrease the purity of the light reaching the eye of the inspector from the specimen, and leave the dominant wavelength nearly constant. Since this is simple optical mixture, we can even write a formula (from equation 2) for the color change associated with a decline in gloss that causes the surface-reflected light to be increased by an amount ΔY . If the tristimulus values of the original color are X, Y, Z for standard daylight (source C), the tristimulus values X', Y', Z' for the modified color are simply:

$$X' = X + \Delta X \quad Y' = Y + \Delta Y \quad Z' = Z + \Delta Z$$

But, since the surface-reflected light has the same chromaticity as the light source itself, we know from the sums in Table 10 that $\Delta X = 0.980\Delta Y$, and $\Delta Z = 1.181\Delta Y$, and so the formula is:

$$\left. \begin{aligned} X' &= X + 0.980\Delta Y \\ Y' &= Y + 1.000\Delta Y \\ Z' &= Z + 1.181\Delta Y \end{aligned} \right\} \quad (39)$$

Plot of the chromaticity coordinates on the (x, y) diagram will disclose that the two colors (X', Y', Z') and (X, Y, Z) differ in purity but are the same in dominant wavelength. The abrading of the surface has caused the color perceived to belong to the specimen to become lighter and less saturated but has left the hue nearly unchanged. This information does not help the shader very much. He knew it already. He still has to make appropriate changes in the pigment formulation to make the color match. To a psychophysicist it looks as though a little less white pigment should be used; but this may not be the whole story.

This influence of color on gloss brings up a frequent cause of confusion between those who work with colorants and those who work with the measurement of light. It is obvious that the color of the reflected light has changed, but, since the colorant formulation is left unchanged by the surface abrasion, the worker in colorants would probably say that the color has stayed the same, too. He might say that the two specimens look different but he knows they are really the same color. Evidences of this use of the term color can easily be cited. Note in the *Color Harmony Manual* that the same designation is used for the glossy and the mat sides of the color chips. The chip labeled 7pa shows by this view two versions of the same color, the glossy version of 7pa and the mat version.

Similarly, in the Standard Color Card of America a single color name and number designation are shown for the satin-finish and ribbed-finish portion of each swatch. This is really a colorant designation. It shows what a particular dye will do on a satin-finish textile, and it also shows what the same dye will do on a ribbed-finish textile. It is no doubt very natural to a dyer to speak of these as two aspects of the same color; but they have far different sets of tristimulus values and to a colorimetrist are different colors produced by the same colorant.

OPACITY OR HIDING POWER

To analyze the hiding power of a colorant layer, we must inquire what happens to the part of an incident light beam that is not re-

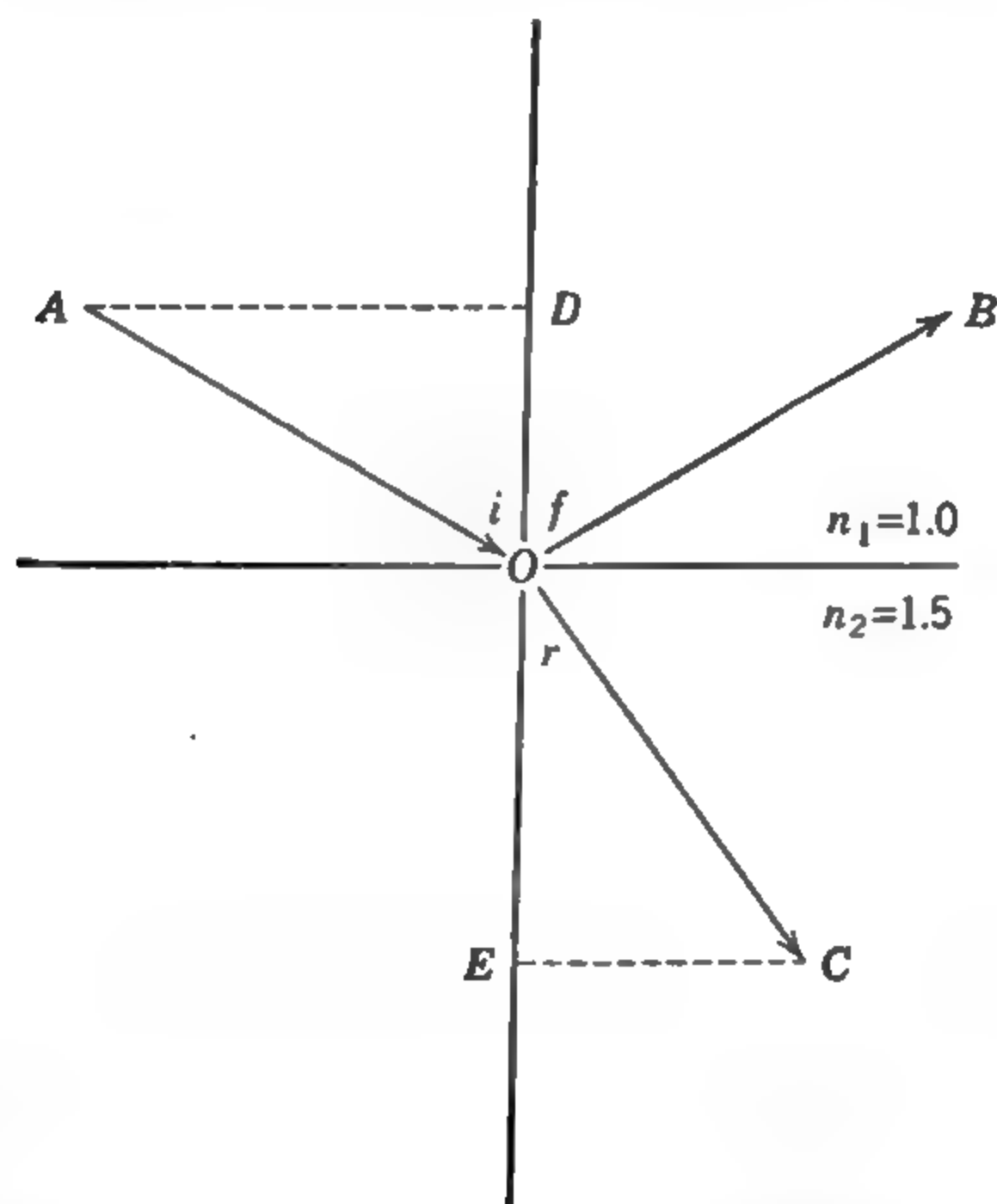


FIG. 96. Illustration of the bending of a ray by refraction toward the perpendicular as it enters the more dense medium (Snell's law).

flected at the surface of the layer. This fraction $(1 - \rho)$ penetrates the surface but is bent somewhat toward the perpendicular; see Fig. 96. The amount of bending is such that the ratio of the sine of the angle of incidence, i , to the sine of the angle of refraction, r , is the inverse ratio of the indices of refraction, n_1 and n_2 , of the respective media, or:

$$(\sin i)/(\sin r) = n_2/n_1 \quad (40)$$

This is known as Snell's law. Thus in Fig. 96 the line AO indicates the incident beam, and the line OB , the reflected beam such that the angle of incidence, i , is equal and of opposite sign to the angle of reflection, f . These two beams are in the medium of index of refraction, n_1 . The line OC represents the refracted beam in the medium of index, n_2 . The angle of refraction, r , is such that:

$$\frac{\sin i(= \overline{AD}/\overline{AO})}{\sin r(= \overline{CE}/\overline{CO})} = n_2/n_1$$

which for Fig. 96 is taken to be 1.5, representative of glass in contact with vacuum or air. The index of refraction of every material depends on the wavelength of the incident beam. For some materials the variation is pronounced; for others, slight. This variation is called the dispersion of the index for that material, and it accounts for the formation of a spectrum (see Fig. 21) when a prism of a transparent material is illuminated by a unidirectional beam of light of mixed wavelength such as sunlight.

After a part, $(1 - \rho)$, of the incident beam has penetrated the bounding surface of a colorant layer, its flux will diminish by a constant fraction for each distance element of unit length traversed in the medium (Bouguer's law; see equation 6). In a perfectly nonturbid medium this decline in flux comes from absorption of radiant energy by the medium which converts it into heat.

Consider now what happens when the refracted beam strikes a pigment particle.

If the particle is a crystal of dimensions larger by a factor of about 10 than the wavelength of the radiant energy, part of the flux of the incident beam will be reflected and part refracted again in accord with Fresnel's law (equations 38 and 38a). But, since the randomly oriented pigment crystals present faces at unknown angles to the refracted beam, the identity of the beam is lost at this point. Part of it is reflected, part penetrates the pigment crystals and emerges modified in spectral character by any selective absorption within the crystals, and these parts go on in random courses striking other pigment crystals. The net effect is a diffusion of flux more or less uniform in all directions at any point in the colorant layer.

If the particle is a crystal, a crystal fragment, or a cluster of such fragments considerably smaller in dimensions than 10 times the wavelength of the incident energy, the Fresnel laws do not apply. That is, as the dimensions of the pigment particles are reduced below about 10 times the wavelength of the incident energy the beam

begins to go around the particle instead of being reflected and refracted by it. In this process, some of the flux is absorbed and the rest is scattered, some backward, some to the sides, but preponderantly in the forward direction. The laws of scattering are complicated (Rayleigh, 1871; Mie, 1908; Shoulejkin, 1924; Stutz, 1930; King, 1940; Oster, 1948) and need not be discussed here except to remark that the amount of the scattering is determined partly by the size of the particles and partly by the ratio of the index of refraction of the pigment particles to that of the vehicle. For very small fragments, those approaching molecular dimensions, even the scattering influence vanishes in large measure and the pigment acts as if it were in solution in the medium. There is, therefore, an optimum size for pigment particles with respect to ability to diffuse incident flux. This optimum diameter is between 0.15 and 0.25 microns (Gardner and Sward, 1946). The existence of this optimum diameter accounts for a large part of the interest of the pigments industry in particle size. The net effect of this scattering and rescattering by successive small pigment particles is the same as for Fresnel reflection by successive large pigment particles. The flux penetrating the upper boundary of the layer is diffused more or less uniformly in all directions. Some of this diffuse flux reaches the bottom face of the colorant layer and is partly returned by reflection from the base material on which the colorant layer is deposited. Some of it reaches the top face.

In order to be seen by the inspector, this flux must again penetrate the top boundary of the colorant layer. Not all the flux reaching the under side of this boundary manages to get out, however. Part is again directed back into the pigment layer by reflection from the under side of the top boundary. There is an important optical relation that permits easy evaluation of this internal reflectance. This is the reciprocal relation which states that all changes suffered by a light beam going in one direction are suffered equally by a light beam formed by reversing the direction of the first. Thus a beam in Fig. 96 going from *C* to *O* would suffer by internal reflection a loss of flux by an amount exactly equal to that suffered by the original beam *AO* because of external reflection in the direction *OB*. Fresnel's law thus applies both to internal and to external reflection, but care must be taken to use as the angle *i* the angle in the less dense medium (refractive index n_1) in both cases. Figure 97 diagrams what happens to light diffused in a number of directions from a pigment particle, *P*, somewhat beneath the top boundary of a colorant layer whose vehicle has an index of refraction equal to

1.5. The light leaving the pigment particle in the direction PC is said to strike the under side of the surface at the critical angle. This is the angle, r , in the denser medium for which the beam just fails to emerge into the air ($i = 0$). By setting $i = 0$ and $n_2/n_1 = 1.5$ in equation 40, we obtain $1/\sin r = 1.5$, and the critical angle for the case diagrammed in Fig. 97 is seen to be the angle whose sine is $2/3$, which is about 42° . A beam incident at this angle takes the course CO in Fig. 97. Any beam striking the under side of the surface

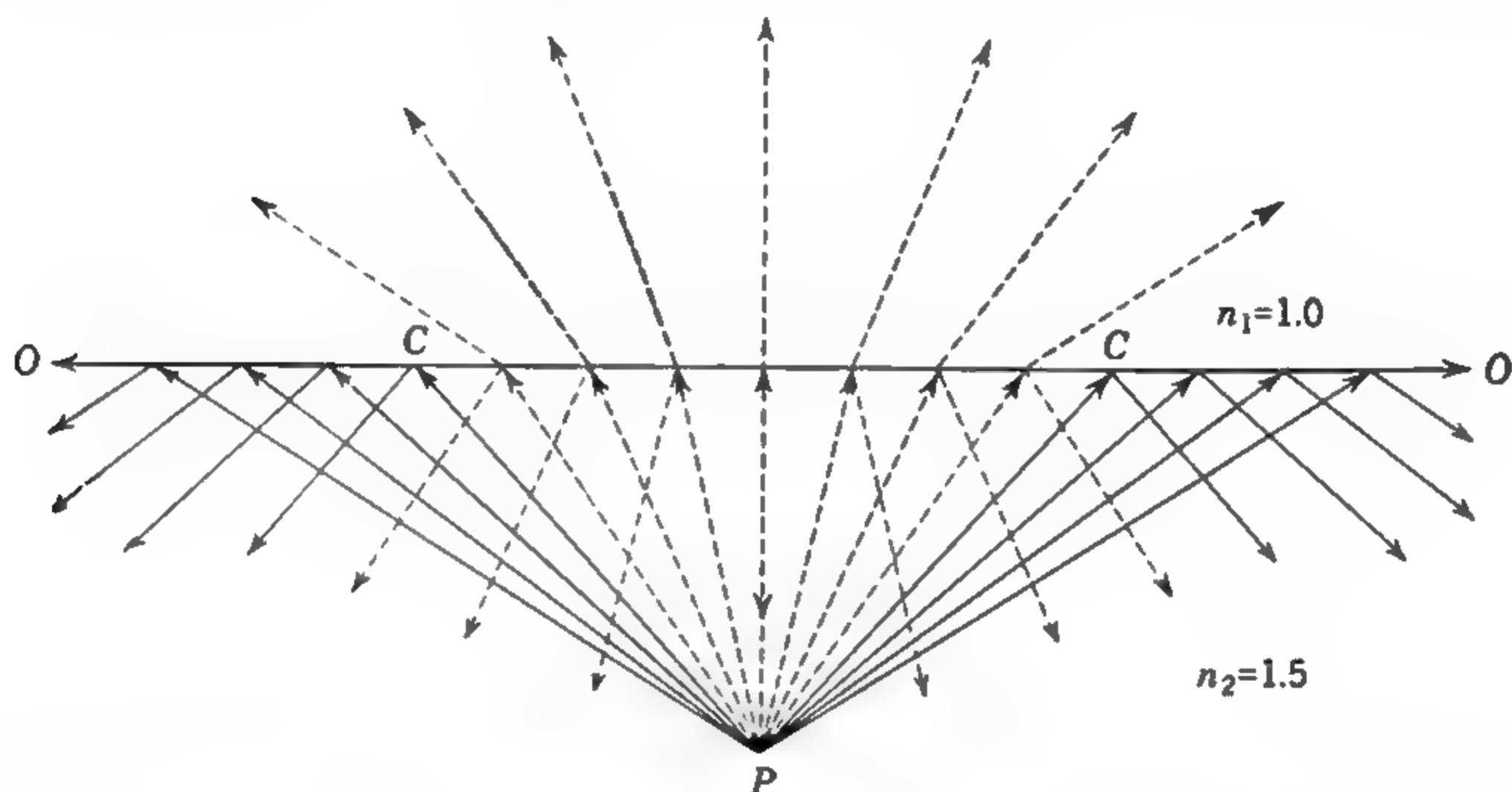


FIG. 97. Illustration of the bending of a ray by refraction away from the perpendicular as it enters the less dense medium, also of the total internal reflection of a ray incident at an angle greater than the critical angle.

more obliquely than the critical angle is totally reflected. These beams are shown by solid lines in Fig. 97. Any beam striking less obliquely is partially reflected back into the denser medium, but the rest emerges into the air and may be seen by the inspector. These beams are shown by dotted lines in Fig. 97.

The flux reflected back into the denser medium by the under side of the boundary ($OCCO$) will, of course, strike other pigment particles, and that part of it that happens to be scattered toward the boundary at angles less than the critical angle will partly emerge so as to be seen by the inspector. It is evident that multiple reflection and scattering play important roles in the appearance of colorant layers.

On the assumption that the flux striking the under side of the boundary of a colorant layer is completely diffused, we may take an average for all angles of the Fresnel reflectances for internal reflection to see how the part emerging compares with that contrib-

uting to multiple reflection within the layer. Figure 98 shows such averages (Judd, 1942) as a function of the refractive-index ratio, n_2/n_1 . For comparison it also shows similar averages for external reflection described explicitly by equations 38 and 38a, and also reflectance for perpendicular incidence in accord with equation 36. It is seen from Fig. 98 that nearly 60 percent of the diffuse flux incident internally on a plane boundary of a colorant layer of $n_2/n_1 = 1.5$ is reflected; little more than 40 percent emerges.

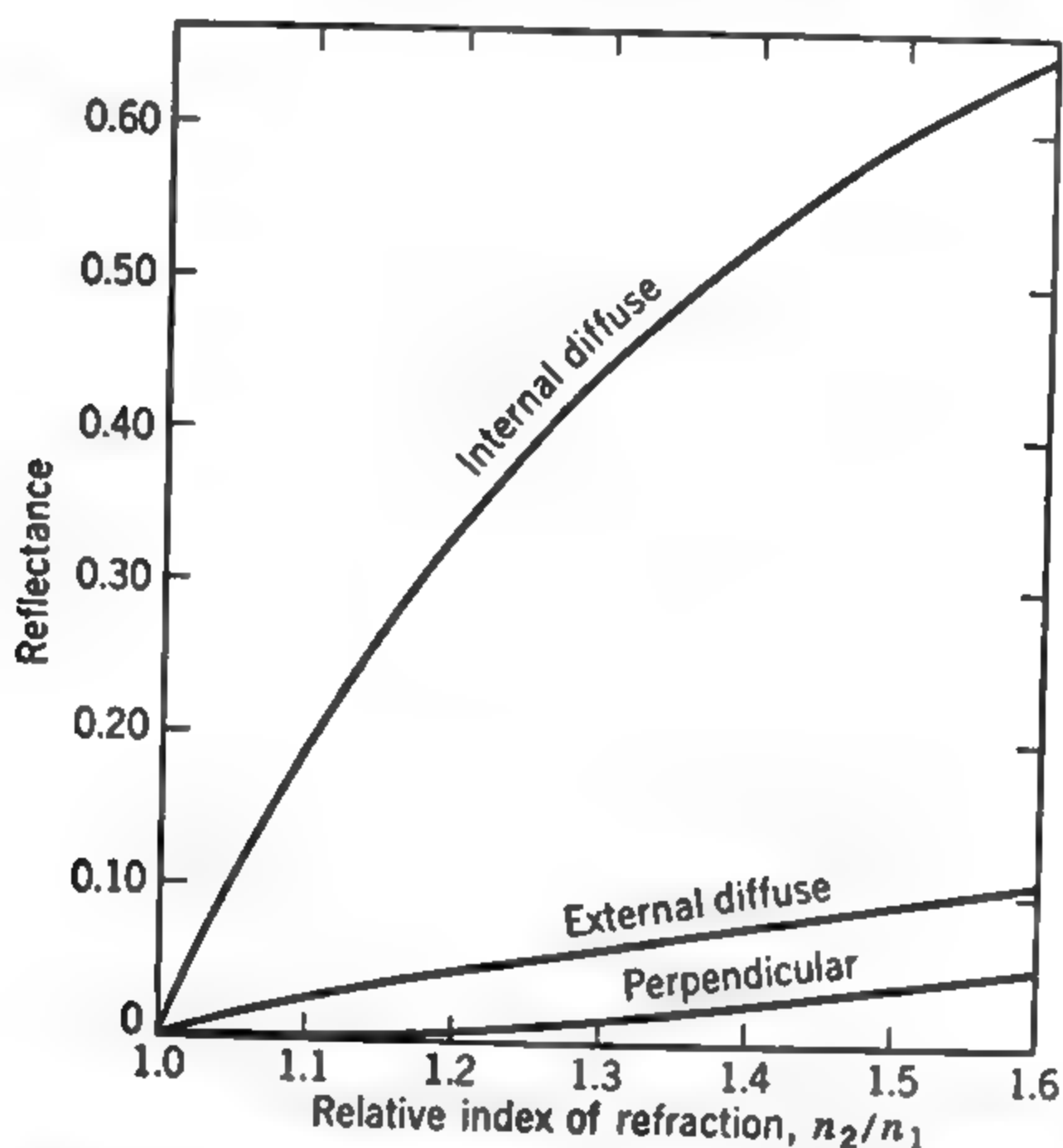


FIG. 98. Internal and external reflectance for completely diffused incident flux as a function of the refractive-index ratio, n_2/n_1 (after Judd, 1942). Reflectance for perpendicular incidence is also shown.

The capacity of a colorant layer to hide the colors beneath is dependent chiefly on the number and size of pigment particles per unit area and on their light-absorbing and light-scattering capacity. Thus, if the colorant layer has no pigment particles in it, its hiding power depends solely on absorption and scattering of light by the vehicle. Most such layers, varnish layers, layers of clear lacquer, clear plastic sheets, are essentially nonhiding. But, if the layer has many pigment particles per unit area, each one of which either strongly absorbs or strongly scatters light, or both, the layer will have considerable hiding power. For light-colored layers the pigment particles must have a high capacity to scatter light combined with a relatively small capacity to absorb it. Since the light-scattering property of pigment particles depends on the ratio, pig-

ment to vehicle, of refractive index, the importance of having pigment particles with indices of refraction as different as possible from that of the medium is quite plain. Pigments with index of refraction not much different from those of usual vehicles (such as silica pigment in linseed oil) result in colorant layers of little or no hiding power. Table 22 gives approximate refractive indices for a few

Table 22. Indices of Refraction for a Few Media Used as Vehicles in Colorant Layers and for Some White Pigments

Substance	Index of refraction $n =$
	$\frac{\text{Speed of light in vacuum}}{\text{Speed of light in substance}}$
Vacuum	1.0000
Air	1.0003
Soybean oil	1.48
Refined linseed oil	1.48
Tung oil	1.52
Vinyl resin	1.48
Shellac	1.52
Rosin	1.52
Phenol-formaldehyde	1.54
Urea	1.66
Diatomaceous earth	1.45
Silica	1.55
Calcium sulfate	1.59
Barytes (BaSO_4)	1.64
Lithopone	1.84
Basic sulfate white lead	1.93
Basic carbonate white lead	1.94 to 2.09
Zinc oxide (ZnO)	2.02
Antimony oxide	2.09 to 2.29
Zinc sulfide (ZnS)	2.37
Titanium oxide (TiO_2), anatase	2.55
Titanium oxide (TiO_2), rutile	2.76

media used for colorant layers and for a number of white pigments (Gardner and Sward, 1946). The high refractive index of titanium dioxide explains the revolutionary effect on the pigmented-coatings industries caused by the introduction of titanium dioxide pigment. This pigment is now widely used in bar soap, paper, vitreous enamel, plastics, and paint. The higher index possessed by the rutile crystal form of TiO_2 also explains the great interest in methods of preparing this pigment so that the anatase crystal form is avoided. The

rutile form also appears slightly yellower than the anatase form to many observers (Jacobsen, 1948), and, since the standard observer applied to the spectrophotometric curves of the two pigments indicates little or no difference, the question of revising the standard observer has been raised (Judd, 1949). New studies have been undertaken both in this country and in Great Britain to see what, if any, revision should be made. For paper and soap, however, the anatase crystal form is preferred, the smaller hiding power being tolerated because of the less yellowish color.

Various tests of the hiding power of colorants have been used. The hiding power of a layer is somewhat correlated with its luminous transmittance; that is, a layer that does not transmit any of the incident light will completely hide the colors of the structures beneath. Such layers are said to be opaque. One early measure of the hiding power of paint is the reciprocal of the thickness of layer necessary to obliterate a glowing filament viewed through the layer. This measure really determines when enough pigment particles are present per unit area of film to prevent any light from coming directly through without being scattered by at least one pigment particle. It does not indicate how well the particles scatter the light incident upon them and has largely been given up. Other methods based on measurement of luminous transmittance suffer from analogous defects. The most used method is the direct test. The paint to be tested is brushed onto a checkerboard (or other) design in black and white. Enough coats are applied to render the design invisible. The hiding power of the paint is the reciprocal of the hiding thickness.

There are a number of drawbacks to this test. It depends upon a judgment of the thickness required to make a pattern just invisible. Even with standardized illumination with specified artificial daylight, some observers can see the design and favor applying another coat; others cannot see it. That is, the end point of the test is rather indefinite. This has led to the development of various methods based on the measurement of the reflectance of the colorant layer over a white backing, R_w , and that over a black backing, R_b , the ratio R_b/R_w being known as the contrast ratio. A contrast ratio of 0.98 is usually taken as being equivalent to complete hiding. If the dependence of contrast ratio on layer thickness is known (Bruce, 1926; Sawyer, 1941), the measurement of contrast ratio for a material at less than hiding thickness permits the thickness required to produce a contrast ratio of 0.98 to be read. The reciprocal of this thickness is taken as the hiding power of the material. Contrast ratio itself indicates for each colorant layer how completely an underlying design

will be hidden. A good review of the various methods is given by Gardner and Sward (1946).

Opacity of a sheet of paper is customarily measured by contrast ratio, R_b/R_x , where the reflectance of the white backing is taken as 0.89 (TAPPI Method T 425 m; ASTM Method D-589-44). Sometimes the ratio R_b/R_x , called printing opacity, is used, R_x being the reflectivity of the paper measured as the reflectance of a nearly opaque stack of the paper. Contrast ratio is also used to specify the opacity of tracing cloth and plastic wall tile. The National Bureau of Standards issues opacity standards made of diffusing glass (Judd, 1934). These are used to calibrate and check the performance of opacimeters, such as the widely used Bausch and Lomb type of opacity meter (Davis, 1933).

KUBELKA-MUNK ANALYSIS

It may seem that optical theory of the hiding power of a colorant layer must stop when the beam loses its identity from striking a small light-scattering particle or a large irregularly shaped pigment particle, but this is not true.

Basic Ideas. If we are satisfied that the net effect of the pigment particles is to produce, whether by Fresnel reflection or by light scattering, essentially perfect diffusion of the light flux in every direction, we can use the methods of calculus to derive theoretical relations of practical interest. First, note that the flux proceeding parallel to the boundaries of the layer will cancel out. Through any element of area in the interior of the layer and perpendicular to the boundaries just as much flux goes to the right as to the left, and no account need be taken of it. Even near the edges of the specimen, where some light flux escapes and prevents exact cancellation, no special account need be taken of flux proceeding parallel to the boundaries, since we are dealing with colorant layers and what we mean by a layer is a solid so extended in two (horizontal) dimensions of space that any light flux lost horizontally through the edges formed in the vertical dimension is too small to worry about compared to what goes up and down.

We may deal, therefore, with the simplified picture of two diffuse light fluxes, one proceeding downward throughout the layer, the other simultaneously proceeding upward. Consider what must happen to the downward-proceeding flux, i , during its passage through any elementary layer parallel to the boundaries of the colorant layer: see Fig. 99. The thickness of this elementary layer is written dx ; this

thickness is small compared to the whole thickness, X ,* of the colorant layer itself, but it is large compared to the diameters of the pigment particles so that we do not have to take account of the action of the individual particles but only of their average effect. This effect is simply to decrease the downward-proceeding flux by absorption by an amount $Ki \, dx$, and also to decrease it by reversal of direction by scattering by an amount $Si \, dx$, where K is the fraction of the downward-proceeding flux lost by absorption in the elementary layer, and S is the fraction lost by having its direction reversed. K and S are

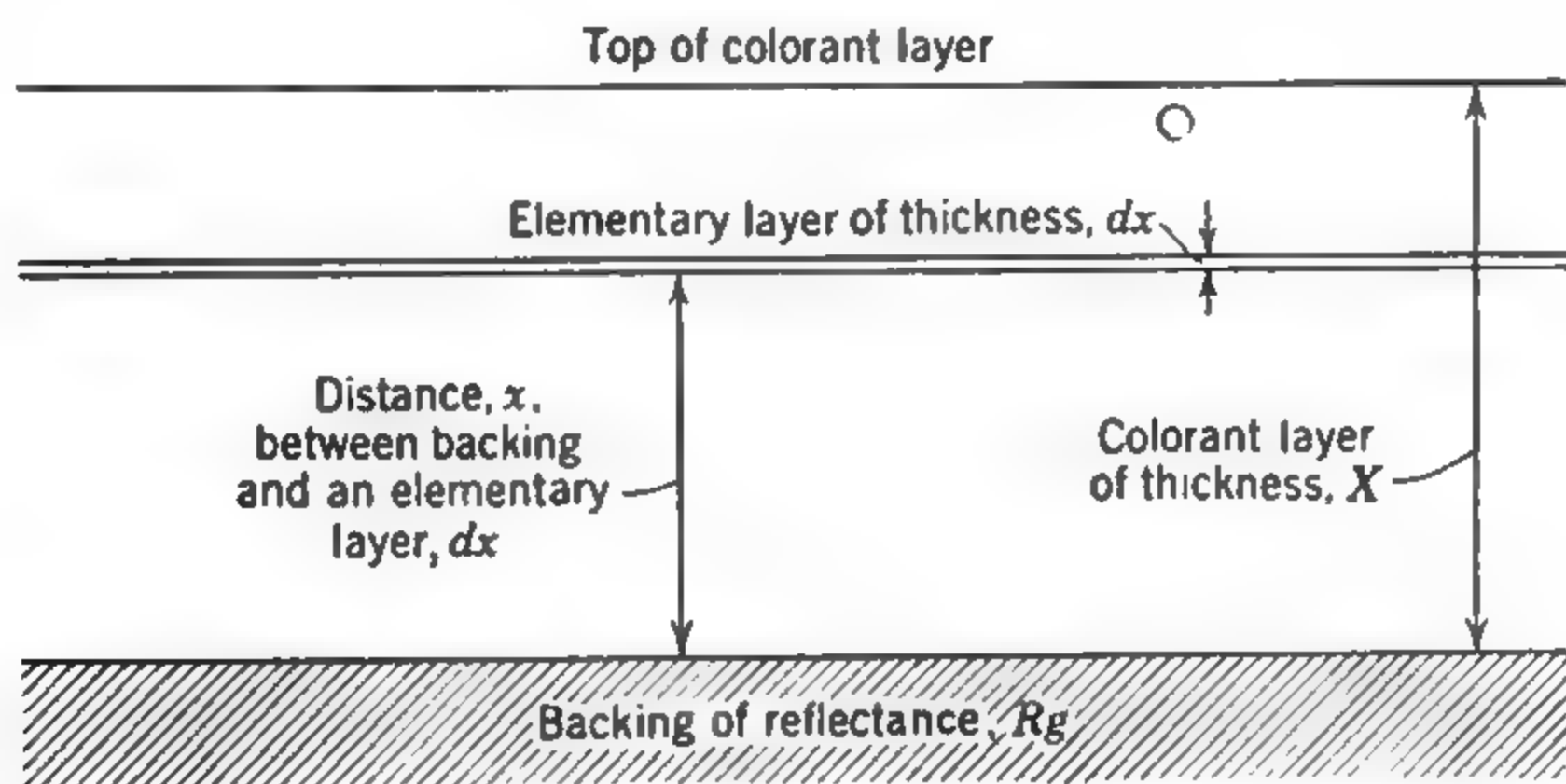


FIG. 99. Basis of the Kubelka-Munk analysis of the reflection and transmission of light by a light-scattering layer.

known as the absorption coefficient and the scattering coefficient, respectively, of the material forming the colorant layer.

Consider now what changes are made in the upward-proceeding flux, j , during its passage through the same elementary layer of thickness, dx . This flux is reduced by the amounts, $Kj \, dx$ and $Sj \, dx$, by absorption and scattering in exactly the same way, but the part $Si \, dx$ reversed in direction by scattering from the downward-proceeding flux is added to the upward-proceeding flux. The total change, dj , of the upward-proceeding flux is thus:

$$dj = -(S + K)j \, dx + Si \, dx \quad (41a)$$

For the total change, $-di$, of the downward-proceeding flux we have the similar expression:

$$-di = -(S + K)i \, dx + Sj \, dx \quad (41b)$$

* The symbols, x and X , referring to thickness are unfortunately the same as those used for one of the chromaticity coordinates, x , y , z , and one of the tristimulus values, X , Y , Z . Similarly, the symbol, i , for the downward-proceeding flux is the same as that used for the angle of incidence. These usual practices, however, seem not to have caused any confusion.

dj and di showing different algebraic signs because of the different directions of the two fluxes; that is, we should always make dx refer to an increase of x , where $x = 0$ corresponds to the unilluminated side of the colorant layer.

Equations 41a and 41b describe completely the changes that take place in the two light fluxes, i and j , in any elementary layer of the colorant film. They are known as differential equations. If we knew what either i or j is at any elementary layer, such as the top layer, and if we knew what are the values of absorption coefficient K and scattering coefficient S , characterizing the pigment-vehicle combination of which the colorant layer is composed, we could figure from one layer to the next both up and down by these equations and so evaluate both i and j as a function of x . For more than two elementary layers, however, these calculations become very long and involved.

The Exponential Solution. The application of calculus methods to the solution of differential equations 41a and 41b by Kubelka and Munk (1931) has resulted in a general expression for the reflectance, R , of any colorant layer of known absorption and scattering coefficients, K and S , laid down on a background of any reflectance, R_0 , as a function of the thickness, X , of the colorant layer. This function is:

$$R = \frac{(R_g - R_\infty)/R_\infty - R_\infty(R_g - 1/R_\infty)e^{SX(1/R_\infty - R_\infty)}}{R_g - R_\infty - (R_g - 1/R_\infty)e^{SX(1/R_\infty - R_\infty)}} \quad (42)$$

where R_∞ is the reflectance of a colorant layer so thick that further increase in thickness, however great, does not significantly change its reflectance, and e is the base of Napierian logarithms ($= 2.71828 \dots$). Note that the absorption coefficient, K , does not appear explicitly in equation 42 but is specified by reflectivity, R_∞ , which depends upon the ratio of the absorption and scattering coefficients, K/S , as follows:

$$K/S = (1 - R_\infty)^2/2R_\infty \quad (43)$$

Equation 43 has been used by Park and Stearns (1944) and by Selling (1947, 1950) as an aid to formulating dyes for papers and fabrics so as to color-match a standard, and it has been used for paint formulations by Duncan (1940, 1949). Foote (1939) used it for dyed papers in the form obtained by solving for R_∞ :

$$R_\infty = 1 + K/S - (K^2/S^2 + 2K/S)^{1/2} \quad (43a)$$

And Saunderson (1942) used it for opaque pigmented plastics in the form:

$$R_{\infty} = 1/[1 + K/S + (K^2/S^2 + 2K/S)^{1/2}] \quad (43b)$$

Foote (1946) published a short table showing the dependence of K/S on R_{∞} and kindly supplied the author with the complete table reproduced as Table *D*, Appendix. Stearns (1951, *a*) has based a color predictor for pigments on equation 43 and (1951, *b*) has recommended it for selection of base stock and new dyes for textiles.

Equation 42 takes no account of reflections from the upper side of the boundary. It applies strictly only to colorant layers immersed in a medium of the same refractive index as the vehicle of the colorant layer itself, such as layers of paint or plastic immersed in glycerine, which are of no practical interest. But cold-water paint films, papers, and textiles are colorant layers whose scattering coefficients are caused chiefly by pigment-air interfaces. The vehicle for these colorant layers may be said, therefore, to be air, which is the same medium in which they are immersed during use. Equation 42 should, therefore, apply well to cold-water paint films, papers, and textiles.

A second restriction on equation 42 is the assumption that the scattering and absorption coefficients, S and K , are the same throughout the entire thickness of the colorant layer. It cannot apply, therefore, to mat and semimat paint films into the top elementary layers of which there is considerable air penetration, nor to paint films exhibiting what is known as floating, a preponderance of one pigment constituent (usually the more finely ground) in the top elementary layers.

A third restriction arises from the assumption that the pigment particles in the colorant layer are oriented in a random way so as to produce essentially perfect diffusion of light flux within the layer. A thin film of paint whose pigment particles are in the form of flakes (as in aluminum and bronze paints) causes the particles to be mostly horizontal and does not follow equation 42.

A fourth restriction is that the incident light flux itself be perfectly diffuse; that is, all elementary layers are treated alike in equations 41a and 41b, and, since an internal elementary layer receives diffuse flux both from above and from below, the top elementary layer must also get this same kind of flux from above. Duntley (1942) has worked out a general theory not requiring these restrictions. By this theory the properties of the material are specified by four constants instead of two as in equation 42.

The final and most important restriction on equation 42 is that it applies only to one wavelength at a time. If neither S nor K varies appreciably with wavelength (as in near-white colorant layers), equa-

tion 42 gives reflectance for light of mixed wavelength directly. All other colorant layers must be evaluated by applying equation 42 for each part of the spectrum separately.

In spite of these restrictions, equation 42 is very powerful and valuable because it is still rather general. Scores of important, less general relations are summarized by it, though they are not obviously derivable from it explicitly. For example, the scattering coefficient, S , cannot be solved for explicitly from equation 42. In view of its awkward form it may be wondered why other previous theoretical solutions (Stokes, 1860–1862; Bruce, 1926; Channon, Renwick, and Storr, 1918; Gurevic, 1930; Smith, 1931–1932; Silberstein, 1927; Ryde and Cooper, 1931) were not used instead. The answer seems to be that the Kubelka-Munk solution deals with three terms already familiar to the colorant-layer technologist: reflectivity, R_∞ , reflectance over a black backing, R_b , and contrast ratio, R_b/R_w . So tables of the exponential function, e^x , were dusted off and used to compute the theoretical information summarized by equation 42. This laboriously won information was then presented in the form of partial graphical solutions following the lead given by Steele (1935).

One form of these graphical solutions (Judd, 1937) had obvious commercial importance and because of its convenience has been considerably used by industry. This form of graphical solution shows lines of constant reflectivity and lines of constant product of scattering coefficient, S , with thickness, X (product called scattering power, SX), on a plot of reflectance, R_0 , of the colorant layer on an ideal black backing of zero reflectance ($R_p = 0$) as ordinate, against contrast ratio, $R_0/R_{R\theta}$, as abscissa. Figure 100 shows this solution for the abscissa taken as TAPPI opacity, $C_{0.89} = R_0/R_{0.89}$. The vertical line at the right of the plot represents opaque colorant layers ($C_{0.89} = 1$). The lines of constant reflectivity, R_∞ , slope upward and show by R_0 how a black backing is progressively hidden by increasing the thickness of the colorant layer. The influence of, for example, doubling the thickness of the layer, both to increase the reflectance, R_0 , of the layer over a black backing and to increase the opacity, $C_{0.89}$, may be read from this double family of curves by running up any line of constant reflectivity until the product, SX , has been doubled. For example, a layer of reflectivity equal to 0.80, and scattering power, SX , equal to 2.0, is seen to correspond to a reflectance, $R_0 = 0.642$ and an opacity, $C_{0.89} = 0.763$. Doubling the scattering power ($SX = 4.0$) while keeping reflectivity constant corresponds to double the thickness of layer of the same colorant and gives $R_0 = 0.747$ and $C_{0.89} = 0.913$; that is, both reflectance and opacity have been increased. The lines of constant scattering power,

SX , correspond to the variations in reflectance and opacity caused by increasing the absorption coefficient, K , as by addition of a non-

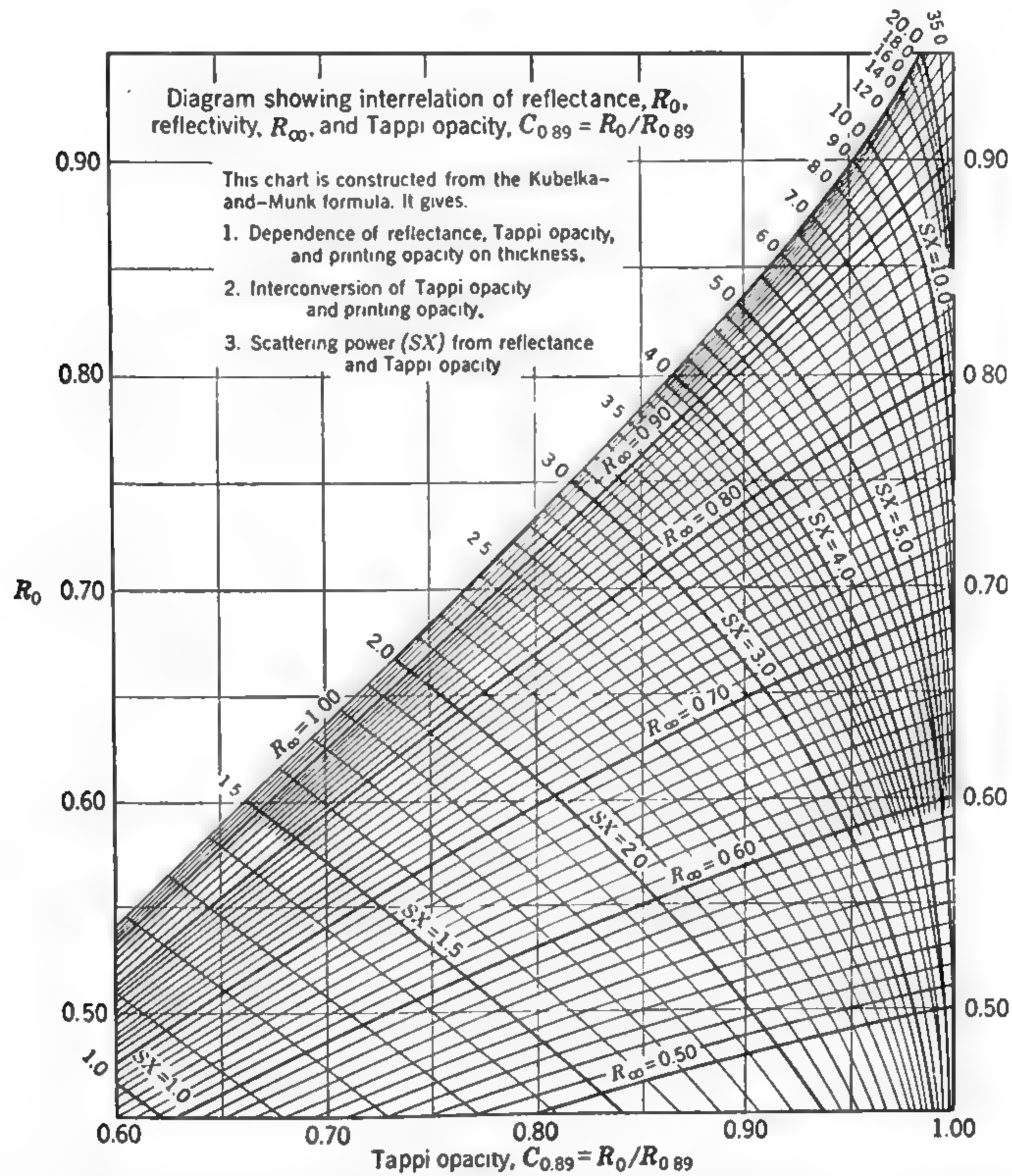


FIG. 100. Reflectance-opacity chart for white backing of reflectance 0.89 (after Judd, 1937). The lines coursing upward to the right refer to constant reflectivity R_∞ ; those coursing upward to the left, to constant scattering power SX . The influence on opacity, $C_{0.89}$, and on reflectance, R_0 , of a specimen layer with a black backing caused by variation in thickness, X , may be read from the former. The influence of adding black dye to the colorant layer may be read from the latter.

light-scattering pigment such as carbon black. It is seen that such an addition decreases the reflectance and increases the opacity of the layer. Two other partial graphical solutions of this form have been published (Judd, 1937), one with abscissa $R_0/R_{0.80}$, the other with the

abscissa $R_0/R_{0.70}$. The first of these is particularly useful for paints because of the widespread use of hiding-power charts in which the white areas have a reflectance close to 0.80. The second applies particularly to dental silicate cements. Graphical methods for determining the influence of white backings of reflectance intermediate to 0.70 and 0.89 have also been published (Judd, 1937). More recently Nordman (1950) has published a partial graphical solution showing loci of constant SX on a plot of R_0 against R_∞ .

Of course, all of this is theoretical information. But studies (Steele, 1937; Judd, 1937; Harrison, 1937; Nolan, 1937; Foote, 1939) showed that the Kubelka-Munk formula applies well to papers and cold-water paint, which it should, and also (Judd, 1937) to vitreous enamel on metal and to dental silicate cements, which must be called a rather lucky thing since equation 42 takes no account of internal reflections from the top boundary of the colorant layer. The commercial applications are obvious. Paper is customarily sold with a reflectance and an opacity requirement. The paper must be of a fairly light color; unless it is, black print is hard to read on it. It also must be fairly opaque; if it is not, print on the back or on the sheet beneath will show through and interfere with reading the print on top. Near-white paint is also often sold with a reflectance and a hiding-power requirement. It must have a certain minimum reflectance; otherwise it will not serve the purposes of near-white paint, and it must have a certain minimum hiding power; otherwise too many coats will have to be applied to hide the unwanted colors of the structures to which it is applied. Suppliers of paint and paper could look to graphical solutions of the Kubelka-Munk formula such as Fig. 100 and see immediately what to do to correct their formulations most economically to meet the stated requirements. If the reflectance requirement was met with something to spare by a formulation previously used, but the opacity requirement was not quite met, the paper manufacturer could see from Fig. 100 whether adding black dye (rather inexpensive) would suffice or whether some titanium dioxide pigment (rather expensive) also had to be added.

The Kubelka-Munk formula seems to apply only in a general way to paints. This failure to apply strictly is probably due to floating in the case of glossy paints (Duncan, 1949) and to a combination of floating and air penetration into the upper elementary layers of the paint film in the case of nonglossy paints. The theory applies sufficiently well to paint films to have considerable technical interest (Gardner and Sward, 1946; Duncan, 1949). As in paper, it is much cheaper to achieve the desired opacity or hiding power by adding a highly light-absorbing constituent, such as carbon black, than it is

to add more of a white constituent of high hiding power. What the customer is really paying for in a near-white paint is scattering coefficient S . Assume that two competing paints have different reflectivities both slightly higher than required. At negligible cost each could be degraded to the minimum reflectivity by addition of black pigment. This addition of black pigment would result in a decrease in R_0 and an increase in contrast ratio $R/R_{0.80}$, in accord with the lines of constant SX on Fig. 100. Other qualities (permanence, consistency, drying time, and so on) being equal, relative value of the two paints to the customer is in inverse proportion to the wet thickness of paint film required for some degree of incomplete hiding; that is, if one paint requires twice the wet thickness of film to produce this degree of hiding, the customer will have to buy twice as much to do the job. But note on Fig. 100 that both paints after adjustment by addition of black pigment are represented on the same line of constant reflectivity. The thickness of film required for any degree of hiding (that is, any value of contrast ratio) is determined solely by scattering power SX . If S for one paint is lower than the other, a corresponding greater thickness, X , will be required to produce this degree of hiding. The scattering coefficient, S , thus determines directly how much area can be hidden by a gallon of this paint, and it is therefore directly proportional to the price the customer would be justified in paying. The best buy is that paint whose ratio of scattering coefficient to price per gallon is highest.

The Hyperbolic Solutions. The interrelation between reflectance and opacity has been the subject of further theoretical studies. Steele (1935) restated the Kubelka-Munk formula in terms of hyperbolic functions. Amy, Sannié, and Sarrat (1937) adapted the Smith (1931) theory to paint formulation. Because of his interest in the formulation of plastics to match a desired color, Ingle (1942) made a comparative mathematical study of the formulas developed by Stokes (1860), Bruce (1926), Smith (1931), Kubelka-Munk (1931), and Amy, Sannié, and Sarrat (1937). He showed that the Smith-Amy and Stokes-Bruce formulas are special cases of the Kubelka-Munk formula. It remained for Kubelka (1948) to free colorant-layer technology from dependence on partial graphical solutions. In an amazing study, Kubelka, then a refugee from Czechoslovakia in the British zone of Germany derived explicit solutions for all the variables so awkwardly locked up in the exponential form of equation 42 and proved that the Gurevic (1930) formula for transmittance and the Judd (1937) formula describing the influence of different white backings on contrast ratio may also be derived from the basic differential equations 41a and 41b. The explicit solutions found by Kubelka make

immediately available nearly all aspects of the relation between reflectance and opacity of colorant layers.

The simple form of these solutions is due to the introduction of hyperbolic functions first used for this purpose by Steele (1935). No doubt the casual reader will be frightened away by the unfamiliar names, \sinh , \cosh , and ctgh , and will skip the rest of this analysis, if indeed he has got this far. But anyone in the business of making, buying, selling, or using colorant layers need not be frightened. The colorant technologist can learn all he needs to know about them right now in five minutes. $\sinh u$ is an abbreviation for $(e^u - 1/e^u)/2$. $\cosh u$ is an abbreviation for $(e^u + 1/e^u)/2$. The sum of these two is obviously, e^u , the exponential function itself. $\operatorname{ctgh} u$ is the ratio: $(\cosh u)/(\sinh u)$. Tables are widely available showing values of $\sinh u$, $\cosh u$, and $\operatorname{ctgh} u$ for various values of u . Read $\sinh u$ as hyperbolic sine of u , \cosh as hyperbolic cosine, and ctgh as hyperbolic cotangent.

Table 23 gives the definitions of the symbols used in the Kubelka explicit solutions. Many of them have already been given earlier,

Table 23. Definition of Symbols Used in Table 24

Symbol	Definition
R_g	Reflectance of background to which the colorant layer is applied
R	Reflectance of layer with background of reflectance, R_g
R_0	Reflectance of layer with ideal black background, $R_g = 0$
R_1	Reflectance of layer with ideal white background, $R_g = 1$
X	Thickness of layer
$R_\infty \equiv (R)_{X \rightarrow \infty}$	Reflectance of layer so thick that further increase in thickness fails to change the reflectance, called reflectivity
T_i	Internal transmittance of layer
$S \equiv (dR_0/dX)_{X \rightarrow 0}$	Coefficient of scatter, rate of increase of reflectance with thickness of a very thin layer for diffusely incident radiant energy
$K \equiv (dT_i/dX)_{X \rightarrow 0}$	Coefficient of absorption, rate of decrease of transmittance with thickness of a very thin layer for diffusely incident radiant energy
$a \equiv (S + K)/S$	Equal to $(1/R_\infty + R_\infty)/2$, whence: $R_\infty = a - (a^2 - 1)^{1/2}$
$b \equiv (a^2 - 1)^{1/2}$	Equal to $(1/R_\infty - R_\infty)/2$, whence: $R_\infty = a - b$
$\sinh u$	Abbreviation for $(e^u - 1/e^u)/2$, hyperbolic sine of u
$\cosh u$	Abbreviation for $(e^u + 1/e^u)/2$, hyperbolic cosine of u ; note that $\sinh u + \cosh u = e^u$
$\operatorname{ctgh} u$	Abbreviation for $(e^u + 1/e^u)/(e^u - 1/e^u)$, hyperbolic cotangent of u ; note that $\operatorname{ctgh} u = (\cosh u)/(\sinh u)$
$Ar \sinh$	Inverse hyperbolic functions: if $v = \sinh u$, then $u = Ar \sinh v$, and the same for $Ar \cosh$ and $Ar \operatorname{ctgh}$
$Ar \cosh$	
$Ar \operatorname{ctgh}$	

but some are abbreviations of expressions that recur frequently in the statements of the solutions. Table 24 summarizes the presently known explicit solutions and is copied directly from Kubelka's paper (1948).

Table 24. Formulas (Kubelka, 1948) Showing the Relationships between the Quantities Defined in Table 23

Characteristic	Formula	Equation Number
$R = f(SX, R_g, R_\infty)$	$R = \frac{1 - R_g(a - b \operatorname{ctgh} bSX)}{a - R_g + b \operatorname{ctgh} bSX}$	44
$R_0 = f(SX, R_\infty)$	$R_0 = 1/(a + b \operatorname{ctgh} bSX)$	45
$T_i = f(SX, R_\infty)$	$T_i = b/(a \sinh bSX + b \cosh bSX)$	46
$SX = f(R, R_g, R_\infty)$	$SX = \frac{1}{b} \left(Ar \operatorname{ctgh} \frac{a - R}{b} - Ar \operatorname{ctgh} \frac{a - R_g}{b} \right)$	47
$SX = f(R_0, R_\infty)$	$SX = \frac{1}{b} Ar \operatorname{ctgh} \frac{1 - aR_0}{bR_0}$	48
$SX = f(T_i, R_\infty)$	$SX = \frac{1}{b} \left(Ar \sinh \frac{b}{T_i} - Ar \sinh b \right)$	49
$R_0 = f(T_i, R_\infty)$	$R_0 = a - (T_i^2 + b^2)^{1/2}$	50
$T_i = f(R_0, R_\infty)$	$T_i^2 = (a - R_0)^2 - b^2$	51
$R_\infty = f(R_0, T_i)$	$a = \frac{1 + R_0^2 - T_i^2}{2R_0} \quad (\text{Gurevic})$	52
$R = f(R_0, R_g, R_\infty)$	$R = \frac{R_0 - R_g(2aR_0 - 1)}{1 - R_0R_g}$	53
$R_0 = f(R, R_g, R_\infty)$	$R_0 = (R - R_g)/[1 - R_g(2a - R)]$	54
$R_\infty = f(R, R_0, R_g)$	$a = \frac{1}{2} \left[R + \frac{R_0 - R + R_g}{R_0R_g} \right]$	55
$R = f(R_0, T_i, R_g)$	$R = R_0 + T_i^2 R_g / (1 - R_0R_g)$	56
$T_i = f(R, R_0, R_g)$	$T_i^2 = (R - R_0)(1/R_g - R_0)$	57
$R = f(R_1, R_0, R_g)$	$R = \frac{R_0(1 - R_g) + R_1R_g(1 - R_0)}{1 - R_0R_g} \quad (\text{Judd})$	58
$R_1 = f(R, R_0, R_g)$	$R_1 = \frac{R - R_0 + R_0R_g(1 - R)}{R_g(1 - R_0)} \quad (\text{Judd})$	59

First, let us see what happens to reflectance and internal transmittance by equations 45 and 46 as the layer becomes less and less light scattering. For scattering coefficient S , small compared to absorption coefficient K , we have from the definitions in Table 23:

$$a = (S + K)/S \text{ which approaches } K/S$$

$$b = (a^2 - 1)^{1/2} \text{ which also approaches } K/S$$

By substituting $a = K/S = b$ in equations 45 and 46, we get:

$$R_0 = S/K(1 + \operatorname{ctgh} KX)$$

$$T_i = 1/(\sinh KX + \cosh KX)$$

The expression for R_0 obviously approaches zero as S approaches zero; so equation 45, as expected, indicates that a nonscattering layer over a black background ($R_g = 0$) itself has a reflectance $R_0 = 0$. Also from the definitions of the hyperbolic sine and cosine (Table

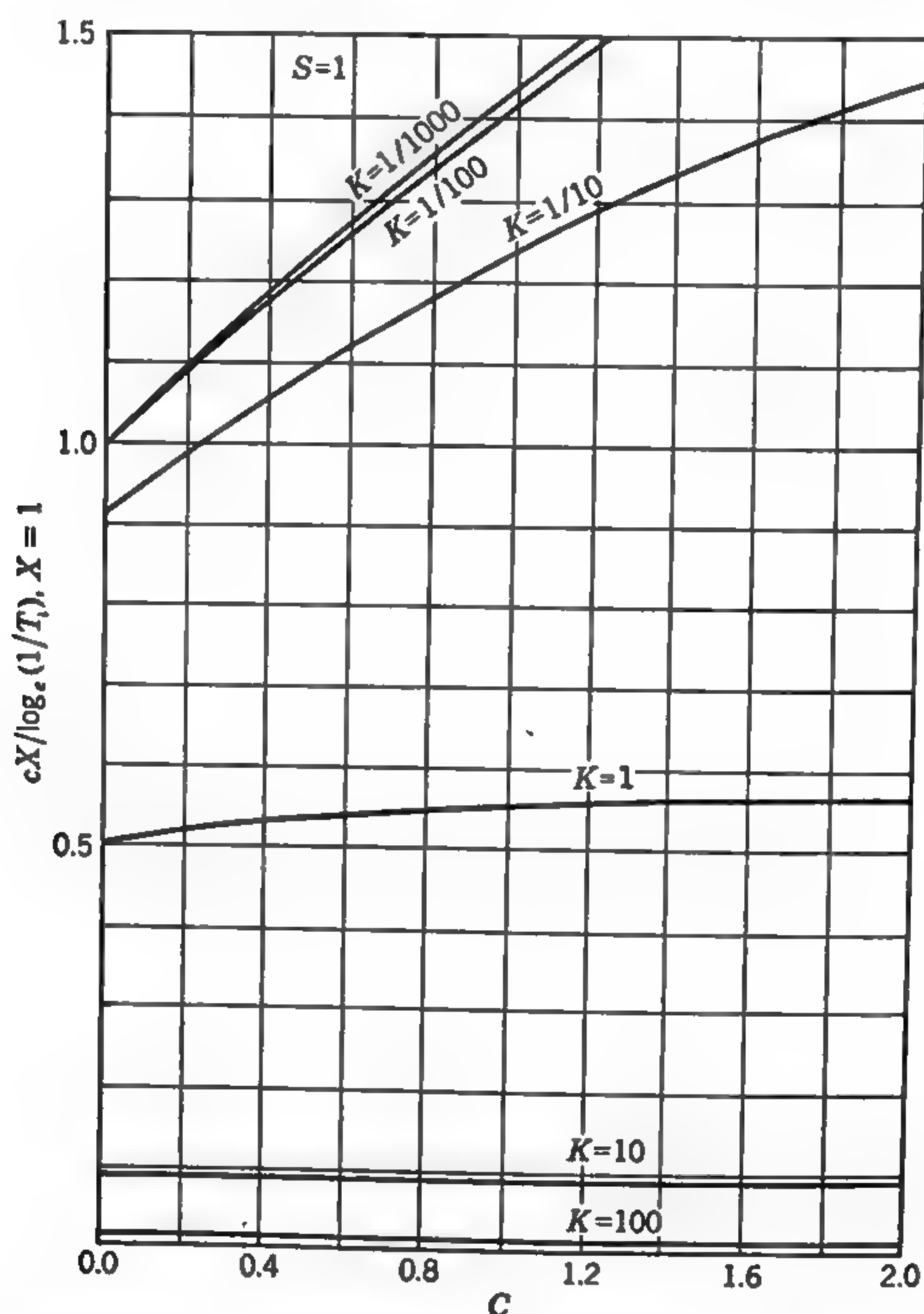


FIG. 101. Variation of the reciprocal of attenuation index, $[\log_e(1/T_i)]/cX$, with concentration, c , of the absorbing and scattering constituent. Each curve refers to unit thickness ($X = 1$) and to a constant ratio (K/S) of absorption to scattering coefficient. Note that the more highly light-scattering layers ($K/S < 1$) transmit more light at high concentrations because the scattered light is not lost, but is rescattered in the forward direction.

23), it is seen that the denominator is simply e^{KX} , whence we see that $T_i = e^{-KX}$, which is simply another way of writing equation 6a.* If the absorption coefficient K is proportional to the concentration of the colorant in the layer, we may write cK_1 instead of K , where K_1 is the absorption coefficient of the layer for unit concentration of the colorant, and the equation becomes: $T_i = e^{-cK_1X}$, or $\log_e(1/T_i) = cK_1X$. For slightly turbid media internal transmittance does not follow this law exactly, and the deviations may be shown on a graph such as Fig. 101. Here the expression, $cX/\log(1/T_i)$, is plotted against the concentration, c , of the colorant in the layer. The horizontal lines correspond to scattering coefficient, small compared to absorption coefficient, and the other lines were computed from equation 46 for the ratio K/S indicated on the graph. By determining the transmittance for diffuse flux incident on two layers differing only in concentration of the colorant, it is thus possible by a plot such as Fig. 101 to determine approximately the ratio, K/S , of the absorption coefficient to scattering coefficient of the colorant. Graphs of this sort showing straight-line approximations to these curves have been used to analyze transmittance data on sugar solutions, and they might be useful also for varnishes, lacquers, and plastics. For turbid media the name, absorption coefficient, for the quantity, $\log_e(1/T_i)$, is not appropriate, because this quantity measures both the absorbing and the scattering power of the body. Some term such as attenuation coefficient should be used. The quantity, $[\log_e(1/T_i)]/cX$, is sometimes used to characterize the absorbing and scattering constituent under the name, attenuation index.

A few examples will show how to use the formulas (44 to 59) listed in Table 24 to solve problems involving reflectance and opacity of colorant layers. A table of $\text{ctgh } u$ is given as Table E of the Appendix.

Example 1. Suppose that the reflectance of a specimen of paper is measured over a black backing ($R_g = 0$) and a white backing ($R_g = 0.89$) and found to have reflectances $R_0 = 0.642$ and $R_{0.89} = 0.841$. Required: to find the absorption coefficient, K , the scattering coefficient, S , and the reflectivity of the paper.

From the first column of Table 24 it may be noted that reflectivity R_∞ is given as a function of R , R_0 , and R_g in equation 55, whence we find:

$$a = \frac{1}{2} \left[0.841 + \frac{0.642 - 0.841 + 0.89}{0.642 \times 0.89} \right]$$

$$= (0.841 + 1.209)/2 = 1.025$$

* Note, however, that the absorption coefficient K for perfectly diffused flux is higher by a factor of 2 (Kubelka, 1948) than that for unidirectional flux.

But from the definition of a (Table 23) it is seen that $R_\infty = a - (a^2 - 1)^{1/2} = 1.025 - 0.051^{1/2} = 0.800$. This is the problem solved graphically by means of Fig. 100; and it is seen that the two results agree.

Now to find the scattering coefficient, S , note from the first column of Table 24 that SX may be found from either equation 47 or equation 48. We shall use both. The first thing to do is to evaluate b . From the definition of b in Table 23, it is seen that $b = (a^2 - 1)^{1/2} = 0.051^{1/2} = 0.225$; or b might be found as: $a - R_\infty = 1.025 - 0.800 = 0.225$. By substituting $R_g = 0$ and $R = 0.642$ into equation 47 we find:

$$\begin{aligned} SX &= \frac{1}{0.225} \left(\text{Ar ctgh} \frac{1.025 - 0.642}{0.225} - \text{Ar ctgh} \frac{1.025}{0.225} \right) \\ &= (\text{Ar ctgh } 1.702 - \text{Ar ctgh } 4.556)/0.225 \\ &= (0.674 - 0.223)/0.225 = 2.00 \end{aligned}$$

Turning now to equation 48, we find:

$$\begin{aligned} SX &= \frac{1}{0.225} \left(\text{Ar ctgh} \frac{1 - 1.025 \times 0.642}{0.225 \times 0.642} \right) \\ &= \frac{\text{Ar ctgh } 2.37}{0.225} \end{aligned}$$

which from Table E, Appendix, is seen to be $0.450/0.225 = 2.00$, which is the same value found from Fig. 100 and from equation 47.

Finally, to find the absorption coefficient, K , we turn in Table 23 to the definition of a , whence we see that $K = S(a - 1) = 2.00(1.025 - 1)/X = 0.050/X$. Another way to find K is from equation 43, whence we see that: $K = 2.00(1 - 0.800)^2/2 \times 0.800 = 2.00 \times 0.040/1.600X = 0.050/X$, which checks. This completes the solution of Example 1.

Example 2. Take Example 1 in reverse. Suppose that we know that the reflectivity of a colorant layer is 0.800, and its scattering power, SX , is 2.00. Required: to find its reflectance, R_0 , for black backing and its TAPPI opacity, $R_0/R_{0.89}$.

From the first column in Table 24 we find that equation 45 gives R_0 as a function of the two quantities given (R_∞, SX). We must first evaluate a and b which we do from the definitions in Table 23:

$$\begin{aligned} a &= (1/0.80 + 0.80)/2 = (1.25 + 0.80)/2 = 1.025 \\ b &= (1/0.80 - 0.80)/2 = (1.25 - 0.80)/2 = 0.225 \end{aligned}$$

Then equation 45 becomes:

$$\begin{aligned} R_0 &= 1/(1.025 + 0.225 \text{ ctgh } 0.225 \times 2.00) \\ &= 1/(1.025 + 0.225 \times 2.370) \\ &= 1/1.558 = 0.642 \end{aligned}$$

which from Example 1 we already know is correct.

To evaluate TAPPI opacity, $R_0/R_{0.89}$, requires us to use equation 44 with R_g set equal to 0.89, thus:

$$\begin{aligned}
 R &= \frac{1 - 0.89(1.025 - 0.225 \operatorname{ctgh} 0.225 \times 2.00)}{1.025 - 0.89 + 0.225 \operatorname{ctgh} 0.225 \times 2.00} \\
 &= \frac{1 - 0.89(1.025 - 0.533)}{1.558 - 0.89} \\
 &= \frac{1 - 0.438}{0.668} = 0.841
 \end{aligned}$$

which from Example 1 we already know is correct. This completes the solution of Example 2.

Example 3. What is the reflectance, R_0 , for black backing and TAPPI opacity of the sheet found by doubling the basis weight of the paper specimen of Examples 1 and 2?

The solution is the same as for Example 2 except that SX is taken as 4.00 instead of as 2.00. From equation 45:

$$\begin{aligned}
 R_0 &= 1/(1.025 + 0.225 \operatorname{ctgh} 0.225 \times 4.00) \\
 &= 1/(1.025 + 0.225 \times 1.396) \\
 &= 1/1.339 = 0.747
 \end{aligned}$$

which is the value previously read from Fig. 100.

Equation 44 becomes:

$$\begin{aligned}
 R &= \frac{1 - 0.89(1.025 - 0.314)}{1.339 - 0.89} \\
 &= \frac{1 - 0.633}{0.449} = 0.817
 \end{aligned}$$

TAPPI opacity is $R_0/R_{0.89} = 0.747/0.817 = 0.914$, which is close to the opacity (0.913) read from Fig. 100. This completes the solution of Example 3.

Example 4. The paper in Example 3 has a printing opacity R_0/R_∞ , equal to $0.747/0.80$, or 0.933 . Suppose the printing opacity requirement to be 88 percent. If a manufacturing tolerance of 2 percent in printing opacity has to be allowed, by how much could the basis weight of the paper be reduced and still be safely above the requirement?

To solve this problem note that R_0 , R_∞ , and S are all known, but we must find X/X_3 , where X_3 is basis weight of paper in Example 3. From Table 24 it is seen that equation 48 applies. For a printing opacity of 88 plus 2 percent tolerance, R_0/R_∞ must equal 0.90. But since $R_\infty = 0.80$, the required value of R_0 is $0.80 \times 0.90 = 0.72$. Furthermore, $SX_3 = 4.0$. Equation 48 becomes:

$$\begin{aligned}
 SX &= 4.0 X/X_3 = \frac{1}{0.225} \left(Ar \operatorname{ctgh} \frac{1 - 1.025 \times 0.72}{0.225 \times 0.72} \right) \\
 X/X_3 &= \frac{1}{0.900} (Ar \operatorname{ctgh} 0.262/0.162) \\
 &= 1.111(Ar \operatorname{ctgh} 1.613)
 \end{aligned}$$

From Table E, Appendix, the value of u for which $\operatorname{ctgh} u = 1.613$ is 0.725; so $X/X_3 = 1.111 \times 0.725 = 0.805$. Nearly a 20 percent reduction in basis weight is

possible. This result can be checked by reading R_0 from Fig. 100 for $SX = 4.0 \times 0.805 = 3.22$. R_0 is seen to be 0.720 making printing opacity $R_0/R_\infty = 0.720/0.800 = 0.90$, as required.

Example 5. Suppose that a film of cold-water paint is characterized by $S = 4.0$, $K = 0.10$, what is its contrast ratio, $C_{0.89} = R_0/R_{0.89}$, for unit thickness of film ($X = 1$)?

First, we find from the definition in Table 23, $a = (S + K)/S = 4.1/4.0 = 1.025$, whence as in Example 1 we find $b = 0.225$ and $R_\infty = 0.800$. The reflectance over a black background, R_0 , is then determined as in Example 3 to be 0.747. The reflectance over a white background of reflectance, $R_g = 0.80$, is found from equation 44 by setting $R_g = 0.80$, and we find:

$$R = \frac{1 - 0.80 \times 0.711}{1.339 - 0.80} = \frac{0.431}{0.539} = 0.800$$

which we can see is correct because we knew already that the reflectivity of this colorant layer, $R_\infty = 0.800$, is the same as the reflectance of any thickness of it backed by an opaque layer of it. The contrast ratio $R_0/R_{0.80}$ is therefore $0.747/0.800 = 0.933$.

This result can also be obtained from equations 58 and 59. First, from equation 59 find R_1 by substituting $R_g = 0.89$, $R_{0.89} = 0.817$ (from Example 3), and $R_0 = 0.747$:

$$\begin{aligned} R_1 &= \frac{0.817 - 0.747 + 0.747 \times 0.89(1 - 0.817)}{0.89(1 - 0.747)} \\ &= 0.192/0.225 = 0.852 \end{aligned}$$

Then equation 58 becomes:

$$\begin{aligned} R &= \frac{0.747(1 - 0.80) + 0.852 \times 0.80(1 - 0.747)}{1 - 0.747 \times 0.80} \\ &= 0.321/0.402 = 0.80 \end{aligned}$$

which checks, but the solution directly by means of equation 44, though involving hyperbolic functions again, is simpler.

Example 6. Suppose that the paint layer of Example 5 is put down on a piece of glass and illuminated diffusely. What fraction of this incident diffuse flux will emerge into the glass?

The problem may be solved by means of equation 51 or by equation 57. Equation 51 becomes:

$$\begin{aligned} T^2 &= (1.025 - 0.747)^2 - 0.225^2 = 0.0267 \\ T &= 0.163 \end{aligned}$$

Equation 57 becomes:

$$\begin{aligned} T^2 &= (0.800 - 0.747)(1/0.800 - 0.747) = 0.0267 \\ T &= 0.163 \end{aligned}$$

which checks.

Example 7. How many coats of unit thickness ($X = 1$) of the cold-water paint in Examples 5 and 6 are required to produce "complete hiding" defined by $R_0/R_{0.80} = 0.98$?

Since $R_{0.80} = R_{\infty} = 0.800$, $R_0 = 0.98 \times 0.80 = 0.784$, and this problem can be explicitly solved by equation 48 by setting $S = 4.0$:

$$\begin{aligned} 4.0 X &= \frac{1}{0.225} \left(Ar \operatorname{ctgh} \frac{1 - 1.025 \times 0.784}{0.225 \times 0.784} \right) \\ X &= 1.111 Ar \operatorname{ctgh} 1.112 \\ &= 1.111 \times 1.468 = 1.6 \end{aligned}$$

and we see that two coats would be more than enough.

Example 8. Suppose that the cold-water paint of Examples 5, 6, and 7 was intended to meet a reflectance requirement of 84 percent at hiding thickness. Obviously, as it stands, it will not meet this requirement, since its reflectivity, R_{∞} , is only 0.80. But the paint manufacturer can buy special white pigment of reflectivity equal to 96 percent such that a cold-water paint made of it will give at one-fourth unit thickness ($X = \frac{1}{4}$) a reflectance for black backing, R_0 , equal to 0.80. What percentage of admixture of the special white paint would have to be made so that the mixed paint would meet the reflectance requirement of 84 percent?

First, the scattering coefficient of the special paint has to be determined by equation 48, and for this a and b must be evaluated from their definitions in Table 23 in terms of reflectivity:

$$\begin{aligned} a &= (1/0.96 + 0.96)/2 = 1.0008 \\ b &= (1/0.96 - 0.96)/2 = 0.0408 \end{aligned}$$

Equation 48 becomes:

$$\begin{aligned} S/4 &= \frac{1}{0.0408} \left(Ar \operatorname{ctgh} \frac{1 - 1.0008 \times 0.80}{0.0408 \times 0.80} \right) \\ S &= 4(Ar \operatorname{ctgh} 6.11)/0.0408 \end{aligned}$$

From Table E, Appendix, we find for $\operatorname{ctgh} u = 6.11$, $u = 0.165$; so $S = 4 \times 0.165/0.0408 = 16.2$.

We now find K from a and S by the definition of a in Table 23:

$$K = S(a - 1) = 16.2 \times 0.0008 = 0.013$$

The special paint is superior to the original paint because it scatters light better (compare the scattering coefficient 16.2 with 4.0) and also because it absorbs very little light (compare the absorption coefficient 0.013 with 0.10). The high ratio of S to K permits the special paint to yield its very high reflectivity ($R_{\infty} = 0.96$). In any mixture of cold-water paints, S and K for the mixture are simply the volume-weighted averages of S and K for the constituents. The problem may be solved by reading R_{∞} from K/S in Table D, Appendix.

Proportion of Special Paint in Mixture	Mixture			
	S	K	K/S	R_{∞}
0.0	4.00	0.1000	0.02500	0.800
0.2	6.44	.0826	.01283	.852
0.4	8.88	.0652	.00734	.886
0.6	11.32	.0478	.00422	.912
0.8	13.76	.0304	.00221	.936
1.0	16.20	.0130	.00080	.961

It will be noted that a 20 percent admixture of the special paint raises the reflectivity of the mixture more than enough to meet the minimum requirement of 84 percent. By interpolation it is seen that the required percentage admixture is about 16. This completes solution of Example 8.

It is evident from these eight examples how the equations of Table 24 should be used to solve colorant-layer formulation problems involving reflectance and opacity. It is also evident that partial graphical solutions of equation 42 like Fig. 100, though convenient for routine calculations, can be dispensed with if only a few problems have to be solved. Kubelka's mathematical analysis (1948) has freed the colorant-layer industries from dependence on graphical solutions; it has also provided equations from which graphical solutions may be much more easily obtained than from equation 42 if colorant-layer industries find them useful.

IDENTIFICATION OF COLORANTS

When a manufacturer of colorant layers (paper, plastics, paint, textiles, ceramics) is given a sample of the color desired by a customer, he can proceed to get a color match in several different ways. The most usual way is to take the pigments and dyes at his disposal and formulate the match by trial and error. If the layer is to be completely hiding, or nearly so, he will need at least four colorant constituents to get a match, unless he finds a match by good luck. Alteration of the proportions of the four constituents provides the requisite three degrees of freedom for a color match. Thus, one pigment, say white, gives no freedom at all. The colorant layer may be white, but no other color. Addition of another pigment, say black, gives one degree of freedom. The colorant layer may then be any one of the series of bluish grays produced by varying proportions of white and black pigment. Addition of a third pigment, say red, permits the colorant layer to be any one of a two-dimensional series of colors all essentially of red hue. But addition of a fourth pigment permits a tridimensional variation, so that by proper adjustment of the proportions of the four pigments the colorant layer may be made to match exactly the specimen submitted if its color lies within the gamut of colors producible by these four constituents.

If the layer is to be put down onto a base which may show through to a greater or lesser extent, the amount of show-through provides one degree of freedom and only three colorant constituents are required. This also holds for applications of dye to textiles. If, as is usual, the colorants chosen because of their availability differ from those used to make the color sample submitted, the match will be

metameric; that is, the formulated color and the sample color will match under the kind of light used in the adjustment of the proportions of the colorants but will differ more or less under other kinds of light.

Another way to achieve the color match is to find what colorants were used to make the color sample, then find what proportions to use to get the desired color. This may require a spectrophotometric study, but the solution is straightforward and the resulting color match will be nonmetameric. It will have the same curve of spectral reflectance or transmittance as the sample and will color-match, regardless of the kind of light used to illuminate the samples or the kind of observer used to judge the match.

The first step in this process is the identification of the colorants used to produce any given colorant layer.

An important, and sometimes unfortunately the chief, function of an industrial research laboratory is to keep track of what the rest of the industry is doing. An identification of the colorants used in the formulations put out by the competitors is a part of this process.

Clear Media. If the layer is nonlight-scattering, the identification of the coloring constituents is relatively simple because the beam does not lose its identity but may be traced. An unattached layer of this sort reflects a fraction, ρ , of the incident flux from its front surface in accord with equations 38 and 38a, where ρ is written for ρ_{\parallel} , ρ_{\perp} , or ρ_T , as the case may be. The remainder, $(1 - \rho)$, penetrates the surface, and a fraction, $T_i(1 - \rho)$, reaches the opposite face, where T_i is the internal transmittance of the layer. Various fractions of this are multiply reflected between the top and the bottom of the layer as shown in Fig. 102. By adding together the

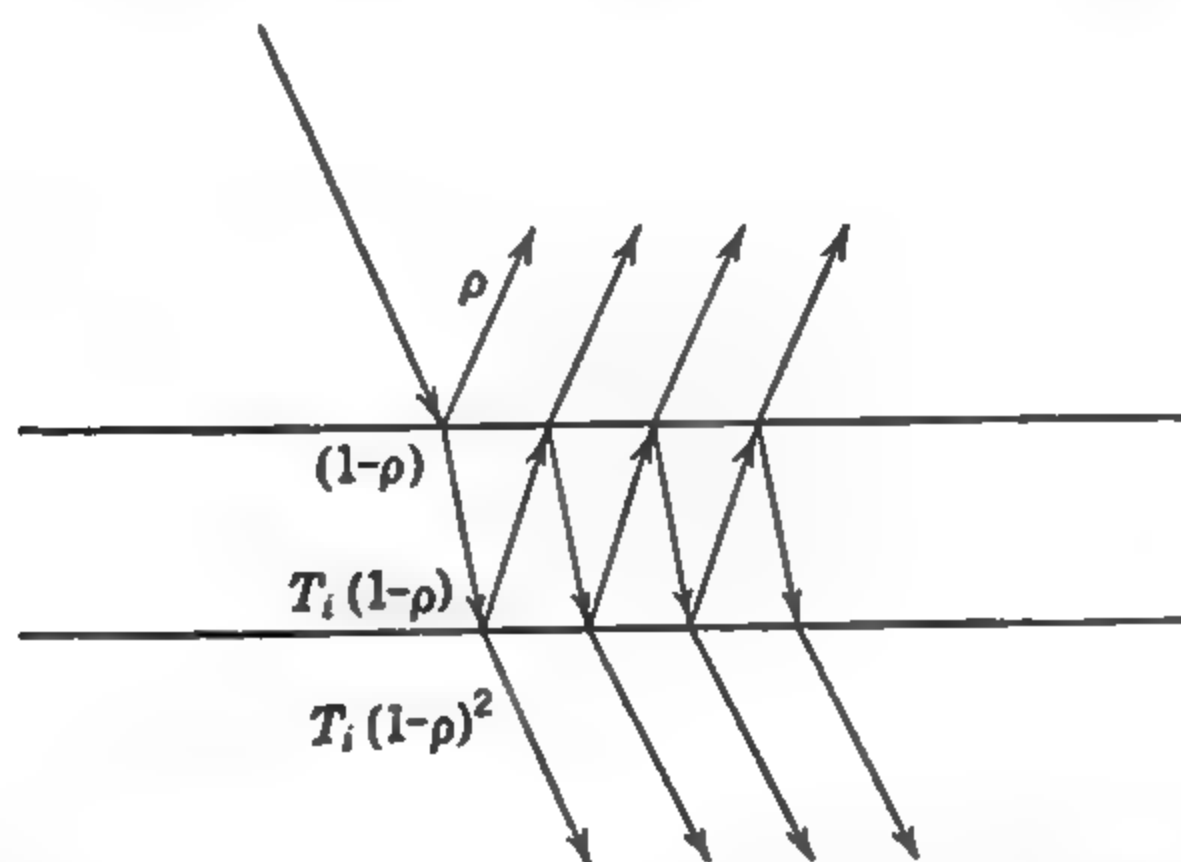


FIG. 102. Multiple reflections between the top and the bottom of a nonlight-scattering layer with plane, parallel faces.

fractions that leave the top and the bottom faces, formulas for the reflectance and transmittance of a nonlight-scattering layer are found for light incident nearly perpendicular to the surface:

$$\begin{aligned} R_0 &= \rho + \rho(1 - \rho)^2 T_i^2 (1 + \rho^2 T_i^2 + \rho^4 T_i^4 + \dots) \\ &= \rho + \rho(1 - \rho)^2 T_i^2 / (1 - \rho^2 T_i^2) \end{aligned} \quad (60)$$

$$\begin{aligned} T &= T_i(1 - \rho)^2 (1 + \rho^2 T_i^2 + \rho^4 T_i^4 + \dots) \\ &= T_i(1 - \rho)^2 / (1 - \rho^2 T_i^2) \end{aligned} \quad (61)$$

Equations 60 and 61 apply to clear sheets of glass, gelatine, or plastic. If the index of refraction of the layer is 1.5, the reflectance, ρ , from the top of the layer for nearly perpendicular incidence is found from equation 36 to be 0.04 or 4 percent. Equation 60 shows that the reflectance of an essentially nonlight-absorbing layer ($T_i = 1$) in front of, but not optically attached to, a black background ($R_g = 0$) is nearly double that, ρ , from the top face alone, or nearly 8 percent. On the other hand, if the layer absorbs most of the flux penetrating the top surface ($T_i \rightarrow 0$), the reflectance, R_0 , corresponds to that reflected from the top surface alone and is only 4 percent.

Equation 61 shows the connection between transmittance T and internal transmittance T_i . For layers of glass and plastic T_i is approximately $T/0.92$, and this approximation is often used. If a cell of solution is compared to a similar cell of solvent by means of the spectrophotometer, the ratio of the two transmittances (called transmittancy) is closely equal to T_i for the solution, and this also is commonly used to evaluate T_i for solutions.

The variation of internal transmittance, T_i , with thickness, X , of layer, and concentration, c , of a nonlight-scattering colorant (dye) is quite simple:

$$T_i = e^{-cK_1X} \quad (62)$$

where K_1 is the absorption coefficient for unit concentration ($c = 1$), and e ($= 2.71828 \dots$) is the base of Napierian logarithms. This is a restatement of equation 6a. For $c = 1$, it is known as Bouguer's law. For thickness X constant it is known as Beer's law. Whenever a change in concentration of a solution of dye or chemical substance of whatever kind causes a change in K_1 , the solution is said "not to obey Beer's law."

The colorants of a clear layer are characterized by the way in which K_1 varies with wavelength. By taking logarithms of both sides of equation 62, K_1 is found to be:

$$K_1 = (-\log_e T_i)/cX \quad (62a)$$

If $-\log_e T_i$ is plotted against cX , a graph, such as Fig. 103, is formed in which the variation of internal transmittance of a clear colorant layer with concentration, c , of colorant and thickness, X , of layer is shown by a family of straight lines passing through the origin. The slope of each line represents the absorption coefficient of the corresponding colorant at unit concentration. The product, cX , is

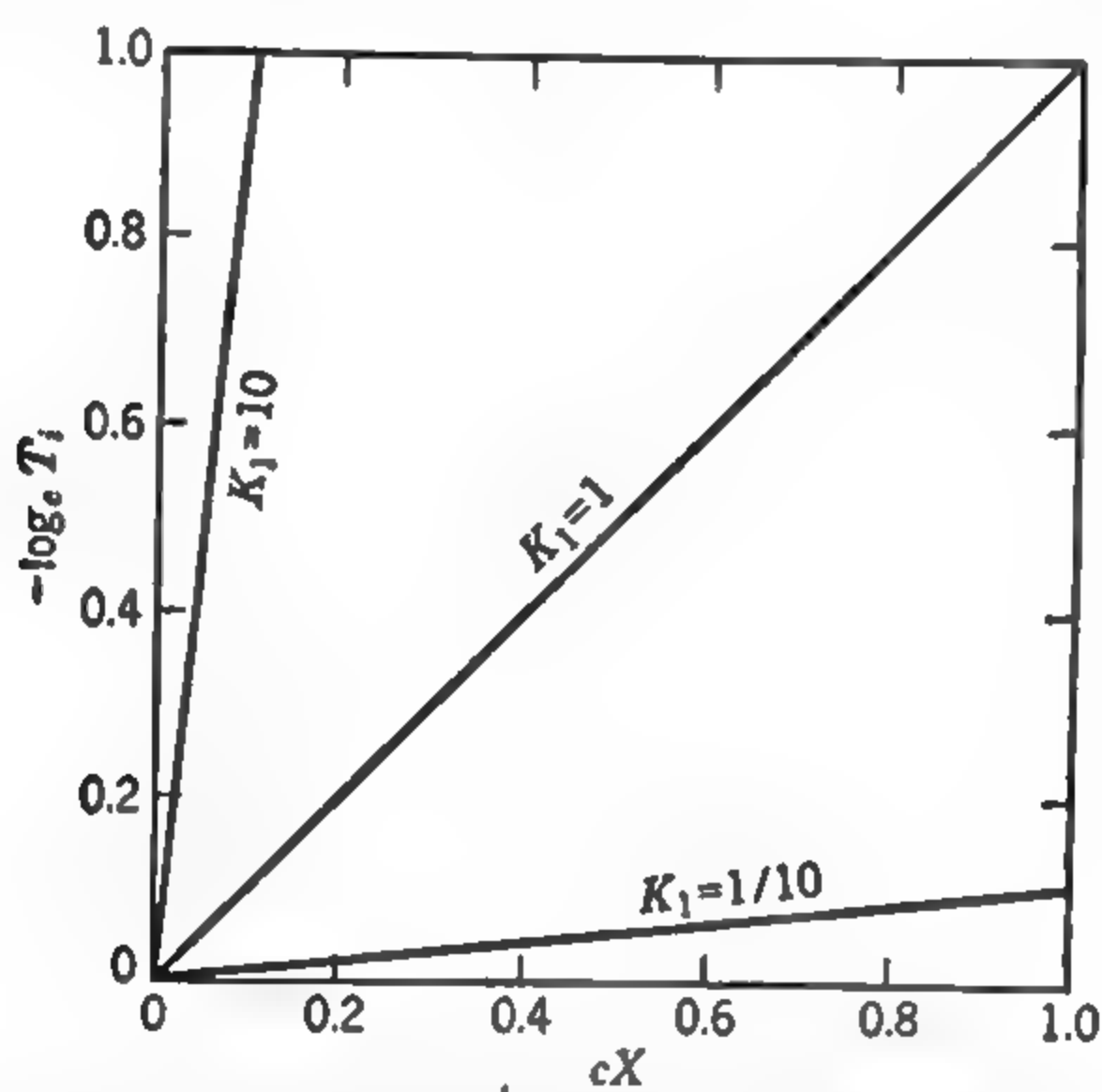


FIG. 103. Graphical determination of the absorption coefficient, K_1 , of a colorant of concentration, c , from the internal transmittance, T_i , of a nonlight-scattering layer of thickness, X . The absorption coefficient is found as the slope of the line on this graph of $-\log_e T_i$ against cX .

a measure of the amount of the colorant in unit area of the layer; therefore, $-\log_e T_i$ is also a measure of it, the proportionality constant relating these two measures being the absorption coefficient, K_1 , for unit concentration. Any deviation of data plotted on this graph from a straight line passing through the origin indicates that K_1 is not a constant independent of concentration, and the colorant is said (Hardy and Young, 1948) "not to obey Beer's law." This kind of plot applies to clear solutions, clear varnish, and lacquer films, and to layers of clear glass and plastic. It is much used in absorptimetry (sometimes miscalled chemical colorimetry; see Mellon, 1950) to determine the concentration of one of the constituents of a solution. It is also often convenient to plot the spectral transmittance on a log scale against wavelength. The GE Recording Spectrophotometer is equipped with a log cam, so that the spectrophotometric data may be recorded directly in this form if desired. If a

cell containing the solvent is placed in the comparison beam, the spectrophotometer records transmittancy, which is a close approximation to internal transmittance, either directly or on a log scale. A skilled colorant technologist may recognize by its characteristic shape the curve corresponding to many colorants, or even two-component mixtures of such colorants. But there is a still more helpful way to plot such data. This may be seen by taking the logarithms of both sides of equation 62a:

$$\log (-\log T_i) = \log K_1 + \log cX \quad (62b)$$

For any one spectrophotometric curve the thickness of colorant layer, X , and the concentration of the colorant, c , are constant independent of wavelength. By plotting $\log (-\log T_i)$ against wavelength we may, therefore, obtain a curve whose shape is independent of the amount of colorant present (Mellon, 1950; Shurcliff, 1942; Stearns, 1944, 1950; Wright, 1944). As may be seen from equation 62b, the shape depends solely on the variation of K_1 with wavelength. By this device the colorant or the colorant combination may be made to write its optical signature. In any other thickness or concentration, the same colorant must repeat its optical signature exactly, and thus reveal its identity.

Figure 104 shows three sets of spectrophotometric data for cobalt glass plotted in three different ways. The first set refers to unit amount of the colorant ($cX = \text{Lovibond blue numeral} = 1$). The second set refers to $cX = 4$ and corresponds to Lovibond 4.0 blue. The third set refers to $cX = 16$ and corresponds to Lovibond 16.0 blue. The first method of plotting is to use internal transmittance T_i directly. This plot reveals that the first set of data corresponds to a pale blue color; the second, to a moderate blue color; and the third, to a deep blue. Data read from this plot could be used to compute the tristimulus values, X, Y, Z , as in Table 8.

The second method of plotting is to express the internal transmittance, T_i , on a log scale. Logarithms to the base 10 are just as suitable for this purpose as those to the Napierian base, e , and are generally used because of their greater convenience. What this plot shows most clearly is that the top curve refers to one-fourth as much coloring material as the middle curve; and the bottom curve refers to four times as much. Every ordinate (measured from the top line of the graph) of the first curve is one-fourth that of the middle curve for the same wavelength, and one-sixteenth that of the bottom curve.

A ceramist working with colorant layers might recognize from any one of the six top curves that the coloring material is cobalt,

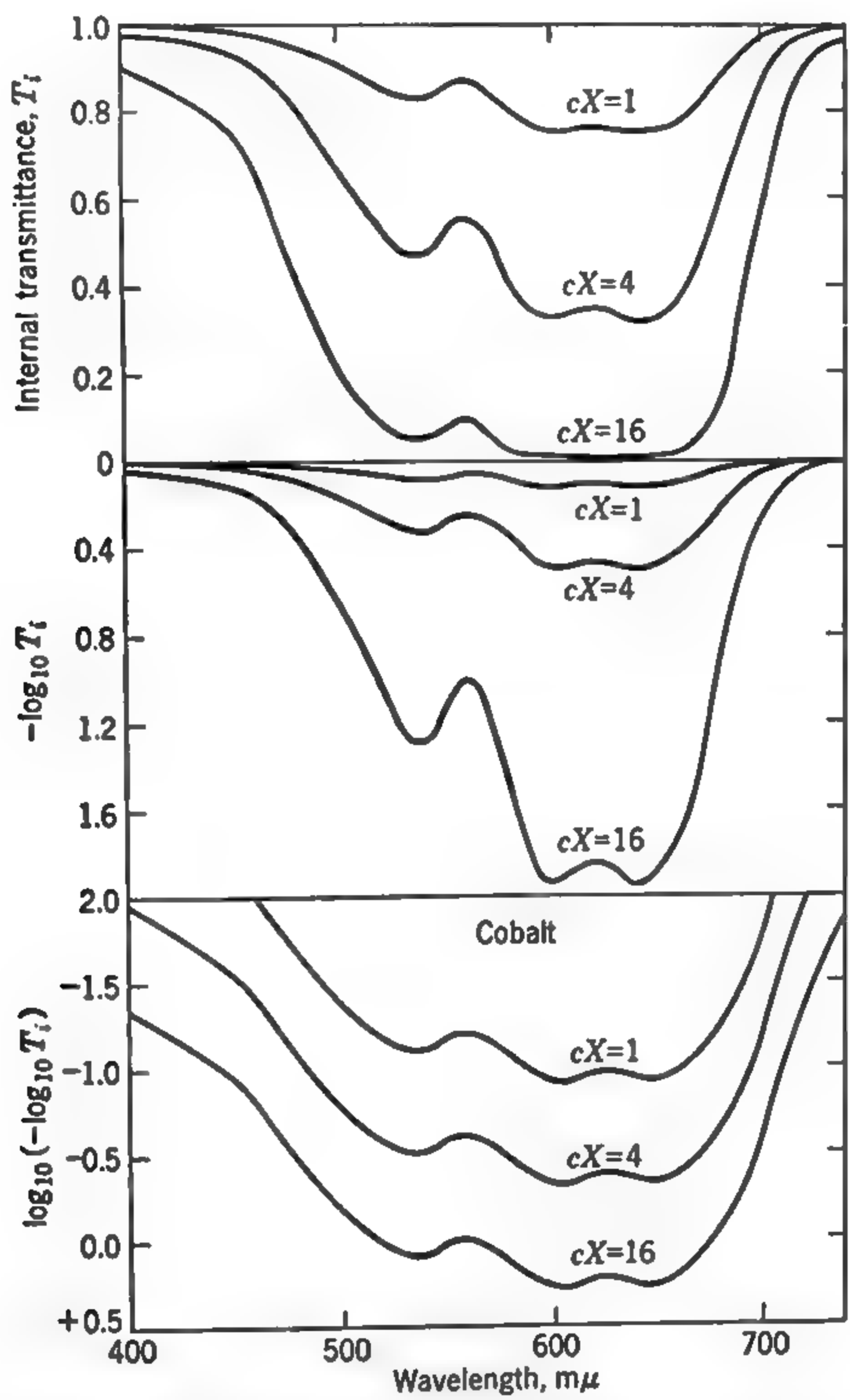


FIG. 104. Spectral internal transmittance, T_i , for three Lovibond blue glasses (Lovibond blue 1.0, 4.0, and 16.0) plotted against wavelength in three ways: first, directly; second, as $-\log T_i$; and third, as $\log(-\log T_i)$. This last plot may be called the optical signature of the coloring material. Note that the shapes of the three curves on this plot are identical.

although the handwriting is somewhat disguised. The bottom plot, however, showing $\log(-\log T_i)$ against wavelength, spells out cobalt unmistakably in three identical optical signatures, and the vertical position of the curve indicates the amounts. Note that at each wavelength the curve separation is $0.602 = \log_{10} 4$.

If there are several constituents in the nonlight-scattering layer, the transmittance of the layer is still found from equation 62. The product, cK_1 , is the sum of the products, $(cK_1)_p$, $(cK_1)_q$, $(cK_1)_r$, and so on, for the separate constituents, p , q , r , and so on, providing that there is no chemical interaction between constituents. What the colorant-layer technologist has to do to identify such a mixture is to try to find colorants, p , q , r , \dots whose absorption coefficients, K_1 , known as functions of wavelength, can be so weighted as to add up to the curve of the unknown on the $(-\log T_i)$ plot. The weights are the desired concentrations, and the best clues to the constituents are the wavelength positions of the transmittance minima, called absorption bands. Stearns (1950) gives detailed instructions for the analysis of layers of clear media by means of a plot of $\log(-\log T_i)$ against wavelength in the form of a photographic transparency.

Turbid Media. Strictly speaking, all colorant layers are turbid; that is, all of them scatter some light. The blue color of the sky and of the tropic sea comes from scattered light. Only a vacuum is strictly nonlight-scattering. The media just discussed, however, scatter sufficiently little light so that equation 62 applies without significant error. We have now to treat layers to which equation 62 does not apply even approximately. We do not refer to slightly turbid media like sugar solutions, but we refer rather to the strongly light-scattering layers (paper, paint, vitreous enamel) of commercial interest.

A nonfluorescent layer that both absorbs and scatters light because of small light-scattering elements in it may be identified by its absorption and scattering coefficients, K and S , as functions of wavelengths throughout the visible spectrum. The ratio of these two, K/S , may be found by measuring with a spectrophotometer the reflectance, R_∞ , of a completely hiding coat of the colorant, and then applying equation 43 by way of Table *D* in the Appendix. The scattering coefficient, S , may be evaluated by the several technics suggested by equations 47, 48, and 49:

(a) Measure reflectance, R , of a layer of known thickness, X , and reflectivity, R_∞ , applied to a background of known reflectance, R_g , and compute S by equation 47.

(b) Measure reflectance, R_0 , of a layer of known thickness, X , and reflectivity, R_∞ , applied to a completely nonreflecting background, $R_g = 0$, and compute S by equation 48. This is a special case of equation 47.

(c) Measure transmittance, T_i , of a layer of known thickness, X , and reflectivity, R_∞ , and compute S from equation 49.

From the reflectivity, R_∞ , the ratio K/S is known by equation 43, and from the scattering coefficient, S , found either by equation 47, 48, or 49, the absorption coefficient, K , is immediately found.

If the absorption and scattering coefficients of the unknown specimen are thus evaluated as functions of wavelength, the task of the colorant-layer technologist is then to find a known colorant or a combination of known colorants whose absorption and scattering coefficients have the same wavelength dependence as the unknown. If this can be done, the identification of the unknown colorant has been completed. The scattering coefficient of the colorant produced by a combination of constituents is, like the absorption coefficient, merely the sum of the coefficients of the constituents (Duncan, 1940; Persoz, 1945). This simple additivity of the coefficients is what makes the Kubelka-Munk analysis applicable to colorant-layer formulation. This additivity was made use of to solve Example 8 following Table 24. It is so important that we shall write it down explicitly:

$$\left. \begin{aligned} K &= (cK_1)_p + (cK_1)_q + (cK_1)_r + \cdots \\ S &= (cS_1)_p + (cS_1)_q + (cS_1)_r + \cdots \end{aligned} \right\} \quad (63)$$

The technologist will thus try to find known colorants such that the weighted sum of the absorption and scattering coefficients is equal for each wavelength to that of the unknown. This is the procedure to be used if opacity and hiding power are of interest, or if the scattering coefficient of the mixture varies importantly with the proportions of the components.

If the colorant layer is completely opaque and has a known fixed amount of light-scattering material in it, a considerable simplification is possible. It is then necessary only to deal with the ratio, K/S , of the absorption to the scattering coefficient. This condition is satisfied by textiles colored by the application of dye; and it is sufficiently well approximated by paints consisting of white paint tinted by addition of toners of low scattering coefficient, that is, so low as to be negligible compared to the scattering coefficient of the white paint. From equation 43 it is seen that the function $(1 - R_\infty)/2R_\infty$ measures at each wavelength the amount of dye or essentially nonscattering pigment added. The optical signature of the dye or pigment is thus found by plotting $\log (1 - R_\infty)/2R_\infty$ against wavelength (Abbott and Stearns, 1944). To identify the colorants in an unknown specimen of this sort, the technologist will make use of clues based on other physical tests (such as index of

refraction) and on chemical tests, but his most useful tool is the spectrophotometer (Stearns, 1951, *b*).

FORMULATION OF COLORANTS

Probably industry's number-one color problem is how to find the proportions of the four or five constituents required to produce a color match for a specimen showing the color the customer wants. A symposium on color held in 1940 was attended by about 150 paint, pigment, and dye men, both executives and technologists, illuminating engineers, and plastics experts. Four nationally known speakers had regaled this audience with two hours of spectrophotometry, CIE system, color standards, effect of illuminant on object color, photoelectric colorimetry, and so on. The audience became drowsy, with here and there someone frankly snoring. The symposium at an end, the chairman called for questions. There were a few polite queries by the more wakeful, but most of the audience remained in a state of suspended animation, and soon the discussion died away practically to zero. The chairman insisted. "Now that you have the experts at your disposal," he said, "this is your chance to find out what you want to know about color." Thus spurred, a paint man got to his feet and said, "What I want is a photoelectric colorimeter that will read the proportions of the pigments required to produce a color match for a specimen." Like magic the somnolence disappeared. The audience snapped to attention, and the ensuing discussion was followed both eagerly and intelligently.

There is no easy answer to this question, of course. The complicated influence of the colorant layer on the radiant energy by which its color is revealed to us prevents this. The essential part of this influence is summarized in the simplest possible way by the differential equations 41a and 41b, and, as we have seen, the integration of these equations shows the net effect to be complicated. How much easier it is to provide an answer if the medium by which the color match is to be produced is light instead of pigment! Color television gives these solutions at the rate of more than a million a second. If the tristimulus values, X , Y , Z , of the desired color are known, the required solution is given by equation 13, where R , G , B are the proportions of the mixture of the red, green, and blue lights of the television receiver. Except for screened-plate printing (Hardy and Wurzburg, 1948), the colorant-layer industries must proceed at a snail's pace to find the answer.

As mentioned before, there are two chief methods of determining what proportions of the colorant constituents are required to pro-

duce a color match for the specimen submitted. The most used is a purely empirical trial-and-error method resulting in a more or less metameric match. The most exact, and probably least used, is to identify the colorants employed for the specimen submitted, to find their exact proportions, and so to produce an exact physical duplicate of it, which, of course, results in a nonmetameric match that will hold for any illuminant and any observer. There are several gradations of method between these two extremes.

Trial and Error. The chief colorant components are selected on the basis of cost, availability, permanence, and so on. If the layer is to be paint or printing ink, a small mix is made up and a record kept of the weights of the ingredients. The color matcher adds each time the ingredient that seems to him to be lacking in the mixture to produce the match. When a sufficiently close approximation to the color of the submitted sample has been obtained, the sum of the weights in the record for each ingredient gives the formula. A large mix is then made up to this formula, and, if the procedure turns out as usual, the large mix will not have quite the color of the submitted sample. Not wishing to add an appreciable amount of the main ingredients, lest more of the material be made up than was ordered, the color matcher seeks to correct the color by addition of small amounts of colorants of high tinting strength. These colorants are called toners, and the operation is known as toning, tinting, or shading. This is a very direct method, and in the hands of an experienced color matcher it often proves to be the least expensive, provided that costs are figured over a short time. An inexperienced color matcher sometimes makes 7- or 8-component mixes instead of 4- or 5-. The greatest drawback of the method is that no permanent record in fundamental terms is kept of the colors produced by the various zigs and zags by which the final color match is achieved. The matcher sees the colors but does not measure them. This information is stored, if at all, in the memory of the color matcher and is a part of the experience whereby he gradually acquires the knack of estimating more accurately how much of which ingredient to add.

Adjustment of dye formulations for paper and textiles and of pigment formulations for plastics and ceramics is similar, each medium offering its own peculiar problems of preliminary testing of small samples.

Use of the Standard Coordinate System for Colorimetry. One obvious answer to the criticism of not keeping adequate records is to study the 4-component colorant-mixture system in advance. If many color matches are to be made with this particular set of four colorants,

such a study may pay off. The method is to develop the color gamut of the system by preparing specimens according to systematic variations in the proportions. These specimens may serve as reference material to aid the color matcher in his formulation, so that his first trial will, itself, be nearly correct; but impermanence of the specimens prevents such use over long periods of time. To provide permanent information each specimen is measured (by spectrophotometry or by suitable colorimetric methods) and its tristimulus values, X , Y , Z , and chromaticity coordinates, x , y , determined. A plot of these data results in a network such as Figs. 37 and 38, from which the proportions required to produce any color specified in CIE terms may be read. Then, when a customer brings in a specimen showing the color desired, this specimen is likewise measured and the required proportions read off.

This method has been applied to glasses by Forrest, Kreidl, and Pett (1948), to papers by Foote (1939), and to paints by Duncan (1949).

Use of the Kubelka-Munk Analysis. If the 4-component colorant mixture behaved in accord with the simple additive law for absorption and scattering coefficients given in equation 63, the network conversion graph could be derived from K and S for the 4 components without actually making the mixtures at all (Duncan, 1949). The reflectivity, R_∞ , would be found as a function of wavelength for each mixture by means of equation 43a or from Table *D* in the Appendix. Steele (1937) showed that the Kubelka-Munk analysis gives a correct account of the addition of filler to paper. Harrison (1937) showed that this analysis is applicable to the retention of dye by paper, and Nolan (1937) showed that it could be extended to predict the color of sheets of paper dyed with combinations of dyestuffs. Selling (1950) has worked out another form of graphical solution based on the Kubelka-Munk analysis that he found applies as a first approximation to dyed papers. All of this is on the plus side. Foote (1939), however, showed that the same amount of dyestuff produced different absorption coefficients, depending on the pulp dyed, that the scattering coefficient of the dyed sheet was independent of the amount of dye added only for moderate and low concentrations of dyestuff, and that the absorption coefficient of paper dyed with a mixture of dyes is less than the sum of the absorption coefficients of papers dyed with the dyes separately. Thus, even if the scattering coefficients of the undyed fibers and the absorption coefficients of the dyes added are known, it is not always possible to compute the absorption and scattering coefficients of the layer of dyed fibers. This holds even if

the absorption coefficients of the dyes are determined by measurement of specimens dyed with them at moderate concentrations. A separate study is required for each fiber-dye combination.

If a white pigment is tinted with small proportions of colored pigment insufficient to alter the scattering coefficient appreciably, or, if various small amounts (c_p , c_q , c_r , and so on) of dye are used to color a textile or paper, these amounts may be determined from a relation derived by combining equations 43 and 63:

$$(1 - R_\infty)^2/2R_\infty = \frac{K}{S} = \frac{(cK_1)_p + (cK_1)_q + (cK_1)_r}{(cS_1)_p + (cS_1)_q + (cS_1)_r} \dots \quad (64)$$

The scattering coefficients may then be set arbitrarily equal to unity (Duncan, 1949), the absorption coefficients $(K_1)_p$, $(K_1)_q$, $(K_1)_r$, and so on, evaluated relative to the scattering coefficients $(S_1)_p$, $(S_1)_q$, $(S_1)_r$, and so on; the denominator of the expression becomes unity, since the proportions c_p , c_q , c_r , and so on, sum to unity. The colorant technologist has then to evaluate equation 64 for several wavelengths and solve a system of simultaneous equations to evaluate the proportions, c_p , c_q , c_r , and so on. Park and Stearns (1944) found it advantageous in applying equation 64 to dyed textiles and papers to substitute for R_∞ the expression $R_\infty - \rho$, thus correcting for surface reflection.

For lightly pigmented thick layers having an approximately plane upper boundary, the internal reflection (see Figs. 97 and 98) must also be taken into account, the actual reflectance measured with the specimen in air, R' , being considerably different from that, R , described by the Kubelka-Munk analysis. The reflectance, R , corresponds to measurements made with the specimen immersed in a liquid of the same index of refraction, such as those carried out by Duntley (1942). Saunderson (1942) has worked out the connection between R and R' in a convenient form:

$$R' = \rho + (1 - \rho)(1 - \rho_i)R/(1 - \rho_i R) \quad (65)$$

where ρ_i is the internal reflectance for more or less completely diffuse flux incident on the under side of the top layer. Note the similarity to equation 60. If the flux incident internally is perfectly diffused and the top layer is optically plane, the value of ρ_i may be found from the index of refraction, n_2 , of the layer from Fig. 98 by taking n_1 for air equal to 1.00. Actual measurements by indirect means of molded plastics indicate values of internal reflectance, ρ_i , equal to about 0.4, somewhat lower than those corresponding to completely

diffused flux incident internally on the under side of the top plane surface. The discrepancy is ascribable to failure of the molded surface to be optically plane, and to a resulting preponderance of the incident flux toward near-perpendicular incidence.

Detailed instructions are given by Saunderson (1942) for the calculation of the spectral reflectivities of opaque plastics containing mixtures in known amounts of pigments of known absorption and scattering coefficients according to equation 64. He also indicates a trial-and-error method of computing the concentrations of the components that is often shorter than solving for them by simultaneous equations.

Use of Other Analyses. An important assumption used in the Kubelka-Munk analysis is that the light-scattering elements are small compared to the elementary layer of thickness, dX , referred to in equations 41a and 41b. We can easily imagine colorant layers in which this is not true. For example, if the pigment particles are so large as to be themselves opaque, the Kubelka-Munk analysis breaks down completely. The reflectance of the top layer of particles is all that counts, and the reflectivity of the layer can be computed by the laws of color mixture by averaging. Thus, the reflectivity of a layer composed of beads of black and white glass is related to the proportions of the black and white beads, f_b and f_w , and their reflectivities, R_b and R_w , simply as:

$$R_{\infty} = f_b R_b + f_w R_w$$

where $f_b + f_w = 1$. This formula also applies to black dots printed on white paper by the screened-plate printing process. For ideal black particles ($R_b = 0$) combined with ideal white particles ($R_w = 1$), we have still more simply:

$$R_{\infty} = f_w$$

How far additive mixture produced by mixing very coarse pigment particles departs from the Kubelka-Munk analysis is shown in Fig. 105 (Duncan, 1949). The straight line corresponds to additive mixtures of ideally black coarse particles with ideally white coarse particles ($R_{\infty} = f_w$). The solid curves correspond to mixtures in various proportions of ideally black and ideally white pigments, according to the Kubelka-Munk analysis (equation 42). An ideally black pigment corresponds to the scattering coefficient, $S_b = 0$; and an ideally white pigment corresponds to the absorption coefficient, $K_w = 0$. The black pigment supplies all the absorption coefficient

of the mixture, $f_b K_b$; the white pigment, all the scattering coefficient, $f_w S_w$. The value of K/S for the mixture is thus $f_b K_b / f_w S_w$ which, since $f_b + f_w = 1$, may be written: $(1 - f_w) K_b / f_w S_w$. Reflectivity,

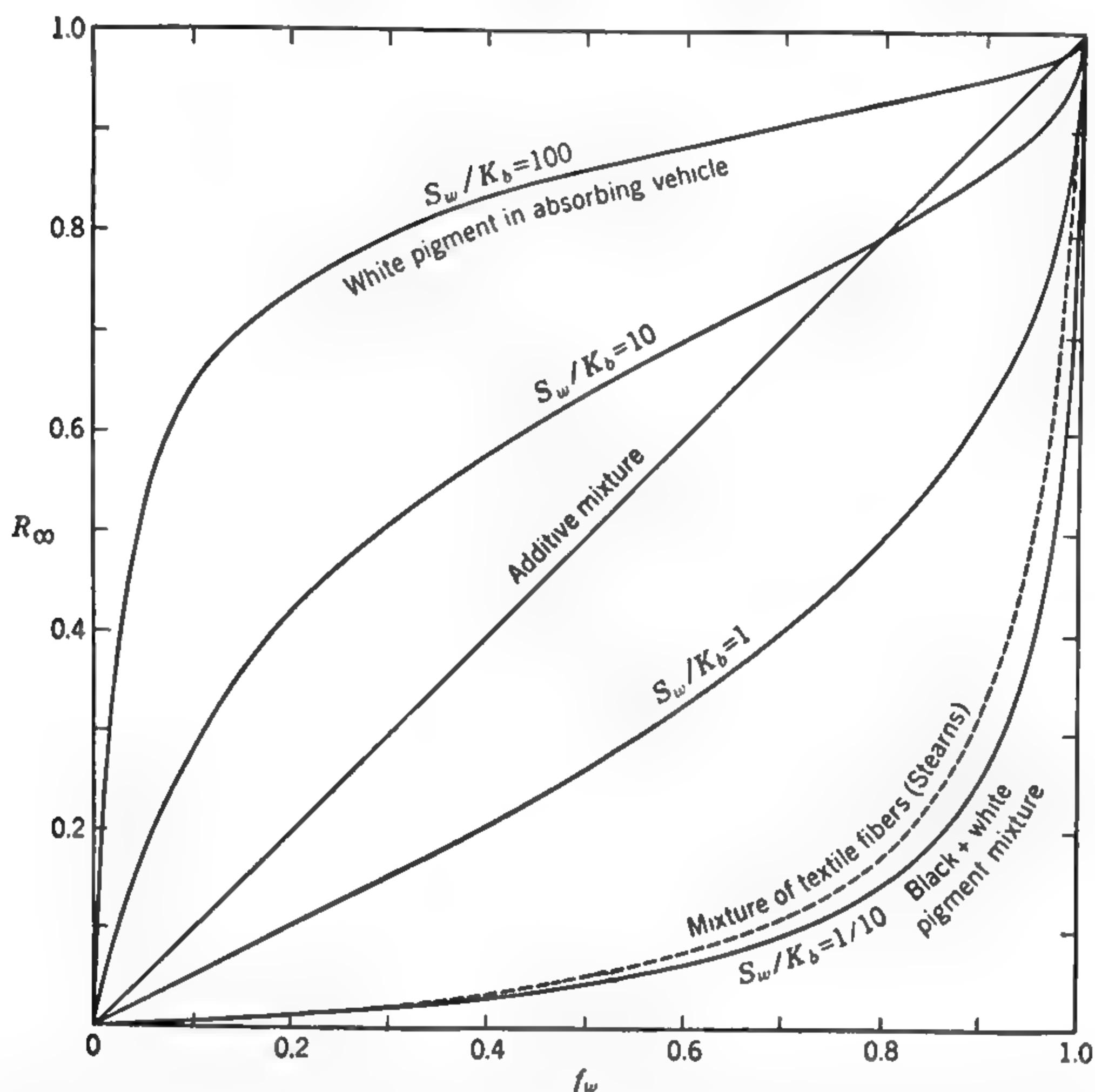


FIG. 105. Reflectivity, R_∞ , as a function of composition for mixtures of a non-light-scattering black colorant with a nonlight-absorbing white colorant. Colorants following the Kubelka-Munk analysis are shown by four curves corresponding to different ratios of absorption coefficient of the black to scattering coefficient of the white. The curves corresponding to mixture by averaging and to mixture of black and white textile fibers according to Stearns (1944) are also shown.

R_∞ , of this ideal paint may be found for each of these values of K/S from Table D in the Appendix. Figure 105 shows the dependence of reflectivity, R_∞ , on proportion of ideal white pigment, f_w , for four ratios of S_w/K_b . The top curve, $S_w/K_b = 100$, corresponds fairly well to reflectivity for short-wave light of a paint composed of linseed oil with white pigment, where K_b refers to the absorption coefficient of the oil. It will be noted that introduction of a small percentage

of white pigment causes the reflectivity of the layer to rise above 50 percent.

The bottom curve, $S_w/K_b = 0.1$, corresponds approximately to mixture of the usual black and white pigments. Note that 50 percent of the white pigment causes the reflectivity to rise to less than 5 percent, whereas only a small fraction of 1 percent of the black pigment has to be mixed with the white pigment to cause the reflectivity to drop from 100 to 95 percent. On this account the usual black pigments are said to have higher tinting strength than the usual white pigments.

For any 2-component colorant layer a plot of reflectivity, R_∞ , against f_w will indicate with some reliability whether the Kubelka-Munk analysis applies. If the curve for the 2-component layer is nearly coincident with one of the family of curves, of which four are shown as solid lines in Fig. 105, then in all probability the Kubelka-Munk analysis applies and formulation of the layer to match the color can proceed as already outlined.

It may not be immediately evident that the plot of reflectivity, R_∞ , against the proportion, f_w , of the white pigment in the black-white mixture as in Fig. 105 shows all the information required to predict also the colors of mixtures of chromatic pigments with white. It must be remembered that for any given type of mixture (whether it is coarse pigment particles following additive mixture or fine pigment particles following the Kubelka-Munk analysis) the relations shown in Fig. 105 hold separately for each wavelength. We take advantage of this fact by developing as an example the variation of color produced by adding various proportions, f_w , of white pigment such that $S_w = 101$ and $K_w = 0.5$, to an ideal reddish orange pigment such that the scattering coefficient $S_0 = 1$ and the absorption coefficient, K_0 , below wavelength $615 \text{ m}\mu$ has the value 1000.5 and above it has the value 4.0. The values of K/S for the mixture are found from equation 63 as:

$$K/S = (1000.5 - 1000f_w)/(100f_w + 1) \quad \lambda < 615 \text{ m}\mu$$

$$K'/S' = (4 - 3.5f_w)/(100f_w + 1) \quad \lambda > 615 \text{ m}\mu$$

Table 25 shows the values of K/S for these mixtures as a function of f_w and also the corresponding values of reflectivity, R_∞ and R_∞' , read from Table D in the Appendix, where the primed values refer to wavelengths greater than $615 \text{ m}\mu$. For each of these mixtures the reflectivities are constant independent of wavelength either at R_∞ for wavelengths less than $615 \text{ m}\mu$ or at R_∞' for wavelengths greater than

Table 25. Color of Mixtures of an Ideal Reddish Orange Pigment with an Ideal White Pigment

Proportion f_w of White Pigment in the Mixture	Characteristics of the Mixture							Munsell	
	K/S (Short Wave)	K'/S' (Long Wave)	R_∞ (Short Wave)	R_∞' (Long Wave)	Tristimulus Values			Value V_y	Adams $V_x - V_y$
					X	Y	Z		
0.00	1000.5	4.00	0.0005	0.101	0.0218	0.0093	0.0006	0.79	0.81
0.05	158.	0.637	.003	.341	.0745	.0323	0.0035	2.04	1.19
0.10	82.	0.332	.006	.452	.1004	.0448	0.0071	2.46	1.26
0.20	38.1	0.157	.013	.575	.1318	.0619	0.0154	2.91	1.30
0.40	14.6	0.0634	.032	.702	.1733	.0902	0.0378	3.51	1.25
0.60	0.57	0.0311	.066	.780	.2160	.1281	0.0779	4.12	1.12
0.80	2.48	0.0148	.147	.842	.2914	.2073	0.1736	5.11	0.86
0.90	1.10	0.00934	.253	.872	.3791	.3068	0.2988	6.05	0.64
0.95	0.526	0.00703	.373	.888	.4748	.4177	0.4405	6.91	0.45
0.97	0.311	0.00617	.463	.895	.5454	.5005	0.5468	7.46	0.34
0.98	0.207	0.00576	.531	.898	.5984	.5629	0.6271	7.84	0.27
0.99	0.105	0.00535	.635	.902	.6791	.6582	0.7499	8.37	0.17
0.995	0.0556	0.00515	.717	.904	.7426	.7332	0.8468	8.75	0.11
0.998	0.0248	0.00503	.801	.905	.8073	.8100	0.9460	9.11	0.06
0.999	0.0149	0.00500	.841	.905	.8381	.8465	0.9932	9.27	0.04
1.000	0.00496	0.00495	.905	.905	.8872	.9050	1.0681	9.52	0.00

615 mμ. The tristimulus values, *X*, *Y*, *Z*, of the mixtures may thus be easily computed from the tristimulus values of the spectrum colors (Table 6) by summation as follows:

$$X = 0.7685R_{\infty} + 0.2119R_{\infty}'$$
$$Y = 0.9131R_{\infty} + 0.0869R_{\infty}'$$
$$Z = 1.1810R_{\infty}$$

Table 25 also shows these tristimulus values. Each of these mixtures has a dominant wavelength of 631 mμ relative to daylight (source C). To assess the meaning of these tristimulus values in terms of perceptibility we may make use of the Adams chromatic-value function, *V_x - V_v*, a correlate of redness with uniform spacing (see equation 34); and for a correlate of lightness with uniform spacing we may use the Munsell value function, *V_v*, defined by equation 20b and evaluated in Table B in the Appendix. Table 25 also shows *V_x - V_v* and *V_v* for each of these mixtures read from Tables A and B. Figure 106 shows by the solid line on a plot of *V_v* against *V_x - V_v* how additions of white pigment up to 20 percent to the reddish orange pigment first make it both lighter and more saturated. Further additions make it lighter and less saturated. The dotted lines show similarly the result of mixture by averaging. Note how much less

saturated the colors produced by mixture by averaging (as in screened-plate printing) are if the pure reddish orange pigment is used as the chromatic ink. But, if the color is developed by adding 40 percent

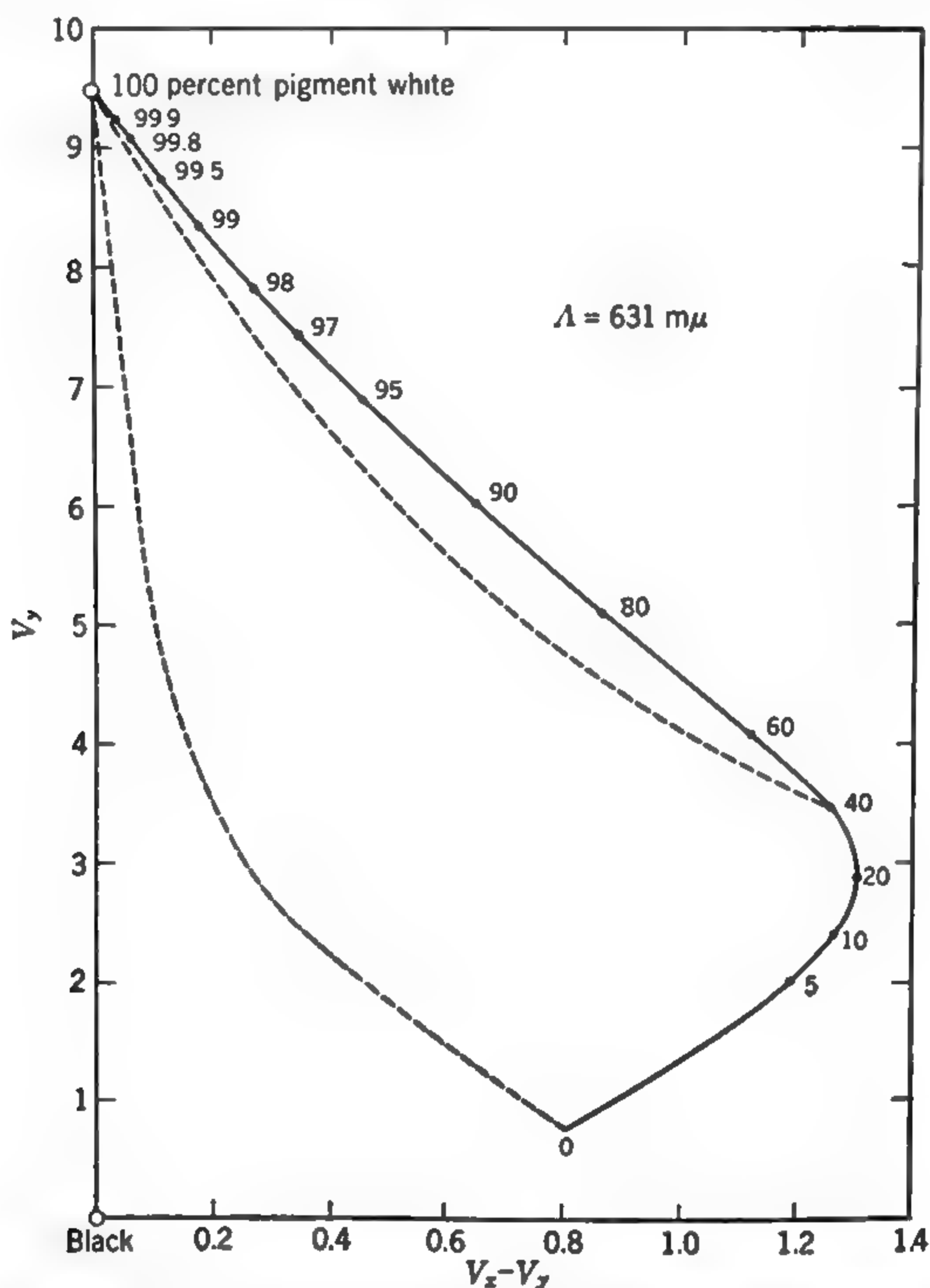


FIG. 106. Extension of (a hypothetical) paint with white paint shown on a plot of Munsell value, V_y , against Adams chromatic-value coordinate, $V_x - V_y$ (measure of redness). Note that additions of the white paint at first cause the mixture to gain in redness as well as lightness. This is called developing the color. If this is not done, the color gamut producible by half-tone screen printing is severely restricted (see dotted lines corresponding to mixture by averaging) and the colorant is wastefully used.

white pigment to it first, then additive mixture produces colors much more nearly the equal in vividness of those produced by pigment mixtures. Figure 106 thus explains how it is possible to extend the gamut of colors producible by screened-plate printing by using three starting inks, as was done by Villalobos (see Fig. 57), instead of only

one. The use of colorants so as to get the most good out of them is the concern of the colorant-layer industries. For many years these technics were developed on an empirical basis; but since 1930 colorant theory has made great strides and is now in a position to give some practical guidance.

Figure 105 gives a curve characterizing still another type of colorant mixture. The retention of dye by various textile fibers is complicated by so many variables that whenever a textile is made up of a mixture of fibers (for example, as in some Army uniforms) it is customary practice to dye large batches of fiber to colors surrounding the color to be matched, and then to make up the final match by mixtures of the fibers in suitable proportions. These mixtures follow the Kubelka-Munk analysis only in a general way, departing from it somewhat toward additive mixture. Stearns (1944) has developed empirically a function of reflectivity that is additive for mixtures of textile fibers, just as the function $(1 - R_\infty)^2/2R_\infty$ is additive for mixtures following the Kubelka-Munk analysis as in equation 64. The function of reflectivity, $F(R_\infty)$, is:

$$F(R_\infty) = \frac{1 - R_\infty}{m(R_\infty - 0.01) + 0.01} \quad (66)$$

where m = approximately 0.15 for wool mixtures. If we have a mixture of ideally black wool fibers of reflectivity $R_b = 0$ with ideally white wool fibers of reflectivity $R_w = 1$ in proportions f_b and f_w , the additivity of the function $F(R_\infty)$ means that:

$$F(R_\infty) = f_b F(R_b) + f_w F(R_w)$$

whence, by setting $m = 0.15$ in equation 66, and noting that $f_b = 1 - f_w$, we obtain:

$$R_\infty = f_w / (18.65 - 17.65f_w) \quad (66a)$$

This is the equation of an hyperbola on the plot of R_∞ against f_w and is shown as a dashed line on Fig. 105. It indicates the dependence of reflectivity, R_∞ , of a mixture of black wool fibers with undyed (white) wool fibers, according to the empirical formula 66, found by Stearns (1951, b) to conform to his measurement of such mixtures. For mixtures containing less than 30 percent of undyed fibers, this empirical relation agrees closely with the Kubelka-Munk analysis for $S_w/K_b = 1/10$. But for higher proportions of undyed fibers the reflectivity is more than is indicated by the Kubelka-Munk analysis.

The method of formulation recommended by Park and Stearns (1944) regardless of the type of mixture is as follows:

(a) Measure for the visible spectrum the reflectivity, R_∞ , of the specimen to be color-matched, and compute $F(R_\infty)$ from equation 66 as a function of wavelength.

(b) Measure also the spectral reflectivities, R_p, R_q, R_r, R_s , of the four constituents to be mixed to produce the match, and compute $F(R_p), F(R_q), F(R_r), F(R_s)$ as functions of wavelength.

(c) Compute summations of these functions, $F(R_\infty), F(R_p), F(R_q), F(R_r), F(R_s)$, for three sets of selected ordinates, one for X , one for Y , and one for Z ; see Table 11.

(d) Solve for the proportions p, q, r, s from the simultaneous equations:

$$p\Sigma_x F(R_p) + q\Sigma_x F(R_q) + r\Sigma_x F(R_r) + s\Sigma_x F(R_s) = \Sigma_x F(R_\infty)$$

$$p\Sigma_y F(R_p) + q\Sigma_y F(R_q) + r\Sigma_y F(R_r) + s\Sigma_y F(R_s) = \Sigma_y F(R_\infty)$$

$$p\Sigma_z F(R_p) + q\Sigma_z F(R_q) + r\Sigma_z F(R_r) + s\Sigma_z F(R_s) = \Sigma_z F(R_\infty)$$

$$p + q + r + s = 1$$

where the indicated summations are those found in step c. This gives a fairly close preliminary formulation.

(e) Make up a mixture of the four batches of dyed fiber in the proportions, p, q, r, s , found in step d, measure its reflectivity, R_∞ , compute its tristimulus values, X, Y, Z , for the illuminant under which the mixture is to be used, such as standard source A, B, or C, and compute the color difference between the specimen to be matched and the preliminary mixture by a color-difference formula such as equations 30 to 35.

(f) If this color difference is too great to be tolerated, the corrections, $\Delta_p, \Delta_q, \Delta_r, \Delta_s$, to the proportions found by the preliminary solution of step d may be approximately evaluated by a second use of the function $F(R_\infty)$, together with its rate of change with reflectivity, R_∞ . See the paper by Park and Stearns (1944) for the details.

The adjustment of the proportions, p, q, r, s , is thus made by successive approximation much as the color matcher adds an estimated amount of the ingredient judged by visual inspection to be lacking. There are several important differences, however. The preliminary formulation found by step d is already a good approximation. A complete spectrophotometric record is kept of all the experimental mixtures leading to the final match. And each step is guided systematically.

Equation 66 found empirically has some generality. It applies to mixtures of fibers other than wool by adjustment of the constant m (Stearns, 1944).

Summary. The systematic formulation of colorant layers is based on the optical laws followed by the particular kind of layer. We are indebted to Park and Stearns (1944) for pointing out explicitly that such formulation must be based upon the discovery of a function, F , of the measurable quantities, transmittance or reflectivity, that is additive in terms of the proportions of the colorants used in the layer. That is, the function F must be such that for every wavelength in the visible spectrum:

$$F = pF_p + qF_q + rF_r \dots$$

For transparent media this is the function of internal transmittance, T_i , suggested by equation 62a:

$$F(T_i) = \log (1/T_i)$$

For mixtures of completely opaque pigment particles this is a function of reflectivity, R_∞ , and is simply:

$$F(R_\infty) = R_\infty$$

For dyed and pigmented layers of paper or textiles, the Kubelka-Munk analysis applies and the function is:

$$F(R_\infty) = (1 - R_\infty)^2/2R_\infty$$

as given by equation 64, in which it may be necessary to subtract a small constant from R_∞ to correspond to external reflection. For pigmented plastics the same function applies except that a correction (Saunderson, 1942) must be applied for internal reflection from the under side of the top surface (see equation 65). For mixtures of large nonopaque elements such as dyed fibers the function is that given as equation 66:

$$F(R_\infty) = \frac{1 - R_\infty}{m(R_\infty - 0.01) + 0.01}$$

where m is a constant to be determined experimentally for each kind of fiber.

If the colorants of the specimen have been identified, a perfect nonmetameric match can be formulated. The solution for the proportions may then be found by means of a set of simultaneous equations referring to but three wavelengths.

If the match is to be made from constituents other than those used in the specimen submitted, a metameric color match is all that can

be expected. That is, the formulated specimen may match the submitted specimen perfectly under one kind of light but will differ more or less from it under other kinds. The solution for the proportions is then most directly found by considering, not a few isolated wavelengths, but averages for three sets, each average corresponding to one of the distribution functions, \bar{x}_λ , \bar{y}_λ , \bar{z}_λ , of the CIE standard observer.

Advances in the formulation of colorant layers since 1930 have been based upon intelligent application of the spectrophotometer.

Appendix

Table A

V_x	X	V_x	X	V_x	X	V_x	X	V_x	X	V_x	X	V_x	X	V_x	X	V_x	X
10.00	100.56	76.91	57.77	6.99	42.08	5.99	29.35	4.99	19.29	3.99	11.701	2.99	6.383	1.99	3.039	0.99	1.173
9.99	100.30	76.70	57.59	8	41.93	8	29.24	8	19.21	8	11.637	8	6.341	8	3.015	8	1.159
8	100.04	76.49	57.42	7	41.79	7	29.13	7	19.12	7	11.574	7	6.299	7	2.990	7	1.145
7	99.79	76.28	57.25	6	41.65	6	29.01	6	19.03	6	11.510	6	6.257	6	2.966	6	1.131
6	99.53	76.07	57.08	5	41.51	5	28.90	5	18.94	5	11.446	5	6.215	5	2.941	5	1.119
5	99.27	75.86	56.91	6.94	41.37	5.94	28.79	4.94	18.85	3.94	11.383	2.94	6.173	1.94	2.917	0.94	1.106
9.94	99.01	75.65	56.74	3	41.24	3	28.68	3	18.77	3	11.321	3	6.131	3	2.892	3	1.092
3	98.76	75.44	56.56	2	41.10	2	28.56	2	18.69	2	11.258	2	6.090	2	2.868	2	1.079
2	98.50	75.24	56.39	1	40.95	1	28.45	1	18.60	1	11.195	1	6.049	1	2.844	1	1.066
1	98.25	75.03	56.22	0	40.81	0	28.34	0	18.51	0	11.133	0	6.008	0	2.821	0	1.053
0	97.99	74.82	56.05	6.89	40.68	5.89	28.23	4.89	18.42	3.89	11.071	2.89	5.967	1.89	2.797	0.89	1.039
9.89	97.74	74.62	55.88	8	40.54	8	28.12	8	18.34	8	11.009	8	5.926	8	2.774	8	1.026
8	97.49	74.41	55.72	7	40.40	7	28.01	7	18.26	7	10.947	7	5.885	7	2.750	7	1.014
7	97.24	74.21	55.55	6	40.27	6	27.89	6	18.17	6	10.886	6	5.845	6	2.727	6	1.001
6	96.98	74.00	55.38	5	40.13	5	27.78	5	18.08	5	10.826	5	5.805	5	2.704	5	0.988
5	96.73	73.80	55.21	6.84	39.99	5.84	27.68	4.84	18.00	3.84	10.765	2.84	5.766	1.84	2.681	0.84	0.976
9.84	96.48	73.59	55.04	3	39.85	3	27.57	3	17.91	3	10.704	3	5.726	3	2.658	3	0.963
3	96.23	73.38	54.87	2	39.72	2	27.46	2	17.83	2	10.643	2	5.686	2	2.635	2	0.950
2	95.98	73.18	54.71	1	39.58	1	27.35	1	17.75	1	10.583	1	5.647	1	2.613	1	0.937
1	95.73	72.98	54.54	0	39.44	0	27.24	0	17.67	0	10.524	0	5.608	0	2.590	0	0.925
0	95.48	72.78	54.37	6.79	39.30	5.79	27.13	4.79	17.58	3.79	10.464	2.79	5.569	1.79	2.568	0.79	0.913
9.79	95.23	72.57	54.21	8	39.17	8	27.02	8	17.50	8	10.404	8	5.530	8	2.547	8	0.900
8	94.98	72.37	54.04	7	39.04	7	26.91	7	17.41	7	10.344	7	5.492	7	2.525	7	0.888
7	94.74	72.17	53.87	6	38.90	6	26.80	6	17.33	6	10.285	6	5.454	6	2.503	6	0.876
6	94.49	71.96	53.71	5	38.77	5	26.70	5	17.25	5	10.227	5	5.416	5	2.481	5	0.864
5	94.25	71.76	53.55	6.74	38.63	5.74	26.59	4.74	17.17	3.74	10.168	2.74	5.378	1.74	2.460	0.74	0.851
9.74	94.00	71.56	53.38	3	38.50	3	26.49	3	17.09	3	10.109	3	5.340	3	2.438	3	0.839
3	93.76	71.36	53.22	2	38.36	2	26.38	2	17.00	2	10.051	2	5.302	2	2.417	2	0.827
2	93.51	71.16	53.05	1	38.23	1	26.27	1	16.92	1	9.993	1	5.265	1	2.395	1	0.815
1	93.27	70.96	52.89	0	38.10	0	26.17	0	16.84	0	9.935	0	5.228	0	2.375	0	0.803
0	93.02	70.77	52.73	6.69	37.96	5.69	26.06	4.69	16.76	3.69	9.878	2.69	5.191	1.69	2.354	0.69	0.791
9.69	92.78	70.57	52.56	8	37.83	8	25.96	8	16.68	8	9.821	8	5.154	8	2.333	8	0.779
8	92.53	70.37	52.40	7	37.70	7	25.85	7	16.60	7	9.764	7	5.118	7	2.313	7	0.768
7	92.29	70.17	52.24	6	37.57	6	25.75	6	16.52	6	9.707	6	5.081	6	2.292	6	0.756
6	92.05	69.97	52.08	5	37.43	5	25.64	5	16.44	5	9.650	5	5.045	5	2.272	5	0.744
5	91.81	69.78	51.91	6.64	37.30	5.64	25.54	4.64	16.36	3.64	9.593	2.64	5.009	1.64	2.251	0.64	0.732
9.64	91.57	69.58	51.75	3	37.17	3	25.43	3	16.28	3	9.537	3	4.973	3	2.231	3	0.721
3	91.33	69.38	51.59	2	37.04	2	25.33	2	16.21	2	9.482	2	4.937	2	2.212	2	0.709
2	91.09	69.19	51.43	1	36.91	1	25.23	1	16.13	1	9.426	1	4.902	1	2.192	1	0.697
1	90.85	68.99	51.27	0	36.78	0	25.12	0	16.05	0	9.370	0	4.867	0	2.173	0	0.685
0	90.61	68.80	51.11	6.59	36.65	5.59	25.02	4.59	15.97	3.59	9.315	2.59	4.831	1.59	2.153	0.59	0.674
9.59	90.37	68.60	50.95	8	36.52	8	24.91	8	15.89	8	9.260	8	4.796	8	2.133	8	0.662
8	90.13	68.41	50.79	7	36.39	7	24.81	7	15.81	7	9.205	7	4.762	7	2.114	7	0.650
7	89.89	68.22	50.63	6	36.27	6	24.71	6	15.74	6	9.150	6	4.728	6	2.094	6	0.638
6	89.66	68.02	50.47	5	36.14	5	24.61	5	15.66	5	9.095	5	4.693	5	2.075	5	0.627

9.54	89.19	8.54	67.83	7.54	50.31	6.54	36.00	5.54	24.51	4.54	15.58	3.54	9.041	2.54	4.659	1.54	2.056	0.54	0.616
3	88.95	3	67.64	3	50.16	2	35.87	3	24.40	3	15.50	3	8.987	3	4.624	3	2.037	3	0.605
2	88.72	2	67.44	2	49.84	1	35.75	2	24.30	2	15.43	2	8.933	2	4.590	2	2.019	2	0.593
1	88.48	1	67.25	1	49.69	0	35.62	1	24.20	1	15.35	1	8.880	1	4.557	1	2.000	1	0.581
0	88.25	0	67.06	0	49.53	0	35.49	0	24.10	0	15.26	0	8.827	0	4.524	0	1.981	0	0.570
9.49	88.01	■.49	66.87	7.49	49.37	6.49	35.36	5.49	24.00	4.49	15.18	3.49	8.774	2.49	4.490	1.49	1.963	0.49	0.559
8	87.78	8	66.68	8	49.22	8	35.24	8	23.90	8	15.11	8	8.721	8	4.457	8	1.944	8	0.548
7	87.55	7	66.48	7	49.06	7	35.11	7	23.80	7	15.04	7	8.668	7	4.424	7	1.926	7	0.536
6	87.32	6	66.29	6	48.90	6	34.98	6	23.70	6	14.96	6	8.615	6	4.391	6	1.909	6	0.525
5	87.08	5	66.11	5	48.75	5	34.86	5	23.60	5	14.88	5	8.563	5	4.359	5	1.891	5	0.514
9.44	86.85	8.44	65.92	7.44	48.59	6.44	34.74	5.44	23.50	4.44	14.81	3.44	8.511	2.44	4.327	1.44	1.873	0.44	0.503
3	86.62	3	65.73	3	48.44	3	34.61	3	23.40	3	14.74	3	8.459	3	4.294	3	1.855	3	0.491
2	86.39	2	65.54	2	48.29	2	34.48	2	23.30	2	14.67	2	8.407	2	4.261	2	1.837	2	0.479
1	86.16	1	65.35	1	48.13	1	34.35	1	23.21	1	14.59	1	8.356	1	4.229	1	1.820	1	0.468
0	85.93	0	65.16	0	47.97	0	34.23	0	23.11	0	14.52	0	8.305	0	4.198	0	1.802	0	0.458
9.39	85.70	8.39	64.97	7.39	47.82	6.39	34.11	5.39	23.01	4.39	14.44	3.39	8.254	2.39	4.167	1.39	1.785	0.39	0.446
8	85.47	8	64.79	8	47.67	8	33.98	8	22.91	8	14.37	8	8.203	8	4.135	8	1.768	8	0.435
7	85.25	7	64.60	7	47.52	7	33.86	7	22.81	7	14.29	7	8.153	7	4.104	7	1.751	7	0.424
6	85.02	6	64.41	6	47.36	6	33.74	6	22.72	6	14.22	6	8.102	6	4.073	6	1.734	6	0.413
5	84.79	5	64.23	5	47.21	5	33.61	5	22.62	5	14.15	5	8.052	5	4.042	5	1.718	5	0.401
9.34	84.56	■.34	64.04	7.34	47.06	6.34	33.49	5.34	22.52	4.34	14.08	3.34	8.002	2.34	4.011	1.34	1.701	0.34	0.390
3	84.33	3	63.85	3	46.91	3	33.36	3	22.42	3	14.00	3	7.952	3	3.980	3	1.684	3	0.378
2	84.10	2	63.67	2	46.76	2	33.24	2	22.33	2	13.93	2	7.902	2	3.950	2	1.668	2	0.367
1	83.88	1	63.49	1	46.61	1	33.12	1	22.24	1	13.86	1	7.853	1	3.920	1	1.651	1	0.356
0	83.66	0	63.31	0	46.45	0	33.00	0	22.14	0	13.79	0	7.804	0	3.890	0	1.634	0	0.345
9.29	83.44	8.29	63.12	7.29	46.30	6.29	32.88	5.29	22.04	4.29	13.72	3.29	7.755	2.29	3.860	1.29	1.618	0.29	0.334
8	83.22	8	62.93	8	46.15	8	32.76	8	21.94	8	13.65	8	7.706	8	3.830	8	1.602	8	0.323
7	82.99	7	62.75	7	46.00	7	32.64	7	21.85	7	13.58	7	7.658	7	3.801	7	1.586	7	0.312
6	82.77	6	62.57	6	45.85	6	32.51	6	21.76	6	13.51	6	7.610	6	3.772	6	1.570	6	0.300
5	82.54	5	62.38	5	45.71	5	32.39	5	21.66	5	13.43	5	7.562	5	3.742	5	1.554	5	0.289
9.24	82.32	8.24	62.20	7.24	45.56	6.24	32.28	5.24	21.57	4.24	13.36	3.24	7.514	2.24	3.713	1.24	1.538	0.24	0.277
3	82.10	3	62.02	3	45.41	3	32.16	3	21.47	3	13.29	3	7.466	3	3.684	3	1.523	3	0.266
2	81.88	2	61.84	2	45.27	2	32.03	2	21.38	2	13.23	2	7.419	2	3.656	2	1.507	2	0.255
1	81.66	1	61.66	1	45.12	1	31.91	1	21.29	1	13.16	1	7.372	1	3.628	1	1.491	1	0.243
0	81.44	0	61.48	0	45.00	0	31.79	0	21.20	0	13.09	0	7.325	0	3.599	0	1.476	0	0.232
9.19	81.22	8.19	61.30	7.19	44.82	6.19	31.68	5.19	21.10	4.19	13.02	3.19	7.278	2.19	3.571	1.19	1.461	0.19	0.221
8	81.00	8	61.12	8	44.68	8	31.56	8	21.01	8	12.95	8	7.231	8	3.542	8	1.446	8	0.210
7	80.78	7	60.94	7	44.53	7	31.44	7	20.91	7	12.88	7	7.184	7	3.514	7	1.430	7	0.198
6	80.56	6	60.76	6	44.38	6	31.32	6	20.82	6	12.81	6	7.138	6	3.487	6	1.416	6	0.187
5	80.34	5	60.58	5	44.24	5	31.21	5	20.73	5	12.75	5	7.092	5	3.460	5	1.401	5	0.175
9.14	80.12	8.14	60.40	7.14	44.09	6.14	31.09	5.14	20.64	4.14	12.68	3.14	7.046	2.14	3.432	1.14	1.385	0.14	0.164
3	79.90	3	60.23	3	43.94	3	30.97	3	20.55	3	12.61	3	7.000	3	3.405	3	1.370	3	0.152
2	79.69	2	60.05	2	43.79	2	30.85	2	20.45	2	12.55	2	6.955	2	3.378	2	1.356	2	0.140
1	79.47	1	59.87	1	43.65	1	30.74	1	20.36	1	12.48	1	6.910	1	3.351	1	1.341	1	0.128
0	79.26	0	59.69	0	43.51	0	30.62	0	20.27	0	12.41	0	6.865	0	3.325	0	1.327	0	0.117
9.09	79.04	8.09	59.51	7.09	43.36	6.09	30.50	5.09	20.18	4.09	12.34	3.09	6.820	2.09	3.298	1.09	1.313	0.09	0.106
8	78.82	8	59.34	8	43.22	8	30.38	8	20.09	8	12.28	8	6.776	8	3.272	8	1.298	8	0.094
7	78.61	7	59.17	7	43.08	7	30.27	7	20.00	7	12.22	7	6.731	7	3.245	7	1.284	7	0.082
6	78.40	6	58.99	6	42.93	6	30.15	6	19.91	6	12.15	6	6.687	6	3.219	6	1.270	6	0.071
5	78.19	5	58.81	5	42.79	5	30.04	5	19.82	5	12.08	5	6.643	5	3.192	5	1.256	5	0.060
9.04	77.97	8.04	58.64	7.04	42.65	6.04	29.92	5.04	19.74	4.04	12.02	3.04	6.599	2.04	3.166	1.04	1.242	0.04	0.048
3	77.76	3	58.46	3	42.50	3	29.80	3	19.65	3	11.95	3	6.556	3	3.141	3	1.228	3	0.035
2	77.55	2	58.29	2	42.36	2	29.69	2	19.56	2	11.88	2	6.513	2	3.116	2	1.214	2	0.023
1	77.33	1	58.12	1	42.22	1	29.58	1	19.47	1	11.82	1	6.470	1	3.090	1	1.200	1	0.012
0	77.12	0	57.94	0	42.07	0	29.46	0	19.38	0	11.76	0	6.427	0	3.064	0	1.186	0	0.000

Table B

V _Y	Y	V _Y	Y	V _Y	Y	V _Y	Y	V _Y	Y	V _Y	Y	V _Y	Y	V _Y	Y	V _Y	Y
10.00	102.57	8.99	78.45	7.99	58.92	6.99	42.92	5.99	29.94	4.99	19.68	3.99	11.935	2.99	6.511	1.99	3.100
9.99	102.30	8	78.23	8	58.74	8	42.77	8	29.82	8	19.59	8	11.870	8	6.468	8	3.075
9.99	102.04	7	78.02	7	58.57	7	42.63	7	29.71	7	19.50	7	11.805	7	6.425	7	3.050
9.99	101.78	6	77.80	6	58.39	6	42.49	6	29.59	6	19.41	6	11.740	6	6.382	6	3.025
9.99	101.52	5	77.59	5	58.22	5	42.34	5	29.48	5	19.32	5	11.675	5	6.339	5	3.000
9.94	101.25	5	77.38	7.94	58.04	6.94	42.20	5.94	29.36	4.94	19.23	3.94	11.611	2.94	6.296	1.94	2.975
9.94	100.99	3	77.16	3	57.87	3	42.06	3	29.25	3	19.14	3	11.547	3	6.254	3	2.950
9.94	100.73	2	76.95	2	57.69	2	41.92	2	29.13	2	19.06	2	11.483	2	6.212	2	2.925
9.94	100.47	1	76.74	1	57.52	1	41.77	1	29.02	1	18.97	1	11.419	1	6.170	1	2.901
9.94	100.21	0	76.53	0	57.35	0	41.63	0	28.90	0	18.88	0	11.356	0	6.128	0	2.877
9.89	99.95	0	76.32	7.89	57.17	6.89	41.49	5.89	28.79	4.89	18.79	3.89	11.292	2.89	6.086	1.89	2.853
9.89	99.69	8	76.11	8	57.00	8	41.35	8	28.68	8	18.70	8	11.229	8	6.045	8	2.829
9.89	99.44	7	75.90	7	56.83	7	41.21	7	28.57	7	18.62	7	11.167	7	6.003	7	2.805
9.89	99.18	6	75.69	6	56.66	6	41.07	6	28.45	6	18.53	6	11.104	6	5.962	6	2.781
9.89	98.92	5	75.48	5	56.48	5	40.93	5	28.34	5	18.44	5	11.042	5	5.921	5	2.758
9.84	98.66	5	75.27	7.84	56.31	6.84	40.79	5.84	28.23	4.84	18.36	3.84	10.980	2.84	5.881	1.84	2.735
9.84	98.41	3	75.06	3	56.14	3	40.65	3	28.12	3	18.27	3	10.918	3	5.841	3	2.712
9.84	98.15	2	74.85	2	55.97	2	40.51	2	28.01	2	18.19	2	10.856	2	5.800	2	2.688
9.84	97.90	1	74.64	1	55.80	1	40.37	1	27.90	1	18.10	1	10.795	1	5.760	1	2.665
9.84	97.64	0	74.44	0	55.63	0	40.23	0	27.78	0	18.02	0	10.734	0	5.720	0	2.642
9.79	97.39	0	74.23	7.79	55.46	6.79	40.09	5.79	27.67	4.79	17.93	3.79	10.673	2.79	5.680	1.79	2.620
9.79	97.14	8	74.02	8	55.29	8	39.95	8	27.56	8	17.85	8	10.612	8	5.641	8	2.598
9.79	96.88	7	73.82	7	55.12	7	39.82	7	27.45	7	17.76	7	10.551	7	5.602	7	2.575
9.79	96.63	6	73.61	6	54.95	6	39.68	6	27.34	6	17.68	6	10.491	6	5.563	6	2.553
9.74	96.38	5	73.40	5	54.78	5	39.54	5	27.23	5	17.60	5	10.431	5	5.524	5	2.531
9.74	96.13	5	73.20	7.74	54.62	6.74	39.40	5.74	27.12	4.74	17.51	3.74	10.371	2.74	5.485	1.74	2.509
9.74	95.88	3	72.99	3	54.45	3	39.27	3	27.02	3	17.43	3	10.311	3	5.447	3	2.487
9.74	95.63	2	72.79	2	54.28	2	39.13	2	26.91	2	17.34	2	10.252	2	5.408	2	2.465
9.74	95.38	1	72.59	1	54.11	1	39.00	1	26.80	1	17.26	1	10.193	1	5.370	1	2.443
9.74	95.13	0	72.38	0	53.94	0	38.86	0	26.69	0	17.18	0	10.134	0	5.332	0	2.422
9.69	94.88	0	72.18	7.69	53.78	6.69	38.72	5.69	26.58	4.69	17.10	3.69	10.075	2.69	5.295	1.69	2.401
9.69	94.63	8	71.98	8	53.61	8	38.59	8	26.48	8	17.02	8	10.017	8	5.257	8	2.380
9.69	94.38	7	71.78	7	53.45	7	38.45	7	26.37	7	16.93	7	9.959	7	5.220	7	2.359
9.69	94.14	6	71.57	6	53.28	6	38.32	6	26.26	6	16.85	6	9.901	6	5.183	6	2.338
9.69	93.89	5	71.37	5	53.12	5	38.18	5	26.15	5	16.77	5	9.843	5	5.146	5	2.317
9.64	93.64	5	71.17	7.64	52.95	6.64	38.05	5.64	26.05	4.64	16.69	3.64	9.785	2.64	5.109	1.64	2.296
9.64	93.40	3	70.97	3	52.79	3	37.92	3	25.94	3	16.61	3	9.728	3	5.072	3	2.276
9.64	93.15	2	70.77	2	52.62	2	37.78	2	25.84	2	16.53	2	9.671	2	5.036	2	2.256
9.64	92.91	1	70.57	1	52.46	1	37.65	1	25.73	1	16.45	1	9.614	1	5.000	1	2.236
9.64	92.66	0	70.37	0	52.30	0	37.52	0	25.62	0	16.37	0	9.557	0	4.964	0	2.216
9.59	92.42	0	70.17	7.59	52.13	6.59	37.38	5.59	25.52	4.59	16.29	3.59	9.501	2.59	4.928	1.59	2.196
9.59	92.18	8	69.97	8	51.97	8	37.25	8	25.41	8	16.21	8	9.445	8	4.892	8	2.176
9.59	91.93	7	69.78	7	51.81	7	37.12	7	25.31	7	16.13	7	9.389	7	4.857	7	2.156
9.59	91.69	6	69.58	6	51.64	6	36.99	6	25.20	6	16.05	6	9.333	6	4.822	6	2.136
9.59	91.45	5	69.38	5	51.48	5	36.86	5	25.10	5	15.97	5	9.277	5	4.787	5	2.116

9.54	3	2	1	0	9.49	8	7	6	5	9.44	3	2	1	0	9.39	8	7	6	5	9.34	3	2	1	0	9.29	8	7	6	5	9.24	3	2	1	0	9.19	8	7	6	5	9.14	3	2	1	0	9.09	8	7	6	5	9.04	3	2	1	0
90.97	90.73	90.49	90.25	90.01	89.77	89.53	89.30	89.06	88.82	88.59	88.35	88.12	87.88	87.65	87.41	87.18	86.95	86.72	86.48	86.25	86.02	85.79	85.56	85.33	85.10	84.88	84.65	84.42	84.19	83.97	83.74	83.52	83.29	83.07	82.84	82.62	82.39	82.17	81.95	81.73	81.50	81.28	81.06	80.84	80.62	80.40	80.18	79.97	79.75	79.53	79.31	79.10	78.88	78.66
69.18	68.99	68.79	68.59	68.40	68.20	68.01	67.81	67.62	67.43	67.23	67.04	66.85	66.66	66.46	66.27	66.08	65.89	65.70	65.51	65.32	65.13	64.94	64.76	64.57	64.38	64.19	64.01	63.82	63.63	63.45	63.26	63.08	62.89	62.71	62.52	62.34	62.16	61.98	61.79	61.61	61.43	61.25	61.07	60.88	60.70	60.52	60.35	60.17	59.99	59.81	59.63	59.45	59.28	59.10
7.54	3	2	1	0	7.49	8	7	6	5	7.44	3	2	1	0	7.39	8	7	6	5	7.34	3	2	1	0	7.29	8	7	6	5	7.24	3	2	1	0	7.19	8	7	6	5	7.14	3	2	1	0	7.09	8	7	6	5	7.04	3	2	1	0
51.32	51.16	51.00	50.84	50.68	50.52	50.36	50.20	50.04	49.88	49.72	49.56	49.41	49.25	49.09	48.93	48.78	48.62	48.47	48.31	48.16	48.00	47.85	47.69	47.54	47.38	47.23	47.08	46.92	46.77	46.62	46.47	46.32	46.17	46.02	45.87	45.72	45.57	45.42	45.27	45.12	44.97	44.82	44.67	44.52	44.38	44.23	44.08	43.94	43.79	43.64	43.50	43.35	43.21	43.06
6.54	3	2	1	0	6.49	8	7	6	5	6.44	3	2	1	0	6.39	8	7	6	5	6.34	3	2	1	0	6.29	8	7	6	5	6.24	3	2	1	0	6.19	8	7	6	5	6.14	3	2	1	0	6.09	8	7	6	5	6.04	3	2	1	0
36.72	36.59	36.46	36.33	36.20	36.07	35.94	35.81	35.68	35.56	35.43	35.30	35.17	35.04	34.92	34.79	34.66	34.54	34.41	34.28	34.16	34.03	33.91	33.78	33.66	33.54	33.41	33.29	33.16	33.04	32.92	32.80	32.67	32.55	32.43	32.31	32.19	32.07	31.95	31.83	31.71	31.59	31.47	31.35	31.23	31.11	30.99	30.87	30.75	30.64	30.52	30.40	30.28	30.17	30.05
5.54	3	2	1	0	5.49	8	7	6	5	5.44	3	2	1	0	5.39	8	7	6	5	5.34	3	2	1	0	5.29	8	7	6	5	5.24	3	2	1	0	5.19	8	7	6	5	5.14	3	2	1	0	5.09	8	7	6	5	5.04	3	2	1	0
25.00	24.89	24.79	24.69	24.58	24.48	24.38	24.28	24.17	24.07	23.97	23.87	23.77	23.67	23.57	23.47	23.37	23.27	23.17	23.07	22.97	22.87	22.78	22.68	22.58	22.48	22.38	22.29	22.19	22.09	22.00	21.90	21.81	21.71	21.62	21.52	21.43	21.33	21.24	21.14	21.05	20.96	20.86	20.77	20.68	20.59	20.49	20.40	20.31	20.22	20.13	20.04	19.95	19.86	19.77
4.54	3	2	1	0	4.49	8	7	6	5	4.44	3	2	1	0	4.39	8	7	6	5	4.34	3	2	1	0	4.29	8	7	6	5	4.24	3	2	1	0	4.19	8	7	6	5	4.14	3	2	1	0	4.09	8	7	6	5	4.04	3	2	1	0
15.89	15.81	15.74	15.65	15.57	15.49	15.42	15.34	15.26	15.18	15.11	15.03	14.96	14.88	14.81	14.73	14.66	14.58	14.51	14.43	14.36	14.28	14.21	14.14	14.07	13.99	13.92	13.85	13.78	13.70	13.63	13.56	13.49	13.42	13.35	13.28	13.21	13.14	13.07	13.00	12.93	12.86	12.80	12.73	12.66	12.59	12.52	12.46	12.39	12.32	12.26	12.19	12.12	12.06	12.00
3.54	3	2	1	0	3.49	8	7	6	5	3.44	3	2	1	0	3.39	8	7	6	5	3.34	3	2	1	0	3.29	8	7	6	5	3.24	3	2	1	0	3.19	8	7	6	5	3.14	3	2	1	0	3.09	8	7	6	5	3.04	3	2	1	0
9.222	9.167	9.112	9.058	9.003	8.949	8.895	8.841	8.787	8.734	8.681	8.628	8.575	8.523	8.471	8.419	8.367	8.316	8.264	8.213	8.162	8.111	8.060	8.010	7.960	7.910	7.860	7.811	7.762	7.713	7.664	7.615	7.567	7.519	7.471	7.423	7.375	7.328	7.281	7.234	7.187	7.140	7.094	7.048	6.992	6.956	6.911	6.866	6.821	6.776	6.731	6.687	6.643	6.599	6.555
2.54	3	2	1	0	2.49	8	7	6	5	2.44	3	2	1	0	2.39	8	7	6	5	2.34	3	2	1	0	2.29	8	7	6	5	2.24	3	2	1	0	2.19	8	7	6	5	2.14	3	2	1	0	2.09	8	7	6	5	2.04	3	2	1	0
4.752	4.717	4.682	4.648	4.614	4.580	4.546	4.512	4.479	4.446	4.413	4.380	4.347	4.314	4.282	4.250	4.218	4.186	4.154	4.123	4.092	4.060	4.029	3.998	3.968	3.938	3.907	3.877	3.847	3.817	3.787	3.758	3.729	3.700	3.671	3.642	3.613	3.585	3.557	3.529	3.501	3.473	3.445	3.418	3.391	3.364	3.337	3.310	3.283	3.256	3.230	3.204	3.178	3.152	3.126
1.54	3	2	1	0	1.49	8	7	6	5	1.44	3	2	1	0	1.39	8	7	6	5	1.34	3	2	1	0	1.29	8	7	6	5	1.24	3	2	1	0	1.19	8	7	6	5	1.14	3	2	1	0	1.09	8	7	6	5	1.04	3	2	1	0
2.097	2.078	2.059	2.040	2.021	2.002	1.983	1.965	1.947	1.929	1.910	1.892	1.874	1.856	1.838	1.821	1.803	1.786	1.769	1.752	1.735	1.718	1.701	1.684	1.667	1.650	1.634	1.618	1.601	1.585	1.569	1.553	1.537	1.521	1.506	1.490	1.475	1.459	1.444	1.429	1.413	1.398	1.383	1.368	1.354	1.339	1.324	1.310	1.295	1.281	1.267	1.253	1.238	1.224	1.210
0.54	3	2	1	0	0.49	8	7	6	5	0.44	3	2	1	0	0.39	8	7	6	5	0.34	3	2	1	0	0.29	8	7	6	5	0.24	3	2	1	0	0.19	8	7	6	5	0.14	3	2	1	0	0.09	8	7	6	5	0.04	3	2	1	0
0.628	0.617	0.605	0.593	0.581	0.570	0.559	0.547	0.535	0.524	0.513	0.501	0.489	0.478	0.467	0.455	0.444	0.432	0.421	0.409	0.398	0.386	0.375	0.363	0.352	0.341	0.329	0.318	0.306	0.295	0.283	0.272	0.260	0.248	0.237	0.225	0.214	0.202	0.191	0.179	0.167	0.155	0.143	0.131	0.120	0.108	0.096	0.084	0.073	0.061	0.049	0.036	0.024	0.012	0.000

Table C

V_z	Z	V_z	Z	V_z	Z	V_z	Z	V_z	Z	V_z	Z	V_z	Z	V_z	Z	V_z	Z		
10.00	121.14	8.99	92.65	7.99	69.59	6.99	50.69	5.99	35.36	4.99	23.24	3.99	14.096	2.99	7.690	1.99	3.661	0.99	1.413
9.99	120.82	8.99	92.39	7.99	69.38	6.99	50.52	5.99	35.22	4.99	23.14	3.99	14.019	2.99	7.639	1.99	3.631	0.99	1.396
9.99	120.51	8.99	92.14	7.99	69.17	6.99	50.35	5.99	35.09	4.99	23.03	3.99	13.942	2.99	7.588	1.99	3.602	0.99	1.379
9.99	119.90	8.99	91.89	7.99	68.96	6.99	50.18	5.99	34.95	4.99	22.92	3.99	13.865	2.99	7.537	1.99	3.573	0.99	1.363
9.99	119.58	8.99	91.64	7.99	68.76	6.99	50.01	5.99	34.82	4.99	22.82	3.99	13.789	2.99	7.487	1.99	3.543	0.99	1.348
9.99	119.27	8.99	91.39	7.99	68.55	6.99	49.84	5.99	34.68	4.99	22.71	3.99	13.713	2.99	7.436	1.99	3.514	0.99	1.332
9.99	118.97	8.99	91.13	7.99	68.35	6.99	49.67	5.99	34.55	4.99	22.60	3.99	13.637	2.99	7.386	1.99	3.484	0.99	1.316
9.99	118.66	8.99	90.88	7.99	68.14	6.99	49.50	5.99	34.41	4.99	22.50	3.99	13.562	2.99	7.337	1.99	3.455	0.99	1.300
9.99	118.35	8.99	90.63	7.99	67.93	6.99	49.33	5.99	34.27	4.99	22.40	3.99	13.486	2.99	7.287	1.99	3.426	0.99	1.284
9.99	118.04	8.99	90.38	7.99	67.73	6.99	49.17	5.99	34.13	4.99	22.30	3.99	13.412	2.99	7.237	1.99	3.398	0.99	1.268
9.99	117.74	8.99	90.14	7.99	67.52	6.99	49.00	5.99	34.00	4.99	22.19	3.99	13.336	2.99	7.188	1.99	3.369	0.99	1.252
9.99	117.44	8.99	89.89	7.99	67.32	6.99	48.84	5.99	33.87	4.99	22.09	3.99	13.262	2.99	7.139	1.99	3.341	0.99	1.237
9.99	117.13	8.99	89.64	7.99	67.12	6.99	48.67	5.99	33.74	4.99	21.99	3.99	13.188	2.99	7.090	1.99	3.312	0.99	1.221
9.99	116.83	8.99	89.39	7.99	66.92	6.99	48.50	5.99	33.60	4.99	21.88	3.99	13.114	2.99	7.041	1.99	3.284	0.99	1.206
9.99	116.52	8.99	89.14	7.99	66.71	6.99	48.34	5.99	33.47	4.99	21.78	3.99	13.041	2.99	6.993	1.99	3.257	0.99	1.190
9.99	116.22	8.99	88.90	7.99	66.50	6.99	48.17	5.99	33.34	4.99	21.68	3.99	12.968	2.99	6.946	1.99	3.230	0.99	1.175
9.99	115.92	8.99	88.65	7.99	66.30	6.99	48.01	5.99	33.21	4.99	21.58	3.99	12.894	2.99	6.898	1.99	3.203	0.99	1.160
9.99	115.62	8.99	88.40	7.99	66.10	6.99	47.84	5.99	33.08	4.99	21.48	3.99	12.821	2.99	6.850	1.99	3.175	0.99	1.144
9.99	115.32	8.99	88.15	7.99	65.90	6.99	47.68	5.99	32.95	4.99	21.38	3.99	12.749	2.99	6.803	1.99	3.147	0.99	1.129
9.99	115.02	8.99	87.91	7.99	65.70	6.99	47.51	5.99	32.81	4.99	21.28	3.99	12.677	2.99	6.755	1.99	3.120	0.99	1.114
9.99	114.72	8.99	87.67	7.99	65.50	6.99	47.35	5.99	32.68	4.99	21.18	3.99	12.605	2.99	6.708	1.99	3.094	0.99	1.099
9.99	114.42	8.99	87.42	7.99	65.30	6.99	47.19	5.99	32.55	4.99	21.08	3.99	12.533	2.99	6.662	1.99	3.068	0.99	1.084
9.99	114.12	8.99	87.18	7.99	65.10	6.99	47.03	5.99	32.42	4.99	20.98	3.99	12.461	2.99	6.616	1.99	3.041	0.99	1.070
9.99	113.83	8.99	86.94	7.99	64.90	6.99	46.86	5.99	32.29	4.99	20.88	3.99	12.390	2.99	6.570	1.99	3.015	0.99	1.055
9.99	113.53	8.99	86.69	7.99	64.70	6.99	46.70	5.99	32.16	4.99	20.78	3.99	12.319	2.99	6.524	1.99	2.989	0.99	1.040
9.99	113.24	8.99	86.45	7.99	64.51	6.99	46.54	5.99	32.03	4.99	20.68	3.99	12.248	2.99	6.478	1.99	2.963	0.99	1.025
9.99	112.94	8.99	86.21	7.99	64.31	6.99	46.38	5.99	31.91	4.99	20.58	3.99	12.178	2.99	6.433	1.99	2.937	0.99	1.011
9.99	112.65	8.99	85.97	7.99	64.11	6.99	46.21	5.99	31.78	4.99	20.48	3.99	12.108	2.99	6.387	1.99	2.911	0.99	0.997
9.99	112.35	8.99	85.73	7.99	63.91	6.99	46.05	5.99	31.65	4.99	20.38	3.99	12.038	2.99	6.342	1.99	2.885	0.99	0.982
9.99	112.06	8.99	85.49	7.99	63.71	6.99	45.89	5.99	31.52	4.99	20.29	3.99	11.969	2.99	6.297	1.99	2.860	0.99	0.967
9.99	111.76	8.99	85.25	7.99	63.51	6.99	45.73	5.99	31.39	4.99	20.20	3.99	11.899	2.99	6.253	1.99	2.836	0.99	0.953
9.99	111.47	8.99	85.01	7.99	63.32	6.99	45.57	5.99	31.27	4.99	20.10	3.99	11.830	2.99	6.209	1.99	2.811	0.99	0.939
9.99	111.18	8.99	84.77	7.99	63.13	6.99	45.41	5.99	31.14	4.99	20.00	3.99	11.762	2.99	6.165	1.99	2.786	0.99	0.925
9.99	110.89	8.99	84.53	7.99	62.93	6.99	45.25	5.99	31.01	4.99	19.90	3.99	11.693	2.99	6.121	1.99	2.761	0.99	0.911
9.99	110.59	8.99	84.29	7.99	62.74	6.99	45.09	5.99	30.88	4.99	19.81	3.99	11.624	2.99	6.078	1.99	2.736	0.99	0.896
9.99	110.30	8.99	84.05	7.99	62.54	6.99	44.94	5.99	30.76	4.99	19.71	3.99	11.556	2.99	6.034	1.99	2.712	0.99	0.882
9.99	110.01	8.99	83.82	7.99	62.35	6.99	44.78	5.99	30.64	4.99	19.62	3.99	11.489	2.99	5.990	1.99	2.688	0.99	0.868
9.99	109.72	8.99	83.58	7.99	62.15	6.99	44.62	5.99	30.52	4.99	19.52	3.99	11.422	2.99	5.947	1.99	2.664	0.99	0.854
9.99	109.43	8.99	83.35	7.99	61.96	6.99	44.47	5.99	30.39	4.99	19.43	3.99	11.354	2.99	5.905	1.99	2.641	0.99	0.840
9.99	109.15	8.99	83.11	7.99	61.77	6.99	44.31	5.99	30.26	4.99	19.33	3.99	11.287	2.99	5.863	1.99	2.617	0.99	0.826
9.99	108.87	8.99	82.87	7.99	61.57	6.99	44.15	5.99	30.14	4.99	19.24	3.99	11.221	2.99	5.820	1.99	2.594	0.99	0.811
9.99	108.58	8.99	82.64	7.99	61.38	6.99	43.99	5.99	30.01	4.99	19.14	3.99	11.155	2.99	5.778	1.99	2.570	0.99	0.797
9.99	108.29	8.99	82.41	7.99	61.19	6.99	43.84	5.99	29.89	4.99	19.05	3.99	11.089	2.99	5.736	1.99	2.546	0.99	0.783
9.99	108.01	8.99	82.18	7.99	60.99	6.99	43.69	5.99	29.76	4.99	18.96	3.99	11.022	2.99	5.695	1.99	2.522	0.99	0.769
9.99	107.71	8.99	81.94	7.99	60.80	6.99	43.53	5.99	29.64	4.99	18.86	3.99	10.956	2.99	5.654	1.99	2.499	0.99	0.756

9.54	107.44	8.54	81.71	7.54	60.61	6.54	43.37	5.54	29.51	4.54	18.77	3.54	10.891	2.54	5.612	1.54	2.477	0.54	0.742
3	107.15	3	81.48	2	60.42	3	43.21	3	29.40	3	18.67	3	10.826	3	5.571	3	2.454	3	0.729
2	106.87	2	81.24	1	60.23	2	43.06	2	29.28	2	18.58	2	10.762	2	5.530	2	2.432	2	0.715
1	106.59	1	81.01	0	60.04	1	42.91	1	29.16	1	18.49	1	10.698	1	5.489	1	2.409	1	0.700
0	106.30	0	80.78	0	59.85	0	42.75	0	29.03	0	18.39	0	10.633	0	5.449	0	2.387	0	0.686
9.49	106.02	8.49	80.55	7.49	59.67	6.49	42.60	5.49	28.91	4.49	18.29	3.49	10.569	2.49	5.409	1.49	2.364	0.49	0.673
8	105.74	8	80.32	8	59.48	8	42.45	8	28.79	8	18.20	8	10.505	8	5.369	8	2.342	8	0.660
7	105.46	7	80.09	7	59.29	7	42.29	7	28.67	7	18.11	7	10.441	7	5.329	7	2.321	7	0.646
6	105.18	6	79.86	6	59.10	6	42.14	6	28.55	6	18.02	6	10.378	6	5.290	6	2.299	6	0.632
5	104.90	5	79.63	5	58.91	5	41.99	5	28.43	5	17.93	5	10.315	5	5.251	5	2.278	5	0.619
9.44	104.62	8.44	79.40	7.44	58.72	6.44	41.84	5.44	28.31	4.44	17.84	3.44	10.253	2.44	5.212	1.44	2.256	0.44	0.606
3	104.34	3	79.18	3	58.53	3	41.69	3	28.19	3	17.75	3	10.190	3	5.173	3	2.235	3	0.592
2	104.07	2	78.95	2	58.35	2	41.54	2	28.07	2	17.67	2	10.127	2	5.134	2	2.213	2	0.578
1	103.79	1	78.72	1	58.17	1	41.39	1	27.95	1	17.58	1	10.065	1	5.095	1	2.192	1	0.565
0	103.52	0	78.49	0	57.98	0	41.24	0	27.84	0	17.49	0	10.004	0	5.057	0	2.171	0	0.551
9.39	103.24	8.39	78.27	7.39	57.79	6.39	41.09	5.39	27.72	4.39	17.40	3.39	9.943	2.39	5.019	1.39	2.150	0.39	0.537
8	102.96	8	78.04	8	57.61	8	40.93	8	27.60	8	17.31	8	9.882	8	4.982	8	2.129	8	0.524
7	102.69	7	77.82	7	57.42	7	40.79	7	27.48	7	17.22	7	9.821	7	4.944	7	2.109	7	0.510
6	102.42	6	77.59	6	57.24	6	40.64	6	27.36	6	17.14	6	9.760	6	4.906	6	2.089	6	0.497
5	102.14	5	77.37	5	57.06	5	40.49	5	27.25	5	17.05	5	9.700	5	4.869	5	2.069	5	0.483
9.34	101.86	8.34	77.14	7.34	56.88	6.34	40.34	5.34	27.13	4.34	16.96	3.34	9.640	2.34	4.832	1.34	2.049	0.34	0.470
3	101.59	3	76.92	3	56.69	3	40.19	3	27.01	3	16.87	3	9.579	3	4.795	3	2.029	3	0.456
2	101.32	2	76.70	2	56.51	2	40.05	2	26.90	2	16.78	2	9.519	2	4.758	2	2.009	2	0.443
1	101.05	1	76.48	1	56.32	1	39.90	1	26.79	1	16.70	1	9.460	1	4.722	1	1.989	1	0.429
0	100.78	0	76.26	0	56.15	0	39.75	0	26.67	0	16.62	0	9.401	0	4.686	0	1.969	0	0.416
9.29	100.51	8.29	76.03	7.29	55.96	6.29	39.61	5.29	26.55	4.29	16.53	3.29	9.342	2.29	4.650	1.29	1.949	0.29	0.403
8	100.24	8	75.81	8	55.78	8	39.46	8	26.43	8	16.44	8	9.283	8	4.614	8	1.930	8	0.389
7	99.97	7	75.60	7	55.60	7	39.32	7	26.33	7	16.36	7	9.225	7	4.579	7	1.911	7	0.375
6	99.70	6	75.37	6	55.42	6	39.17	6	26.21	6	16.27	6	9.167	6	4.543	6	1.891	6	0.361
5	99.43	5	75.15	5	55.24	5	39.02	5	26.09	5	16.18	5	9.109	5	4.508	5	1.872	5	0.348
9.24	99.17	8.24	74.93	7.24	55.06	6.24	38.88	5.24	25.98	4.24	16.10	3.24	9.051	2.24	4.473	1.24	1.853	0.24	0.334
3	98.90	3	74.71	3	54.88	3	38.74	3	25.86	3	16.01	3	8.994	3	4.438	3	1.834	3	0.321
2	98.64	2	74.50	2	54.71	2	38.59	2	25.75	2	15.93	2	8.987	2	4.404	2	1.815	2	0.307
1	98.37	1	74.28	1	54.53	1	38.44	1	25.64	1	15.85	1	8.880	1	4.370	1	1.796	1	0.293
0	98.11	0	74.06	0	54.35	0	38.30	0	25.53	0	15.77	0	8.823	0	4.336	0	1.778	0	0.280
9.19	97.84	8.19	73.84	7.19	54.17	6.19	38.16	5.19	25.42	4.19	15.68	3.19	8.766	2.19	4.301	1.19	1.760	0.19	0.266
8	97.58	8	73.63	8	54.00	8	38.02	8	25.31	8	15.60	8	8.710	8	4.267	8	1.742	8	0.253
7	97.31	7	73.41	7	53.82	7	37.88	7	25.19	7	15.52	7	8.655	7	4.234	7	1.723	7	0.239
6	97.05	6	73.20	6	53.64	6	37.73	6	25.08	6	15.44	6	8.599	6	4.201	6	1.705	6	0.225
5	96.79	5	72.98	5	53.46	5	37.59	5	24.97	5	15.35	5	8.544	5	4.168	5	1.688	5	0.211
9.14	96.53	8.14	72.76	7.14	53.29	6.14	37.45	5.14	24.86	4.14	15.27	3.14	8.488	2.14	4.135	1.14	1.669	0.14	0.197
3	96.26	3	72.55	3	53.11	3	37.31	3	24.75	3	15.19	3	8.433	3	4.102	3	1.651	3	0.183
2	95.99	2	72.34	2	52.93	2	37.17	2	24.64	2	15.11	2	8.378	2	4.069	2	1.633	2	0.169
1	95.73	1	72.13	1	52.76	1	37.03	1	24.53	1	15.03	1	8.324	1	4.037	1	1.616	1	0.155
0	95.47	0	71.91	0	52.58	0	36.88	0	24.42	0	14.95	0	8.269	0	4.005	0	1.599	0	0.142
9.09	95.21	8.09	71.69	7.09	52.41	6.09	36.74	5.09	24.31	4.09	14.87	3.09	8.215	2.09	3.973	1.09	1.581	0.09	0.128
8	94.95	8	71.48	8	52.24	8	36.60	8	24.20	8	14.79	8	8.162	8	3.941	8	1.564	8	0.113
7	94.69	7	71.20	7	52.06	7	36.46	7	24.09	7	14.71	7	8.109	7	3.909	7	1.547	7	0.099
6	94.44	6	71.06	6	51.89	6	36.32	6	23.99	6	14.63	6	8.056	6	3.877	6	1.529	6	0.086
5	94.19	5	70.85	5	51.72	5	36.18	5	23.88	5	14.55	5	8.003	5	3.845	5	1.512	5	0.072
9.04	93.93	8.04	70.64	7.04	51.54	6.04	36.04	5.04	23.77	4.04	14.48	3.04	7.950	2.04	3.814	1.04	1.496	0.04	0.058
3	93.67	3	70.42	3	51.37	3	35.90	3	23.67	3	14.40	3	7.898	3	3.784	3	1.479	3	0.043
2	93.42	2	70.21	2	51.20	2	35.76	2	23.56	2	14.32	2	7.846	2	3.753	2	1.462	2	0.028
1	93.16	1	70.01	1	51.03	1	35.63	1	23.46	1	14.24	1	7.794	1	3.723	1	1.445	1	0.014
0	92.90	0	69.80	0	50.86	0	35.49	0	23.35	0	14.17	0	7.742	0	3.692	0	1.429	0	0.000

Table D. Ratio of Absorption Coefficient to Scattering Coefficient (K/S) as a Function of Reflectivity in Percent ($100R_{\infty}$)

$100R_{\infty}$	K/S	$100R_{\infty}$	K/S	$100R_{\infty}$	K/S	$100R_{\infty}$	K/S
0.0		5.0	9.02	10.0	4.050	15.0	2.408
.1	449.0	.1	8.83	.1	4.001	.1	2.387
.2	249.0	.2	8.64	.2	3.953	.2	2.365
.3	165.7	.3	8.46	.3	3.906	.3	2.344
.4	124.0	.4	8.29	.4	3.860	.4	2.324
.5	99.0	.5	8.12	.5	3.814	.5	2.303
.6	82.3	.6	7.957	.6	3.770	.6	2.283
.7	70.4	.7	7.800	.7	3.726	.7	2.263
.8	61.5	.8	7.650	.8	3.684	.8	2.244
.9	54.6	.9	7.504	.9	3.642	.9	2.224
1.0	49.0	6.0	7.363	11.0	3.600	16.0	2.205
.1	44.5	.1	7.227	.1	3.560	.1	2.186
.2	40.7	.2	7.096	.2	3.520	.2	2.167
.3	37.5	.3	6.968	.3	3.481	.3	2.149
.4	34.7	.4	6.844	.4	3.443	.4	2.131
.5	32.3	.5	6.725	.5	3.405	.5	2.113
.6	30.3	.6	6.609	.6	3.368	.6	2.095
.7	28.4	.7	6.496	.7	3.332	.7	2.078
.8	26.79	.8	6.387	.8	3.296	.8	2.060
.9	25.33	.9	6.281	.9	3.261	.9	2.043
2.0	24.01	7.0	6.178	12.0	3.227	17.0	2.026
.1	22.82	.1	6.078	.1	3.193	.1	2.009
.2	21.74	.2	5.980	.2	3.159	.2	1.9930
.3	20.75	.3	5.886	.3	3.127	.3	1.9767
.4	19.85	.4	5.794	.4	3.094	.4	1.9606
.5	19.01	.5	5.704	.5	3.062	.5	1.9446
.6	18.24	.6	5.617	.6	3.031	.6	1.9289
.7	17.53	.7	5.532	.7	3.001	.7	1.9134
.8	16.87	.8	5.449	.8	2.970	.8	1.8980
.9	16.26	.9	5.369	.9	2.940	.9	1.8828
3.0	15.68	8.0	5.290	13.0	2.911	18.0	1.8678
.1	15.14	.1	5.213	.1	2.882	.1	1.8529
.2	14.64	.2	5.139	.2	2.854	.2	1.8382
.3	14.17	.3	5.066	.3	2.826	.3	1.8237
.4	13.72	.4	4.994	.4	2.798	.4	1.8094
.5	13.30	.5	4.925	.5	2.771	.5	1.7952
.6	12.91	.6	4.857	.6	2.744	.6	1.7812
.7	12.53	.7	4.791	.7	2.718	.7	1.7673
.8	12.18	.8	4.726	.8	2.692	.8	1.7536
.9	11.84	.9	4.662	.9	2.667	.9	1.7400
4.0	11.52	9.0	4.601	14.0	2.641	19.0	1.7266
.1	11.22	.1	4.540	.1	2.617	.1	1.7133
.2	10.93	.2	4.481	.2	2.592	.2	1.7002
.3	10.65	.3	4.423	.3	2.568	.3	1.6872
.4	10.39	.4	4.366	.4	2.544	.4	1.6743
.5	10.13	.5	4.311	.5	2.521	.5	1.6616
.6	9.89	.6	4.256	.6	2.498	.6	1.6490
.7	9.66	.7	4.203	.7	2.475	.7	1.6366
.8	9.44	.8	4.151	.8	2.452	.8	1.6242
.9	9.23	.9	4.100	.9	2.430	.9	1.6121

Table D. Ratio of Absorption Coefficient to Scattering Coefficient (K/S) as a Function of Reflectivity in Percent ($100R_\infty$)—Continued

$100R_\infty$	K/S	$100R_\infty$	K/S	$100R_\infty$	K/S	$100R_\infty$	K/S
20.0	1.6000	25.0	1.1250	30.0	0.8167	35.0	0.6036
.1	1.5881	.1	1.1175	.1	.8116	.1	.6000
.2	1.5763	.2	1.1101	.2	.8066	.2	.5964
.3	1.5646	.3	1.1028	.3	.8017	.3	.5929
.4	1.5530	.4	1.0955	.4	.7967	.4	.5894
.5	1.5415	.5	1.0883	.5	.7918	.5	.5860
.6	1.5302	.6	1.0811	.6	.7870	.6	.5825
.7	1.5190	.7	1.0740	.7	.7822	.7	.5791
.8	1.5078	.8	1.0670	.8	.7774	.8	.5756
.9	1.4968	.9	1.0600	.9	.7726	.9	.5723
21.0	1.4860	26.0	1.0531	31.0	0.7679	36.0	0.5689
.1	1.4752	.1	1.0462	.1	.7632	.1	.5655
.2	1.4645	.2	1.0394	.2	.7586	.2	.5622
.3	1.4539	.3	1.0326	.3	.7539	.3	.5589
.4	1.4434	.4	1.0259	.4	.7494	.4	.5556
.5	1.4331	.5	1.0193	.5	.7448	.5	.5524
.6	1.4228	.6	1.0127	.6	.7403	.6	.5491
.7	1.4126	.7	1.0062	.7	.7358	.7	.5459
.8	1.4026	.8	.9997	.8	.7313	.8	.5427
.9	1.3926	.9	.9933	.9	.7269	.9	.5395
22.0	1.3827	27.0	0.9868	32.0	0.7225	37.0	0.5364
.1	1.3729	.1	.9805	.1	.7181	.1	.5332
.2	1.3632	.2	.9742	.2	.7138	.2	.5301
.3	1.3536	.3	.9680	.3	.7095	.3	.5270
.4	1.3441	.4	.9618	.4	.7052	.4	.5239
.5	1.3347	.5	.9557	.5	.7010	.5	.5208
.6	1.3254	.6	.9496	.6	.6967	.6	.5178
.7	1.3161	.7	.9436	.7	.6926	.7	.5148
.8	1.3070	.8	.9376	.8	.6884	.8	.5118
.9	1.2979	.9	.9316	.9	.6843	.9	.5088
23.0	1.2889	28.0	.9257	33.0	0.6802	38.0	0.5058
.1	1.2800	.1	.9199	.1	.6761	.1	.5028
.2	1.2712	.2	.9140	.2	.6720	.2	.4999
.3	1.2624	.3	.9083	.3	.6680	.3	.4970
.4	1.2538	.4	.9026	.4	.6640	.4	.4941
.5	1.2452	.5	.8969	.5	.6600	.5	.4912
.6	1.2366	.6	.8912	.6	.6561	.6	.4883
.7	1.2282	.7	.8857	.7	.6521	.7	.4855
.8	1.2198	.8	.8801	.8	.6483	.8	.4827
.9	1.2116	.9	.8746	.9	.6444	.9	.4798
24.0	1.2033	29.0	.8691	34.0	0.6406	39.0	0.4770
.1	1.1952	.1	.8637	.1	.6368	.1	.4743
.2	1.1871	.2	.8583	.2	.6330	.2	.4715
.3	1.1791	.3	.8530	.3	.6292	.3	.4688
.4	1.1712	.4	.8477	.4	.6255	.4	.4660
.5	1.1633	.5	.8424	.5	.6218	.5	.4633
.6	1.1555	.6	.8372	.6	.6181	.6	.4606
.7	1.1478	.7	.8320	.7	.6144	.7	.4580
.8	1.1401	.8	.8268	.8	.6108	.8	.4553
.9	1.1325	.9	.8217	.9	.6072	.9	.4526

Table D. Ratio of Absorption Coefficient to Scattering Coefficient (K/S) as a Function of Reflectivity in Percent ($100R_\infty$)—Continued

$100R_\infty$	K/S	$100R_\infty$	K/S	$100R_\infty$	K/S	$100R_\infty$	K/S
40.0	0.4500	45.0	0.3361	50.0	0.25000	55.0	0.18409
.1	.4474	.1	.3342	.1	.24850	.1	.18294
.2	.4448	.2	.3322	.2	.24702	.2	.18180
.3	.4422	.3	.3302	.3	.24554	.3	.18066
.4	.4396	.4	.3283	.4	.24406	.4	.17953
.5	.4371	.5	.3264	.5	.24260	.5	.17840
.6	.4345	.6	.3245	.6	.24114	.6	.17728
.7	.4320	.7	.3226	.7	.23969	.7	.17617
.8	.4295	.8	.3207	.8	.23825	.8	.17506
.9	.4270	.9	.3188	.9	.23682	.9	.17396
41.0	0.4245	46.0	0.3170	51.0	0.23539	56.0	0.17286
.1	.4220	.1	.3151	.1	.23397	.1	.17177
.2	.4196	.2	.3132	.2	.23256	.2	.17068
.3	.4172	.3	.3114	.3	.23116	.3	.16960
.4	.4147	.4	.3096	.4	.22976	.4	.16852
.5	.4123	.5	.3078	.5	.22837	.5	.16746
.6	.4099	.6	.3060	.6	.22699	.6	.16639
.7	.4075	.7	.3042	.7	.22562	.7	.16533
.8	.4052	.8	.3024	.8	.22425	.8	.16428
.9	.4028	.9	.3006	.9	.22289	.9	.16323
42.0	0.4005	47.0	0.2988	52.0	0.22154	57.0	0.16219
.1	.3982	.1	.2971	.1	.22019	.1	.16116
.2	.3958	.2	.2953	.2	.21885	.2	.16013
.3	.3935	.3	.2936	.3	.21752	.3	.15910
.4	.3912	.4	.2918	.4	.21620	.4	.15808
.5	.3890	.5	.2901	.5	.21488	.5	.15707
.6	.3867	.6	.2884	.6	.21357	.6	.15606
.7	.3845	.7	.2867	.7	.21227	.7	.15505
.8	.3822	.8	.2850	.8	.21097	.8	.15405
.9	.3800	.9	.2833	.9	.20968	.9	.15306
43.0	0.3778	48.0	0.2817	53.0	0.20840	58.0	0.15207
.1	.3756	.1	.2800	.1	.20712	.1	.15109
.2	.3734	.2	.2783	.2	.20585	.2	.15011
.3	.3712	.3	.2767	.3	.20459	.3	.14913
.4	.3691	.4	.2751	.4	.20333	.4	.14816
.5	.3669	.5	.2734	.5	.20208	.5	.14720
.6	.3648	.6	.2718	.6	.20084	.6	.14624
.7	.3627	.7	.2702	.7	.19960	.7	.14529
.8	.3606	.8	.2686	.8	.19837	.8	.14434
.9	.3584	.9	.2670	.9	.19714	.9	.14340
44.0	0.3564	49.0	0.26541	54.0	0.19593	59.0	0.14246
.1	.3543	.1	.26383	.1	.19471	.1	.14152
.2	.3522	.2	.26226	.2	.19351	.2	.14059
.3	.3502	.3	.26070	.3	.19231	.3	.13967
.4	.3481	.4	.25915	.4	.19112	.4	.13875
.5	.3461	.5	.25760	.5	.18993	.5	.13784
.6	.3441	.6	.25606	.6	.18875	.6	.13693
.7	.3421	.7	.25454	.7	.18758	.7	.13602
.8	.3401	.8	.25302	.8	.18641	.8	.13512
.9	.3381	.9	.25150	.9	.18525	.9	.13422

Table D. Ratio of Absorption Coefficient to Scattering Coefficient (K/S) as a Function of Reflectivity in Percent ($100R_\infty$)—Continued

$100R_\infty$	K/S	$100R_\infty$	K/S	$100R_\infty$	K/S	$100R_\infty$	K/S
60.0	0.13333	65.0	0.09423	70.0	0.06429	75.0	0.04167
.1	.13245	.1	.09355	.1	.06377	.1	.04128
.2	.13156	.2	.09287	.2	.06325	.2	.04089
.3	.13069	.3	.09220	.3	.06274	.3	.04051
.4	.12981	.4	.09153	.4	.06223	.4	.04013
.5	.12895	.5	.09086	.5	.06172	.5	.03975
.6	.12808	.6	.09020	.6	.06122	.6	.03938
.7	.12722	.7	.08954	.7	.06071	.7	.03900
.8	.12637	.8	.08888	.8	.06021	.8	.03863
.9	.12552	.9	.08823	.9	.05972	.9	.03826
61.0	0.12467	66.0	0.08758	71.0	0.05923	76.0	0.03789
.1	.12383	.1	.08693	.1	.05873	.1	.03753
.2	.12299	.2	.08629	.2	.05825	.2	.03717
.3	.12216	.3	.08565	.3	.05776	.3	.03681
.4	.12133	.4	.08501	.4	.05728	.4	.03645
.5	.12051	.5	.08438	.5	.05680	.5	.03609
.6	.11969	.6	.08375	.6	.05632	.6	.03574
.7	.11887	.7	.08313	.7	.05585	.7	.03539
.8	.11806	.8	.08250	.8	.05538	.8	.03504
.9	.11725	.9	.08188	.9	.05491	.9	.03470
62.0	0.11645	67.0	0.08127	72.0	0.05444	77.0	0.03435
.1	.11565	.1	.08066	.1	.05398	.1	.03401
.2	.11486	.2	.08005	.2	.05352	.2	.03367
.3	.11407	.3	.07944	.3	.05306	.3	.03333
.4	.11328	.4	.07884	.4	.05261	.4	.03299
.5	.11250	.5	.07824	.5	.05216	.5	.03266
.6	.11172	.6	.07764	.6	.05171	.6	.03233
.7	.11095	.7	.07705	.7	.05126	.7	.03200
.8	.11018	.8	.07646	.8	.05081	.8	.03167
.9	.10941	.9	.07588	.9	.05037	.9	.03135
63.0	0.10865	68.0	0.07529	73.0	0.04993	78.0	0.03103
.1	.10789	.1	.07471	.1	.04949	.1	.03070
.2	.10714	.2	.07414	.2	.04906	.2	.03039
.3	.10639	.3	.07356	.3	.04863	.3	.03007
.4	.10564	.4	.07299	.4	.04820	.4	.02976
.5	.10490	.5	.07243	.5	.04777	.5	.02944
.6	.10416	.6	.07186	.6	.04735	.6	.02913
.7	.10343	.7	.07130	.7	.04693	.7	.02882
.8	.10270	.8	.07074	.8	.04651	.8	.02852
.9	.10197	.9	.07019	.9	.04609	.9	.02821
64.0	0.10125	69.0	0.06964	74.0	0.04568	79.0	0.02791
.1	.10053	.1	.06909	.1	.04526	.1	.02761
.2	.09982	.2	.06854	.2	.04485	.2	.02731
.3	.09910	.3	.06800	.3	.04445	.3	.02702
.4	.09840	.4	.06746	.4	.04404	.4	.02672
.5	.09769	.5	.06692	.5	.04364	.5	.02643
.6	.09699	.6	.06639	.6	.04324	.6	.02614
.7	.09630	.7	.06586	.7	.04284	.7	.02585
.8	.09560	.8	.06533	.8	.04245	.8	.02557
.9	.09492	.9	.06481	.9	.04206	.9	.02528

Table D. Ratio of Absorption Coefficient to Scattering Coefficient (K/S) as a Function of Reflectivity in Percent ($100R_x$)—Continued

$100R_{\infty}$	K/S	$100R_{\infty}$	K/S	$100R_{\infty}$	K/S	$100R_{\infty}$	K/S
80.0	0.02500	85.0	0.01324	90.0	0.005556	95.0	0.001316
.1	.02472	.1	.01304	.1	.005439	.1	.001262
.2	.02444	.2	.01285	.2	.005324	.2	.001210
.3	.02417	.3	.01267	.3	.005210	.3	.001159
.4	.02389	.4	.01248	.4	.005097	.4	.001109
.5	.02362	.5	.01230	.5	.004986	.5	.001060
.6	.02335	.6	.01211	.6	.004876	.6	.001013
.7	.02308	.7	.01193	.7	.004768	.7	.000966
.8	.02281	.8	.01175	.8	.004661	.8	.000921
.9	.02255	.9	.01157	.9	.004555	.9	.000876
81.0	0.02228	86.0	0.01140	91.0	0.004451	96.0	0.000833
.1	.02202	.1	.01122	.1	.004347	.1	.000791
.2	.02176	.2	.01105	.2	.004246	.2	.000751
.3	.02151	.3	.01087	.3	.004145	.3	.000711
.4	.02125	.4	.01070	.4	.004046	.4	.000672
.5	.02100	.5	.01054	.5	.003948	.5	.000635
.6	.02075	.6	.01037	.6	.003852	.6	.000598
.7	.02050	.7	.01020	.7	.003756	.7	.000563
.8	.02025	.8	.01004	.8	.003662	.8	.000529
.9	.02000	.9	.00987	.9	.003570	.9	.000496
82.0	0.01976	87.0	0.009713	92.0	0.003478	97.0	0.000464
.1	.01951	.1	.009553	.1	.003388	.1	.000433
.2	.01927	.2	.009394	.2	.003299	.2	.000403
.3	.01903	.3	.009238	.3	.003212	.3	.000375
.4	.01880	.4	.009082	.4	.003126	.4	.000347
.5	.01856	.5	.008929	.5	.003041	.5	.000321
.6	.01833	.6	.008776	.6	.002957	.6	.000295
.7	.01809	.7	.008625	.7	.002874	.7	.000271
.8	.01786	.8	.008476	.8	.002793	.8	.000247
.9	.01764	.9	.008328	.9	.002713	.9	.000225
83.0	0.01741	88.0	0.008182	93.0	0.002634	98.0	0.000204
.1	.01718	.1	.008037	.1	.002557	.1	.000184
.2	.01696	.2	.007893	.2	.002481	.2	.000165
.3	.01674	.3	.007751	.3	.002406	.3	.000147
.4	.01652	.4	.007611	.4	.002332	.4	.000130
.5	.01630	.5	.007472	.5	.002259	.5	.000114
.6	.01609	.6	.007334	.6	.002188	.6	.0000994
.7	.01587	.7	.007198	.7	.002118	.7	.0000856
.8	.01566	.8	.007063	.8	.002049	.8	.0000729
.9	.01545	.9	.006930	.9	.001981	.9	.0000612
84.0	0.01524	89.0	0.006798	94.0	0.001915	99.0	0.0000505
.1	.01503	.1	.006667	.1	.001850	.1	.0000409
.2	.01482	.2	.006538	.2	.001786	.2	.0000323
.3	.01462	.3	.006410	.3	.001723	.3	.0000247
.4	.01442	.4	.006284	.4	.001661	.4	.0000181
.5	.01422	.5	.006159	.5	.001601	.5	.0000126
.6	.01402	.6	.006036	.6	.001541	.6	.0000080
.7	.01382	.7	.005914	.7	.001483	.7	.0000045
.8	.01362	.8	.005793	.8	.001426	.8	.0000020
.9	.01343	.9	.005674	.9	.001370	.9	.0000005
						100.0	0.0000000

Table E. Hyperbolic Cotangents

u	$\text{ctgh } u$	u	$\text{ctgh } u$	u	$\text{ctgh } u$	u	$\text{ctgh } u$
0.000	∞	0.050	20.02	0.100	10.033	0.150	6.717
.001	1000.0	.051	19.62	.101	9.935	.151	6.673
.002	500.0	.052	19.25	.102	9.838	.152	6.630
.003	333.3	.053	18.89	.103	9.743	.153	6.587
.004	250.0	.054	18.54	.104	9.650	.154	6.545
.005	200.0	.055	18.20	.105	9.559	.155	6.503
.006	166.7	.056	17.88	.106	9.469	.156	6.462
.007	142.9	.057	17.56	.107	9.381	.157	6.422
.008	125.0	.058	17.26	.108	9.295	.158	6.382
.009	111.1	.059	16.97	.109	9.211	.159	6.342
0.010	100.0	0.060	16.69	0.110	9.128	0.160	6.303
.011	90.0	.061	16.41	.111	9.046	.161	6.265
.012	83.3	.062	16.15	.112	8.966	.162	6.227
.013	76.9	.063	15.89	.113	8.887	.163	6.189
.014	71.4	.064	15.65	.114	8.810	.164	6.152
.015	66.7	.065	15.41	.115	8.734	.165	6.115
.016	62.5	.066	15.17	.116	8.659	.166	6.079
.017	58.8	.067	14.95	.117	8.586	.167	6.044
.018	55.6	.068	14.73	.118	8.514	.168	6.008
.019	52.6	.069	14.52	.119	8.443	.169	5.973
0.020	50.01	0.070	14.31	0.120	8.373	0.170	5.939
.021	47.63	.071	14.11	.121	8.305	.171	5.905
.022	45.46	.072	13.91	.122	8.237	.172	5.871
.023	43.49	.073	13.72	.123	8.171	.173	5.838
.024	41.68	.074	13.54	.124	8.106	.174	5.805
.025	40.01	.075	13.36	.125	8.042	.175	5.772
.026	38.47	.076	13.18	.126	7.978	.176	5.740
.027	37.05	.077	13.01	.127	7.916	.177	5.709
.028	35.72	.078	12.85	.128	7.855	.178	5.677
.029	34.49	.079	12.68	.129	7.795	.179	5.646
0.030	33.34	0.080	12.527	0.130	7.736	0.180	5.615
.031	32.27	.081	12.373	.131	7.677	.181	5.585
.032	31.26	.082	12.222	.132	7.620	.182	5.555
.033	30.31	.083	12.076	.133	7.563	.183	5.525
.034	29.42	.084	11.933	.134	7.507	.184	5.496
.035	28.58	.085	11.793	.135	7.452	.185	5.467
.036	27.79	.086	11.657	.136	7.398	.186	5.438
.037	27.04	.087	11.523	.137	7.345	.187	5.410
.038	26.33	.088	11.393	.138	7.292	.188	5.382
.039	25.65	.089	11.266	.139	7.241	.189	5.354
0.040	25.01	0.090	11.141	0.140	7.189	0.190	5.326
.041	24.40	.091	11.019	.141	7.139	.191	5.299
.042	23.82	.092	10.900	.142	7.090	.192	5.272
.043	23.27	.093	10.784	.143	7.041	.193	5.246
.044	22.74	.094	10.670	.144	6.992	.194	5.219
.045	22.24	.095	10.558	.145	6.945	.195	5.193
.046	21.75	.096	10.449	.146	6.898	.196	5.167
.047	21.29	.097	10.342	.147	6.852	.197	5.142
.048	20.85	.098	10.237	.148	6.806	.198	5.116
.049	20.42	.099	10.134	.149	6.761	.199	5.091

Table E. Hyperbolic Cotangents—Continued

<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>
0.200	5.066	0.250	4.0830	0.300	3.4327	0.350	2.9729
.201	5.042	.251	4.0674	.301	3.4220	.351	2.9651
.202	5.018	.252	4.0519	.302	3.4113	.352	2.9573
.203	4.994	.253	4.0365	.303	3.4007	.353	2.9496
.204	4.970	.254	4.0213	.304	3.3902	.354	2.9419
.205	4.946	.255	4.0062	.305	3.3797	.355	2.9343
.206	4.923	.256	3.9912	.306	3.3693	.356	2.9267
.207	4.900	.257	3.9763	.307	3.3590	.357	2.9191
.208	4.877	.258	3.9616	.308	3.3488	.358	2.9116
.209	4.854	.259	3.9470	.309	3.3386	.359	2.9042
0.210	4.832	0.260	3.9324	0.310	3.3285	0.360	2.8968
.211	4.809	.261	3.9180	.311	3.3184	.361	2.8894
.212	4.787	.262	3.9037	.312	3.3085	.362	2.8821
.213	4.766	.263	3.8895	.313	3.2985	.363	2.8748
.214	4.744	.264	3.8755	.314	3.2887	.364	2.8675
.215	4.723	.265	3.8615	.315	3.2789	.365	2.8603
.216	4.701	.266	3.8476	.316	3.2692	.366	2.8532
.217	4.680	.267	3.8339	.317	3.2595	.367	2.8460
.218	4.660	.268	3.8203	.318	3.2499	.368	2.8390
.219	4.639	.269	3.8067	.319	3.2404	.369	2.8319
0.220	4.619	0.270	3.7933	0.320	3.2309	0.370	2.8249
.221	4.598	.271	3.7799	.321	3.2215	.371	2.8180
.222	4.578	.272	3.7667	.322	3.2122	.372	2.8110
.223	4.558	.273	3.7536	.323	3.2029	.373	2.8042
.224	4.539	.274	3.7405	.324	3.1937	.374	2.7973
.225	4.519	.275	3.7276	.325	3.1845	.375	2.7905
.226	4.500	.276	3.7147	.326	3.1754	.376	2.7837
.227	4.481	.277	3.7020	.327	3.1663	.377	2.7770
.228	4.462	.278	3.6893	.328	3.1573	.378	2.7703
.229	4.443	.279	3.6768	.329	3.1484	.379	2.7637
0.230	4.424	0.280	3.6643	0.330	3.1395	0.380	2.7570
.231	4.406	.281	3.6519	.331	3.1307	.381	2.7505
.232	4.387	.282	3.6396	.332	3.1219	.382	2.7439
.233	4.369	.283	3.6274	.333	3.1132	.383	2.7374
.234	4.351	.284	3.6153	.334	3.1045	.384	2.7309
.235	4.333	.285	3.6033	.335	3.0959	.385	2.7245
.236	4.316	.286	3.5913	.336	3.0874	.386	2.7181
.237	4.298	.287	3.5795	.337	3.0789	.387	2.7117
.238	4.281	.288	3.5677	.338	3.0704	.388	2.7054
.239	4.263	.289	3.5560	.339	3.0620	.389	2.6991
0.240	4.246	0.290	3.5444	0.340	3.0536	0.390	2.6928
.241	4.229	.291	3.5329	.341	3.0453	.391	2.6866
.242	4.213	.292	3.5214	.342	3.0371	.392	2.6804
.243	4.196	.293	3.5101	.343	3.0289	.393	2.6742
.244	4.179	.294	3.4988	.344	3.0207	.394	2.6681
.245	4.163	.295	3.4876	.345	3.0126	.395	2.6620
.246	4.147	.296	3.4765	.346	3.0046	.396	2.6559
.247	4.131	.297	3.4654	.347	2.9966	.397	2.6499
.248	4.115	.298	3.4545	.348	2.9886	.398	2.6438
.249	4.099	.299	3.4436	.349	2.9807	.399	2.6379

Table E. Hyperbolic Cotangents—Continued

<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>
0.400	2.6319	0.450	2.3702	0.500	2.1640	0.550	1.9979
.401	2.6260	.451	2.3656	.501	2.1603	.551	1.9949
.402	2.6201	.452	2.3610	.502	2.1566	.552	1.9920
.403	2.6143	.453	2.3565	.503	2.1530	.553	1.9890
.404	2.6085	.454	2.3519	.504	2.1493	.554	1.9860
.405	2.6027	.455	2.3474	.505	2.1457	.555	1.9831
.406	2.5969	.456	2.3429	.506	2.1421	.556	1.9802
.407	2.5912	.457	2.3384	.507	2.1386	.557	1.9773
.408	2.5855	.458	2.3340	.508	2.1350	.558	1.9744
.409	2.5798	.459	2.3295	.509	2.1314	.559	1.9715
0.410	2.5742	0.460	2.3251	0.510	2.1279	0.560	1.9686
.411	2.5686	.461	2.3207	.511	2.1244	.561	1.9657
.412	2.5630	.462	2.3164	.512	2.1209	.562	1.9629
.413	2.5574	.463	2.3120	.513	2.1174	.563	1.9600
.414	2.5519	.464	2.3077	.514	2.1139	.564	1.9572
.415	2.5464	.465	2.3033	.515	2.1105	.565	1.9544
.416	2.5409	.466	2.2991	.516	2.1070	.566	1.9515
.417	2.5355	.467	2.2948	.517	2.1036	.567	1.9487
.418	2.5301	.468	2.2905	.518	2.1002	.568	1.9459
.419	2.5247	.469	2.2863	.519	2.0968	.569	1.9432
0.420	2.5193	0.470	2.2821	0.520	2.0934	0.570	1.9404
.421	2.5140	.471	2.2779	.521	2.0900	.571	1.9376
.422	2.5087	.472	2.2737	.522	2.0866	.572	1.9349
.423	2.5034	.473	2.2695	.523	2.0833	.573	1.9321
.424	2.4982	.474	2.2654	.524	2.0799	.574	1.9294
.425	2.4929	.475	2.2613	.525	2.0766	.575	1.9267
.426	2.4877	.476	2.2572	.526	2.0733	.576	1.9240
.427	2.4826	.477	2.2531	.527	2.0700	.577	1.9213
.428	2.4774	.478	2.2490	.528	2.0668	.578	1.9186
.429	2.4723	.479	2.2450	.529	2.0635	.579	1.9159
0.430	2.4672	0.480	2.2409	0.530	2.0602	0.580	1.9133
.431	2.4621	.481	2.2369	.531	2.0570	.581	1.9106
.432	2.4571	.482	2.2329	.532	2.0538	.582	1.9080
.433	2.4520	.483	2.2289	.533	2.0506	.583	1.9053
.434	2.4470	.484	2.2250	.534	2.0474	.584	1.9027
.435	2.4421	.485	2.2210	.535	2.0442	.585	1.9001
.436	2.4371	.486	2.2171	.536	2.0410	.586	1.8975
.437	2.4322	.487	2.2132	.537	2.0378	.587	1.8949
.438	2.4273	.488	2.2093	.538	2.0347	.588	1.8923
.439	2.4224	.489	2.2054	.539	2.0316	.589	1.8897
0.440	2.4175	0.490	2.2016	0.540	2.0284	0.590	1.8872
.441	2.4127	.491	2.1978	.541	2.0253	.591	1.8846
.442	2.4079	.492	2.1939	.542	2.0222	.592	1.8821
.443	2.4031	.493	2.1901	.543	2.0192	.593	1.8795
.444	2.3983	.494	2.1863	.544	2.0161	.594	1.8770
.445	2.3936	.495	2.1826	.545	2.0130	.595	1.8745
.446	2.3889	.496	2.1788	.546	2.0100	.596	1.8720
.447	2.3842	.497	2.1751	.547	2.0070	.597	1.8695
.448	2.3795	.498	2.1714	.548	2.0039	.598	1.8670
.449	2.3749	.499	2.1676	.549	2.0009	.599	1.8645

Table E. Hyperbolic Cotangents—Continued

<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>	<i>u</i>	ctgh <i>u</i>
0.600	1.8620	0.650	1.7493	0.700	1.6546	0.750	1.5744
.601	1.8596	.651	1.7472	.701	1.6529	.751	1.5730
.602	1.8571	.652	1.7452	.702	1.6512	.752	1.5715
.603	1.8547	.653	1.7431	.703	1.6494	.753	1.5700
.604	1.8522	.654	1.7411	.704	1.6477	.754	1.5686
.605	1.8498	.655	1.7391	.705	1.6460	.755	1.5671
.606	1.8474	.656	1.7370	.706	1.6443	.756	1.5656
.607	1.8450	.657	1.7350	.707	1.6426	.757	1.5642
.608	1.8426	.658	1.7330	.708	1.6409	.758	1.5628
.609	1.8402	.659	1.7310	.709	1.6392	.759	1.5613
0.610	1.8378	0.660	1.7290	0.710	1.6375	0.760	1.5599
.611	1.8354	.661	1.7270	.711	1.6358	.761	1.5584
.612	1.8331	.662	1.7251	.712	1.6342	.762	1.5570
.613	1.8307	.663	1.7231	.713	1.6325	.763	1.5556
.614	1.8284	.664	1.7211	.714	1.6308	.764	1.5542
.615	1.8260	.665	1.7192	.715	1.6292	.765	1.5528
.616	1.8237	.666	1.7172	.716	1.6275	.766	1.5514
.617	1.8214	.667	1.7153	.717	1.6259	.767	1.5500
.618	1.8191	.668	1.7133	.718	1.6242	.768	1.5486
.619	1.8168	.669	1.7114	.719	1.6226	.769	1.5472
0.620	1.8145	0.670	1.7095	0.720	1.6210	0.770	1.5458
.621	1.8122	.671	1.7075	.721	1.6194	.771	1.5444
.622	1.8099	.672	1.7056	.722	1.6177	.772	1.5430
.623	1.8076	.673	1.7037	.723	1.6161	.773	1.5416
.624	1.8054	.674	1.7018	.724	1.6145	.774	1.5402
.625	1.8031	.675	1.6999	.725	1.6129	.775	1.5389
.626	1.8009	.676	1.6980	.726	1.6113	.776	1.5375
.627	1.7986	.677	1.6962	.727	1.6097	.777	1.5361
.628	1.7964	.678	1.6943	.728	1.6081	.778	1.5348
.629	1.7942	.679	1.6924	.729	1.6065	.779	1.5334
0.630	1.7919	0.680	1.6906	0.730	1.6050	0.780	1.5321
.631	1.7897	.681	1.6887	.731	1.6034	.781	1.5307
.632	1.7875	.682	1.6869	.732	1.6018	.782	1.5294
.633	1.7853	.683	1.6850	.733	1.6003	.783	1.5281
.634	1.7832	.684	1.6832	.734	1.5987	.784	1.5267
.635	1.7810	.685	1.6813	.735	1.5972	.785	1.5254
.636	1.7788	.686	1.6795	.736	1.5956	.786	1.5241
.637	1.7767	.687	1.6777	.737	1.5941	.787	1.5228
.638	1.7745	.688	1.6759	.738	1.5925	.788	1.5214
.639	1.7724	.689	1.6741	.739	1.5910	.789	1.5201
0.640	1.7702	0.690	1.6723	0.740	1.5895	0.790	1.5188
.641	1.7681	.691	1.6705	.741	1.5879	.791	1.5175
.642	1.7660	.692	1.6687	.742	1.5864	.792	1.5162
.643	1.7639	.693	1.6669	.743	1.5849	.793	1.5149
.644	1.7618	.694	1.6652	.744	1.5834	.794	1.5136
.645	1.7597	.695	1.6634	.745	1.5819	.795	1.5123
.646	1.7576	.696	1.6616	.746	1.5804	.796	1.5110
.647	1.7555	.697	1.6599	.747	1.5789	.797	1.5098
.648	1.7534	.698	1.6581	.748	1.5774	.798	1.5085
.649	1.7513	.699	1.6564	.749	1.5759	.799	1.5072

Table E. Hyperbolic Cotangents—Continued

u	$\operatorname{ctgh} u$	u	$\operatorname{ctgh} u$	u	$\operatorname{ctgh} u$	u	$\operatorname{ctgh} u$
0.80	1.5059	1.20	1.1995	1.60	1.08499	2.0	1.03731
.81	1.4935	.21	1.1952	.61	1.08325	2.1	1.03045
.82	1.4813	.22	1.1910	.62	1.08152	2.2	1.02486
.83	1.4696	.23	1.1868	.63	1.07984	2.3	1.02031
.84	1.4581	.24	1.1828	.64	1.07820	2.4	1.01660
.85	1.4470	.25	1.1789	.65	1.07659	2.5	1.01357
.86	1.4362	.26	1.1750	.66	1.07502	2.6	1.01109
.87	1.4258	.27	1.1712	.67	1.07348	2.7	1.00907
.88	1.4156	.28	1.1676	.68	1.07197	2.8	1.00742
.89	1.4057	.29	1.1640	.69	1.07049	2.9	1.00607
0.90	1.3961	1.30	1.1605	1.70	1.06905	3.0	1.00497
.91	1.3867	.31	1.1570	.71	1.06764	3.1	1.00407
.92	1.3776	.32	1.1537	.72	1.06625	3.2	1.00333
.93	1.3687	.33	1.1504	.73	1.06490	3.3	1.00272
.94	1.3601	.34	1.1472	.74	1.06357	3.4	1.00223
.95	1.3517	.35	1.1441	.75	1.06228	3.5	1.00183
.96	1.3436	.36	1.1410	.76	1.06100	3.6	1.00149
.97	1.3356	.37	1.1381	.77	1.05976	3.7	1.00122
.98	1.3279	.38	1.1351	.78	1.05854	3.8	1.00100
.99	1.3204	.39	1.1323	.79	1.05735	3.9	1.00082
1.00	1.3130	1.40	1.1295	1.80	1.05618	4.0	1.000671
.01	1.3059	.41	1.1268	.81	1.05504	4.1	1.000549
.02	1.2989	.42	1.1241	.82	1.05392	4.2	1.000450
.03	1.2921	.43	1.1215	.83	1.05282	4.3	1.000368
.04	1.2855	.44	1.1189	.84	1.05175	4.4	1.000302
.05	1.2791	.45	1.1165	.85	1.05070	4.5	1.000247
.06	1.2728	.46	1.1140	.86	1.04967	4.6	1.000202
.07	1.2667	.47	1.1116	.87	1.04866	4.7	1.000165
.08	1.2607	.48	1.1093	.88	1.04768	4.8	1.000135
.09	1.2549	.49	1.1070	.89	1.04671	4.9	1.000110
1.10	1.2492	1.50	1.1048	1.90	1.04577		
.11	1.2437	.51	1.1026	.91	1.04484		
.12	1.2383	.52	1.1005	.92	1.04393		
.13	1.2330	.53	1.0984	.93	1.04304		
.14	1.2279	.54	1.0963	.94	1.04217		
.15	1.2229	.55	1.0943	.95	1.04132		
.16	1.2180	.56	1.0924	.96	1.04049		
.17	1.2132	.57	1.0905	.97	1.03967		
.18	1.2085	.58	1.0886	.98	1.03887		
.19	1.2040	.59	1.0868	.99	1.03808		

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